There is no spooky action-at-a-distance in quantum correlations: Resolution of the Einstein-Podolsky-Rosen nonlocality puzzle

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Abstract

The long-standing puzzle of the nonlocal Einstein-Podolsky-Rosen correlations is resolved. The correct quantum mechanical correlations arise for the case of entangled particles when strict locality is assumed for the probability amplitudes instead of locality for probabilities. Locality of amplitudes implies that measurement on one particle does not collapse the companion particle to a definite state.

Sixty five years ago, the most significant paper questioning a fundamental aspect of quantum phenomena was written by Einstein, Podolsky and Rosen (EPR) [1]. They addressed the question whether the wave-function represented a complete description of reality in quantum mechanics, and argued that it didn't. Bohr's reply to this paper [2] was not sufficient to resolve the fundamental issues raised by EPR. Decades later, Bohm rephrased the EPR problem [3] in terms of particles correlated in their spin and this helped enormously in analyzing the problem with clarity. John Bell analyzed the EPR problem in the early sixties and established the Bell's inequalities obeyed by any local hidden variable theory for the correlations of entangled particles [4]. Quantum mechanical correlations calculated using the entangled wave-function and spin operators violate these inequalities. Experiments, the first of which was by Freedman and Clauser [5] and the most remarkable by A. Aspect and collaborators [6], have established beyond doubt that there cannot be a viable local realistic hidden variable description of quantum mechanics [7]. Further, these results also have been interpreted as evidence for nonlocal influences in quantum measurements involving entangled particles. Since no instruction set carried by the particles from their source of origin (possibly with the addition of several local hidden variables) can manage to create the correct correlations observed in experiments, the only way out seems to be that measurement of an observable on one of the particles in an entangled pair seems to convey the result of this measurement instantaneously to the other particle resulting in the correct behaviour of the other particle during a measurement on the second particle. Of course, the no signalling theorems in this context prohibit any faster than light signalling using this feature. Nevertheless, we seem to be stuck with the puzzling nonlocality which is probably the deepest mystery in the behaviour of entangled systems. In the quantum mechanical terminology, the measurement of an observable on one of the particles collapses the entire wave-function instantaneously and nonlocally and the second particle acquires a definite value for the same observable, consistent with the correlation determined by the relevant conservation law.

Apart from the disturbing aspect of accepting the concept of nonlocality without being able to understand its nature, there is serious conflict with the spirit of relativity. As soon as we bring in the concept of one measurement being influenced nonlocally by the other, the notion of simultaneity becomes important since both measurements can be labelled by local times. So, if one measurement precede the other in one frame, one can always find a moving frame in which the converse it true, the second measurement preceding the first [8].

In this paper we discuss the resolution of the quantum nonlocality puzzle. The crucial new idea is to assume locality at the level of probability amplitudes instead of at the level of probabilities. For quantum systems which show wave-like behaviour represented by complex numbers, this seems to be the physically correct assumption to make. The quantum correlation is encoded in the difference of an internal variable for the problem.

Consider the breaking up of a correlated state as in the standard Bohm version of the EPR problem [3]. The two particle go off in opposite directions and are in space-like regions. Two observers make measurements on these particles individually at space like separated regions with time stamps such that these results can be correlated later through a classical channel. We assume that strict locality is valid at the level of probability amplitudes. A measurement changes probability amplitudes only locally. *Measurements performed in one region do not change the magnitude or phase of the complex amplitude for the companion particle in a space-like separated region.*

We assign local rules (probability amplitudes) for the outcome of a particular measurement on each of the two particles. We also assume the existence of an internal variable for each of these two particles. The correlation at source is encoded in the relative value, or the difference, of this internal variable for the two particles. For simplicity let us call this internal variable a “phase”, ϕ. Note that it is not a dynamical phase evolving as the particle propagates. It is an internal variable whose difference (possibly zero) remains constant for the particles of the correlated pair. The value of ϕ can vary from particle to particle, but the relative phase between the two particles in all correlated pairs is constant. Consider ϕ as a reference for the particles to determine the angle of a polarizer or analyzer encountered on their way, *locally* (we use the terms polarizer and analyzer in a generic way. They could be Stern-Gerlach like analyzers for spin 1/2 particles). The first particle encounters analyzer #1 kept at an angle θ_1 with respect to some global direction. We denote this angle of the analyzer with reference to ϕ as θ. Similarly, the second particle which has the internal phase angle $\phi + \phi_o$, where ϕ_o is a constant, encounters the second analyzer oriented at angle θ_2 at another space-like separated point. Let the orientation of this analyzer with respect to the internal phase angle of the second particle is θ'. We have $\theta - \theta' = \theta_1 - \theta_2 + \phi_o$. (The constant ϕ_o characterizes the correlation.)

An experiment in which each particle is analyzed by orienting the analyzers at various angles θ_1 and θ_2 is considered next. At each location the result is two-valued denoted by $(+1)$ for transmission and (-1) for absorption of each particle, for any angle of orientation. The classical correlation function $P(\mathbf{a}, \mathbf{b}) = \frac{1}{N}\sum(A_i B_i)$ satisfies $-1 \leq P(\mathbf{a}, \mathbf{b}) \leq 1$. Here $(\mathbf{a}, \mathbf{b})$ denotes the two directions along which the analyzers are oriented and A_i and B_i are the two valued results. The Bell correlation $P(\mathbf{a}, \mathbf{b})$ denotes the average of the quantity (*number of detections in coincidence* $-$ *number of detections in anticoincidence*), where ‘coincidence’ denotes both particles showing same value for the measurement and 'anticoincidence' denotes those with opposite values. Our aim is to calculate the Bell correlation from our formalism employing local amplitudes and compare it with the quantum mechanical prediction obtained from the nonlocal entangled wave function and spin operators.

We specify the *local rule for transmission as a complex number*, *whose square gives the probability of transmission*. The complex amplitude associated with particle #1 is $C_1 = \frac{1}{\sqrt{2}} \exp(i\theta s)$ for measurements at analyzer #1, and for particle #2 is $C_2 = \frac{1}{\sqrt{2}} \exp(i\theta' s)$ at analyzer #2. (For the maximally entangled particles, the amplitude for the alternate outcome at the analyzer differs only by a phase). In these expressions, the quantity s is the spin (in units of $\hbar$) of the particle, 1 for photons and $\frac{1}{2}$ for spin-$\frac{1}{2}$ particles. The locality assumption is strictly enforced since the two complex functions depend only on local variables and on an internal variable determined at source and then individually carried by the particles without any subsequent communication of any sort. The probabilities for the outcomes of measurements at each end are now correctly reproduced, for any angle of orientation. These probabilities are $Re(C_1 C_1^*) = Re(C_2 C_2^*) = \frac{1}{2}$. The correlation function for amplitudes is of the form $Re(C_1 C_2^*)$. The correlation amplitude for an outcome of either $(++)$ or $(--)$ of two maximally entangled particles is

$$\begin{aligned} U(\theta, \theta') &= Re 2(C_1 C_2^*) = Re\{\exp is(\theta - \theta')\} \\ &= \cos\{s(\theta - \theta')\} = \cos\{s(\theta_1 - \theta_2) + s\phi_o\}. \end{aligned} \tag{1}$$

We rewrite this as $U(\theta_1, \theta_2, \phi_o)$ since all references to the individual values of the hidden variable ϕ has dropped out. The square of $U(\theta_1, \theta_2, \phi_o)$ is the probability for coincidence detection of the two particles through the analyzers kept at angles θ_1 and θ_2. (A distinction is made between what the quantum system uses as a rule for transmission, and what we can measure after the act of transmission. The correlation function is analogous to the two-point amplitude correlations of two independent electromagnetic fields).

Next we calculate the Bell correlation function $P(\mathbf{a}, \mathbf{b})$ from the correlation function $U(\theta_1, \theta_2, \phi_o)$. Since $U^2(\theta_1, \theta_2, \phi_o)$ is the probability for a coincidence detection ($++$ or $--$), the quantity $(1 - U^2(\theta_1, \theta_2, \phi_o))$ is the probability for an anticoincidence (events of the type $+-$ and $-+$). Since the average of the quantity (number of coincidences $-$ number of anticoincidences) $=$

$$U^2(\theta_1, \theta_2, \phi_o) - (1 - U^2(\theta_1, \theta_2, \phi_o)) = 2U^2(\theta_1, \theta_2, \phi_o) - 1, \tag{2}$$

the correspondence between $P(\mathbf{a}, \mathbf{b})$ and $U(\theta_1, \theta_2, \phi_o)$ is given by the general expression,

$$P(\mathbf{a}, \mathbf{b}) = 2U^2(\theta_1, \theta_2, \phi_o) - 1 \tag{3}$$

Let us consider for discussion, the case of a correlated state of photons breaking up into *orthogonal polarization states.* This means that if one photon is transmitted through an analyzer on one side, the other one will not transmitted for the same orientation of the analyzer on the other side. So, perfect anti-correlation is implied for $\theta_1 - \theta_2 = 0$. The Bell correlation calculated from quantum mechanics for this case is given by $-\cos(2(\ (\theta_1 - \theta_2))$. That is, if the analyzers are oriented at a relative angle of $\pi/2$, perfect correlation is obtained. When the relative angle is $\pi/4$, the quantum mechanical correlation defined in the Bell way is zero, since there are as many coincidences as anticoincidences.

The correlation function we derived give, for the case of the photons discussed above,

$$U(\theta_1, \theta_2, \phi_o) = \cos\{(\theta_1 - \theta_2) + \phi_o\} \tag{4}$$

We set $\phi_o = \pi/2$ for denoting the correlation of the two orthogonal photons at source . Then we get

$$\begin{aligned} U(\theta_1, \theta_2, \phi_o) &= \cos\{(\theta_1 - \theta_2) + \pi/2\} \\ &= -\sin(\theta_1 - \theta_2) \end{aligned} \tag{5}$$

The probability for coincidence detection is

$$U^2(\theta_1, \theta_2, \phi_o) = \sin^2(\theta_1 - \theta_2) \tag{6}$$

Correspondingly, the probability for anticoincidence is $1 - \sin^2(\theta_1 - \theta_2)$. We get for the Bell correlation,

$$P(\mathbf{a}, \mathbf{b}) = 2\sin^2(\theta_1 - \theta_2) - 1 = -\cos(2((\theta_1 - \theta_2)) \tag{7}$$

This agrees completely with the usual quantum mechanical prediction derived by applying the relevant spin operators on the correct entangled state of the two photons.

Another important example is the case of the singlet state breaking up into two spin 1/2 particles propagating in opposite directions to spatially separated regions. We set $\phi_o = \pi$. Then our correlation function is

$$\begin{aligned} U(\theta_1, \theta_2, \phi_o) &= \cos\{s(\theta_1 - \theta_2) + s\phi_o\} \\ &= \cos\{\frac{1}{2}(\theta_1 - \theta_2) + \pi/2\} \\ &= -\sin\frac{1}{2}(\theta_1 - \theta_2) \end{aligned} \tag{8}$$

The probability for joint detection through two Stern-Gerlach analyzers oriented at relative angle $\theta_1 - \theta_2$ is

$$U^2(\theta_1, \theta_2, \phi_o) = \sin^2(\frac{1}{2}(\theta_1 - \theta_2)) \tag{9}$$

For the case of the two particles of the singlet state,

$$\begin{aligned} 2U^2(\theta_1, \theta_2, \phi_o) - 1 &= 2\sin^2(\frac{1}{2}(\theta_1 - \theta_2)) - 1 \\ &= -\cos(\theta_1 - \theta_2) = -\mathbf{a} \cdot \mathbf{b} \end{aligned} \tag{10}$$

This is again exactly same as the correct Bell correlation $P(\mathbf{a}, \mathbf{b})$ for the quantum mechanical predictions obtained from the singlet entangled wavefunction and the Pauli spin operators. Perfect correlation is obtained for oppositely oriented analyzers and perfect anticorrelation for similarly oriented analyzers. When the analyzers are orthogonal, the correlation is zero.

We have correctly reproduced the quantum mechanical correlation using local probability amplitudes. Bell's theorem prohibiting local realistic theories is not violated since we used the concept of locality for probability amplitudes instead of locality at the level of probabilities. The correct correlation emerges from combining two local complex functions. Single events consisting of two independent measurements at the two analyzers obey the correlation we derived, and the probability for joint detection is given by the square of the correlation function. *In particular if the two analyzers happen to be in the same orientation, perfect correlation is reproduced every time within the strict locality assumption.* It is important to note that we have not used any information on the internal variable ϕ even in terms of distributions. It may be considered as a hidden variable appearing in the measurement prescriptions only through a complex number and has the nature of the origin of a non-dynamical phase associated with the quantum system. In fact, such a variable is not an external input additional to what is already available in the quantum mechanical description, since the zero of the phase of a wave-function is unobservable.

All probabilities are guaranteed to be positive definite in our formalism since the correlation function is real. The nonlocality puzzle in the EPR correlations is resolved. Strict locality including Einstein locality is valid. An answer to the EPR query regarding the completeness of quantum description is found. *It seems clear that even after performing a measurement on one of the particles of an entangled pair, the companion particle cannot be ascribed a reality in the sense of Einstein.* The companion particle's quantum properties remain as unmeasured and as 'un-collapsed' as ever, though the result of a measurement if performed, in the same direction, can be predicted with absolute certainty. Wave-function collapse in the sense of Copenhagen interpretation and realization of an outcome happens only during actually performed measurements and not as a consequence of a measurement on a subsystem of an entangled system. (I will argue in another paper that the results of the Popper's experiment [9, 10, 11] support this view).

The solution presented here resolves the problem, pointed out by EPR, of simultaneous reality of noncommuting observables. In fact the solution denies any reality to an actually unmeasured system. This suggests that there are physical systems in nature that are beyond the scope of the intuitive definition of EPR reality, just as the Copenhagen school maintained. The approach we have taken here gives predictions for correlations which are exactly the same as that would be obtained from the quantum wave-function and operators, without the apparent nonlocal influence of one measurement on the other. The nonlocality apparent in entanglement correlation in quantum mechanics is not an inherent feature, but a conclusion forced on us when using a restrictive definition of physical reality.

The same analysis works for particles entangled in other sets of variables like momentum and coordinate, and energy and time. The results follow from the fact that all these cases of two particle entanglement can be mapped on to the spin-$\frac{1}{2}$ singlet problem with two-valued outcomes. An experiment in which the particles entangled in momentum and position are used, with double slits for each of the particles, the amplitudes are

$$C_1 = \frac{1}{\sqrt{2}} \exp(i\alpha k(x_1 - x_o)/2),$$

$$C_1 \;=\; \frac{1}{\sqrt{2}}\exp(i\alpha k(x_2 - x_o)/2) \tag{11}$$

where x_1 and x_2 are the coordinates of the two detectors separated by a space-like interval. k is the wave vector and α is a scaling factor for the angle subtended by the two slits at the detectors, source etc. The factor 2 dividing the angular variable comes from the mapping with the spin-$\frac{1}{2}$ problem. The single particle data on either side separately do not show any interference. The correlation function is

$$U(x_1, x_2) = \cos(\alpha k(x_1 - x_2)/2) \tag{12}$$

Probability for coincidence detection is

$$P(x_1, x_2) = \cos^2(\alpha k(x_1, x_2)/2) = \frac{1}{2}(1 + \cos k\alpha(x_1 - x_2)) \tag{13}$$

This is the two photon correlation pattern with 100% visibility, derived assuming locality of probability amplitudes. This agrees with the quantum mechanical prediction from the relevant two-particle wave function.

We have also constructed local amplitudes for the Hardy experiment [12] in which quantum mechanics predicts three particular zero joint probabilities are one nonzero joint probability (the other possible joint probabilities in the problem can be nonzero and are not relevant for the demonstration of nonlocality). Local complex amplitudes that reproduce the four relevant joint probabilities can be constructed easily. It is impossible to achieve this if locality at the level of probabilities are assumed, as in a local realistic theory.

Quantum entanglement swapping [13] is understood within this frame work by noting that Bell state measurements choose subensembles of particle pairs that show a particular joint outcome. Particles entangled independently with the pair of particles that are subjected to the Bell state measurement will show a joint outcome consistent with swapped entanglement due to the correlation encoded in the internal variable. *But the Bell state measurement does not collapse the distant particle into a definite state.* Yet all correlations are correctly reproduced. This has important implication to the interpretation of quantum teleportation.

In summary, the long standing puzzle of nonlocality in the EPR correlations is resolved. There is no nonlocal influence between correlated particles separated into space-like regions. The solution has new physical and philosophical implications regarding the nature of reality, measurement and state reduction in quantum systems. Our approach shows that the EPR paradox of simultaneous reality for noncommuting physical variables arise from their restrictive definition of physical reality.

By restoring locality into the quantum measurements of entangled system and removing the undesirable ‘spooky action-at-a-distance’, one of Einstein’s deepest wishes is realized. But his desire for a tangible concept of reality of unmeasured quantum systems does not look tenable.

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