New 3-mode squeezing operator and squeezed vacuum state in 3-wave mixing *

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Abstract

In a 3-wave mixing process occurring in some nonlinear optical medium when $a_1^\dagger$mode interacts with both $a_2^\dagger$mode and $a_3^\dagger$mode, we theoretically study the squeezing effect generated by the operator $S_3 \equiv \exp[\mu(a_1a_2 - a_1^\dagger a_2^\dagger) + \nu(a_1a_3 - a_1^\dagger a_3^\dagger)]$. The new 3-mode squeezed vacuum state in Fock space is derived, and the uncertainty relation for it is demonstrated, It turns out that S_3may exhibit enhanced squeezing. By virtue of the technique of integration within an ordered product (IWOP) of operators, we also derive S_3's normally ordered expansion. The Wigner function of new 3-mode squeezed vacuum state is calculated by using the Weyl ordering invariance under similar transformations.

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1 Introduction

Nowadays quantum entanglement is the focus of quantum information research and attracts many interests due to its wide applications in quantum communication [1, 2]. Entangled states have brought much attention and interests of physicists [3, 4]. The usual two-mode squeezed state, generated from a parametric amplifier [5], not only exhibits squeezing, but also quantum entanglement between the idle-mode and the signal-mode in frequency domain. Therefore, it is simultaneously a typical entangled state of continuous variable. Theoretically, the two-mode squeezed state is constructed by acting a two-mode squeezing operator $S_2 = \exp[\lambda(a_1a_2 - a_1^\dagger a_2^\dagger)]$ [6, 7] on the two-mode vacuum state $|00\rangle$, i.e. $S_2|00\rangle = \text{sech}\lambda \exp[-a_1^\dagger a_2^\dagger \tanh\lambda]|00\rangle$, where λ is a squeezing parameter, and $a_i(a_j^\dagger)$ Bose annihilation (creation) operator satisfying $[a_i, a_j^\dagger] = \delta_{ij}$. Using the relation between Bose operators $(a_i, a_i^\dagger)$ and the coordinate and momentum operators

$$Q_i = \frac{a_i + a_i^\dagger}{\sqrt{2}},\ P_i = \frac{a_i - a_i^\dagger}{\sqrt{2}\mathtt{i}}, \tag{1}$$

one can recast S_2 into the form

$$S_2 = \exp\left[\mathtt{i}\lambda\left(Q_1P_2 + Q_2P_1\right)\right], \tag{2}$$

noting

$$\begin{aligned}[][Q_1P_2, Q_2P_1] &= \mathtt{i}\left(Q_2P_2 - Q_1P_1\right),\\ [Q_1P_2, \mathtt{i}\left(Q_2P_2 - Q_1P_1\right)] &= 2Q_1P_2,\\ [Q_2P_1, \mathtt{i}\left(Q_2P_2 - Q_1P_1\right)] &= -2Q_2P_1,\end{aligned} \tag{3}$$

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thus there involves a $SU(1,1)$ algebraic stricture. In the state $S_2|00\rangle$, the variances of the two-mode quadrature operators of light field,

$$\mathfrak{X}=\frac{Q_1+Q_2}{2},\ \mathfrak{P}=\frac{P_1+P_2}{2}, \tag{4}$$

satisfying the commutation relation $[\mathfrak{X},\mathfrak{P}]=\frac{\mathtt{i}}{2}$, exhibiting the standard squeezing, i.e.,

$$\langle 00|\, S_2^{\dagger}\mathfrak{X}^2 S_2\,|00\rangle=\frac{1}{4}e^{-2\lambda},\langle 00|\, S_2^{\dagger}\mathfrak{P}^2 S_2\,|00\rangle=\frac{1}{4}e^{2\lambda}, \tag{5}$$

which satisfy $(\Delta\mathfrak{X})(\Delta\mathfrak{P})=\frac{1}{4}$.

An interesting question naturally arises: if $a_1^{\dagger}$ mode in a nonlinear optical medium, interacting with both $a_2^{\dagger}$ mode and $a_3^{\dagger}$ mode (e.g., a three-wave mixing), and the corresponding three-mode exponential operator is introduced as

$$S_3\equiv\exp[\mu(a_1a_2-a_1^{\dagger}a_2^{\dagger})+\nu(a_1a_3-a_1^{\dagger}a_3^{\dagger})]. \tag{6}$$

Using Eq.(1) we can recast S_3 into the form

$$S_3\equiv\exp\left[\mathtt{i}\mu\left(Q_2P_1+Q_1P_2\right)+\mathtt{i}\nu\left(Q_3P_1+Q_1P_3\right)\right], \tag{7}$$

where μ, ν are two different interaction parameters, then what is its squeezing effect for the 3-mode quadratures of light field?

To answer this question we must know what is the state $S_3|000\rangle$ ($|000\rangle$ is the 3-mode vacuum state) in Fock space, for this aim, we should know what is the normally ordered expansion of S_3. But how to disentangle the exponential operator S_3? Because there is no simple $SU(1,1)$ algebraic structure among Q_2P_1, Q_1P_2, Q_3P_1 and Q_1P_3, the disentangling seems hard. Thus we turn to appeal to Dirac's coordinate representation and the technique of integration within an ordered product (IWOP) of operators [8, 9, 10, 11] to solve this problem. Our work is arranged as follows: firstly we derive the explicit form of $S_3|000\rangle$, then we demonstrate that it really satisfies the Heisenberg uncertainty relation and may exhibit squeezing enhancement. We also employ the technique of integration within an ordered product (IWOP) of operators to derive the normally ordered expansion of S_3. The Wigner function of $S_3|000\rangle$ is calculated by using the Weyl ordering invariance under similar transformations [12, 13, 14].

2 New 3-mode squeezed vacuum state

For the sake of convenience, we rewrite S_3 in Eq.(7) as the following compact form,

$$S_3=\exp[\mathtt{i}Q_i\Lambda_{ij}P_j], i,j=1,2,3, \tag{8}$$

where the repeated indices imply the Einstein summation notation, and

$$\Lambda=\begin{pmatrix}0&\mu&\nu\\ \mu&0&0\\ \nu&0&0\end{pmatrix}, \tag{9}$$

thus

$$e^{\Lambda}=\begin{pmatrix}\cosh r&\cos\theta\sinh r&\sin\theta\sinh r\\ \cos\theta\sinh r&\sin^2\theta+\cos^2\theta\cosh r&\frac{\sin 2\theta}{2}(\cosh r-1)\\ \sin\theta\sinh r&\frac{\sin 2\theta}{2}(\cosh r-1)&\sin^2\theta\cosh r+\cos^2\theta\end{pmatrix}, \tag{10}$$

its inverse is

$$e^{-\Lambda}=\begin{pmatrix}\cosh r&-\cos\theta\sinh r&-\sin\theta\sinh r\\ -\cos\theta\sinh r&\sin^2\theta+\cos^2\theta\cosh r&\frac{\sin 2\theta(\cosh r-1)}{2}\\ -\sin\theta\sinh r&\frac{\sin 2\theta(\cosh r-1)}{2}&\sin^2\theta\cosh r+\cos^2\theta\end{pmatrix}, \tag{11}$$

where we have set

$$r=\sqrt{\mu^2+\nu^2}, \cos\theta=\frac{\mu}{r}, \sin\theta=\frac{\nu}{r}, \tag{12}$$

noting that Λ is a symmetric matrix. Using the Baker-Hausdorff formula,

$$\begin{aligned} e^{A}Be^{-A} &= B+[A,B]+\frac{1}{2!}[A,[A,B]] \\ &+\frac{1}{3!}[A,[A,[A,B]]]+\cdots, \end{aligned} \tag{13}$$

we see that S_3 causes the following transformation

$$S_3^{-1}Q_kS_3=(e^{-\Lambda})_{ki}Q_i,\ S_3^{-1}P_kS_3=(e^{\Lambda})_{ki}P_i. \tag{14}$$

It then follows that $S_3^{-1}a_kS_3=(e^{-\lambda\Lambda})_{ki}a_i$, i.e.

$$\begin{aligned} S_3^{-1}a_1S_3 &= a_1\cosh r-a_2^{\dagger}\cos\theta\sinh r-a_3^{\dagger}\sin\theta\sinh r, \\ S_3^{-1}a_2S_3 &= -a_1^{\dagger}\cos\theta\sinh r+a_2\left(\sin^2\theta+\cos^2\theta\cosh r\right) \\ &+\frac{1}{2}a_3\left(\cosh r-1\right)\sin 2\theta, \\ S_3^{-1}a_3S_3 &= -a_1^{\dagger}\sin\theta\sinh r+\frac{1}{2}a_2\left(\cosh r-1\right)\sin 2\theta \\ &+a_3\left(\sin^2\theta\cosh r+\cos^2\theta\right). \end{aligned} \tag{15}$$

Noticing that $S_3^{\dagger}=S_3^{-1}$ and $S_3^{\dagger}\left(\mu,\nu\right)=S_3\left(-\mu,-\nu\right)$, from Eq.(15) we also have

$$\begin{aligned} S_3a_1S_3^{-1} &= a_1\cosh r+a_2^{\dagger}\cos\theta\sinh r+a_3^{\dagger}\sin\theta\sinh r, \\ S_3a_2S_3^{-1} &= a_1^{\dagger}\cos\theta\sinh r+a_2\left(\sin^2\theta+\cos^2\theta\cosh r\right) \\ &+\frac{1}{2}a_3\left(\cosh r-1\right)\sin 2\theta \\ S_3a_3S_3^{-1} &= a_1^{\dagger}\sin\theta\sinh r+\frac{1}{2}a_2\left(\cosh r-1\right)\sin 2\theta \\ &+a_3\left(\sin^2\theta\cosh r+\cos^2\theta\right). \end{aligned} \tag{16}$$

For convenience to write, we set $S_3\left|000\right\rangle=\left\|000\right\rangle$. In order to obtain the explicit form of $\left\|000\right\rangle$, using Eq.(15) and $a_1\left|000\right\rangle=0$, we operate a_1 on $\left\|000\right\rangle$ and obtain

$$\begin{aligned} a_1\left\|000\right\rangle &= S_3S_3^{-1}a_1S_3\left|000\right\rangle \\ &= S_3(a_1\cosh r-a_2^{\dagger}\cos\theta\sinh r-a_3^{\dagger}\sin\theta\sinh r)\left|000\right\rangle \\ &= S_3(-a_2^{\dagger}\cos\theta\sinh r-a_3^{\dagger}\sin\theta\sinh r)S_3^{-1}S_3\left|000\right\rangle \\ &= -S_3(a_2^{\dagger}\cos\theta\sinh r+a_3^{\dagger}\sin\theta\sinh r)S_3^{-1}\left\|000\right\rangle, \end{aligned} \tag{17}$$

then we continue to use Eq.(16) to derive

$$\begin{aligned} a_1\left\|000\right\rangle &= -\{[a_1\cos\theta\sinh r+a_2^{\dagger}\left(\sin^2\theta+\cos^2\theta\cosh r\right) \\ &+\frac{1}{2}a_3^{\dagger}\left(\cosh r-1\right)\sin 2\theta]\cos\theta\sinh r \\ &+[a_1\sin\theta\sinh r+\frac{1}{2}a_2^{\dagger}\left(\cosh r-1\right)\sin 2\theta \\ &+a_3^{\dagger}\left(\sin^2\theta\cosh r+\cos^2\theta\right)]\sin\theta\sinh r\}\left\|000\right\rangle \\ &= -(a_1\sinh^2 r+a_2^{\dagger}\cos\theta\cosh r\sinh r \\ &+\frac{1}{2}a_3^{\dagger}\sin\theta\sinh 2r)\left\|000\right\rangle, \end{aligned} \tag{18}$$

so we reach the equation

$$a_1 \left\|000\right\rangle = -\tanh r(a_2^\dagger \cos\theta + a_3^\dagger \sin\theta) \left\|000\right\rangle . \tag{19}$$

Similarly, operating a_2 on $\left\|000\right\rangle$ and using Eqs.(15) and (16) yields

$$\begin{aligned} a_2 \left\|000\right\rangle &= S_3 S_3^{-1} a_2 S_3 \left|000\right\rangle = S_3(-a_1^\dagger \cos\theta \sinh r) \left|000\right\rangle \\ &= S_3(-a_1^\dagger \cos\theta \sinh r) S_3^{-1} \left\|000\right\rangle \\ &= -(a_1^\dagger \cosh r + a_2 \cos\theta \sinh r \\ &+ a_3 \sin\theta \sinh r) \cos\theta \sinh r \left\|000\right\rangle , \end{aligned} \tag{20}$$

which leads to

$$\begin{aligned} &[a_2 \left(1 + \cos^2\theta \sinh^2 r\right) + \frac{1}{2} a_3 \sin 2\theta \sinh^2 r] \left\|000\right\rangle \\ &= -\frac{1}{2} a_1^\dagger \cos\theta \sinh 2r \left\|000\right\rangle . \end{aligned} \tag{21}$$

On the other hand, operating a_3 on $\left\|000\right\rangle$ and using Eqs.(15) and (16) yields

$$\begin{aligned} a_3 \left\|000\right\rangle &= S_3 S_3^{-1} a_3 S_3 \left|000\right\rangle = S_3(-a_1^\dagger \sin\theta \sinh r) \left|000\right\rangle \\ &= S_3(-a_1^\dagger \sin\theta \sinh r) S_3^{-1} \left\|000\right\rangle \\ &= -(a_1^\dagger \cosh r + a_2 \cos\theta \sinh r \\ &+ a_3 \sin\theta \sinh r) \sin\theta \sinh r \left\|000\right\rangle , \end{aligned} \tag{22}$$

i.e.,

$$\begin{aligned} &[a_3 \left(1 + \sin^2\theta \sinh^2 r\right) + \frac{1}{2} a_2 \sin 2\theta \sinh^2 r] \left\|000\right\rangle \\ &= -\frac{1}{2} a_1^\dagger \sin\theta \sinh 2r \left\|000\right\rangle . \end{aligned} \tag{23}$$

Combining Eqs.(21) and (23) we have

$$a_2 \left\|000\right\rangle = -a_1^\dagger \tanh r \cos\theta \left\|000\right\rangle , \tag{24}$$

and

$$a_3 \left\|000\right\rangle = -a_1^\dagger \tanh r \sin\theta \left\|000\right\rangle . \tag{25}$$

From Eqs.(19),(24) and (25), we may predict that $\left\|000\right\rangle$ has the following explicit form:

$$\left\|000\right\rangle = N \exp[-(a_2^\dagger \cos\theta + a_3^\dagger \sin\theta) a_1^\dagger \tanh r] \left|000\right\rangle , \tag{26}$$

where N is the normalization constant, which can be determined by $\left\langle 000 \right\|000\rangle = 1$, and we calculate $N = \mathrm{sech} r$.

3 Squeezing property and quantum fluctuation in $\left\|000\right\rangle$

Squeezing is an important phenomenon in quantum theory and has many applications in various areas in quantum optics and quantum information [15]. In this section, we examine the quadrature squeezing effects of $\left\|000\right\rangle$. The quadratures in the 3-mode case are defined as

$$X_1 = \frac{1}{\sqrt{6}} \sum_{i=1}^{3} Q_i, \; X_2 = \frac{1}{\sqrt{6}} \sum_{i=1}^{3} P_i, \tag{27}$$

which satisfy the relation $[X_1, X_2] = \frac{\mathrm{i}}{2}$. Their variances are $(\Delta X_i)^2 = \langle X_i^2 \rangle - \langle X_i \rangle^2$, $i = 1, 2$. Noting the expectation values of X_1 and X_2 in the state $\|000\rangle$ is $\langle X_1 \rangle = \langle X_2 \rangle = 0$. With the help of Eq.(15), we can calculate that the corresponding variances in the state $\|000\rangle$: (noting Λ is symmetric)

$$\begin{aligned}(\triangle X_1)^2 &= \langle 000| S_3^{-1} X_1^2 S_3 |000\rangle \\ &= \frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}(e^{-\Lambda})_{ki}(e^{-\Lambda})_{jl}\langle 000| Q_k Q_l |000\rangle \\ &= \frac{1}{12}\sum_{i=1}^{3}\sum_{j=1}^{3}(e^{-\Lambda})_{ki}(e^{-\Lambda})_{jl}\langle 000| a_k a_l^\dagger |000\rangle \\ &= \frac{1}{12}\sum_{i=1}^{3}\sum_{j=1}^{3}(e^{-\Lambda})_{ki}(e^{-\Lambda})_{jl}\delta_{kl} \\ &= \frac{1}{12}\sum_{i,j}^{3}(e^{-2\Lambda})_{ij},\end{aligned} \tag{28}$$

and

$$(\triangle X_2)^2 = \langle 000| S_3^{-1} X_2^2 S_3 |000\rangle = \frac{1}{12}\sum_{i,j}^{3}(e^{2\Lambda})_{ij}. \tag{29}$$

The explicit form of the matrices $e^{2\Lambda}$ and $e^{-2\Lambda}$ can be derived from Eq.(10) and (11), so we can obtain

$$\begin{aligned}(\triangle X_1)^2 = \frac{1}{12}[&(2\cosh 2r + 1) + \sin 2\theta(\cosh 2r - 1) \\ &+ 2(\cos\theta + \sin\theta)\sinh 2r],\end{aligned} \tag{30}$$

and

$$\begin{aligned}(\triangle X_2)^2 = \frac{1}{12}[&(2\cosh 2r + 1) + \sin 2\theta(\cosh 2r - 1) \\ &- 2(\cos\theta + \sin\theta)\sinh 2r].\end{aligned} \tag{31}$$

We can successfully verify

$$\begin{aligned}&(\triangle X_1)(\triangle X_2) \\ &= \frac{1}{12}\sqrt{(4\cosh 2r + 4) + \left(1 - 2\sinh^2 r \sin 2\theta\right)^2} \\ &\geqslant \frac{1}{12}\sqrt{(4\cosh 2r + 4) + \left(1 - 2\sinh^2 r\right)^2} \\ &= \frac{1}{12}\sqrt{\frac{1}{2}\cosh 4r + \frac{17}{2}} \geqslant \frac{1}{4},\end{aligned} \tag{32}$$

which confirms the uncertainty relation of quantum mechanics.

To see the trend of squeezing effects in the X_1- or X_2-direction, we plot $(\Delta X_1)^2$ and $(\Delta X_2)^2$ as the function of parameter μ for different ν in Fig.1. When $\nu = 0$, it exhibits the usual two mode squeezing effect depending on the varying μ, $(\Delta X_1)^2$ increases accompanying $(\Delta X_2)^2$ decreases; when $\nu = 0.5$, $(\Delta X_1)^2$ increases more than the case of $\nu = 0$, which exhibits enhanced squeezing in certain domain of μ. In Fig.2, we plot the uncertainty value $(\triangle X_1)(\triangle X_2)$ as the function of r for different θ.

4 Normally ordered form of S_3

We calculate the normally ordered form (denoted by : :) of S_3 by inserting the completeness relation of coherent state

$$S_3 = \int \frac{d^2 z_1 d^2 z_2 d^2 z_3}{\pi^3} S_3 \left| z_1 z_2 z_3 \right\rangle \left\langle z_1 z_2 z_3 \right| . \tag{33}$$

where $|z_1 z_2 z_3\rangle$ is the three-mode coherent state and $|z_i\rangle = \exp[-\frac{|z_i|^2}{2} + z_i a_i^\dagger] |0_i\rangle$, $i = 1, 2, 3$.

Using the relations in Eqs.(15) and (16), we have the explicit relation of $S_3 |z_1 z_2 z_3\rangle$

$$\begin{aligned}
& S_3 \left| z_1 z_2 z_3 \right\rangle \\
& = \exp(-\sum_{i=1}^{3} \frac{|z_i|^2}{2}) S_3 \exp\left(z_1 a_1^\dagger + z_2 a_2^\dagger + z_3 a_3^\dagger \right) S_3^{-1} S_3 \left| 000 \right\rangle \\
& = \frac{1}{\cosh r} \exp(-\sum_{i=1} \frac{|z_i|^2}{2} + z_1 z_2 \cos\theta \tanh r + z_1 z_3 \sin\theta \tanh r) \\
& \times \exp\{ \frac{1}{\cosh r} [a_1^\dagger z_1 + (a_2^\dagger \left(\sin^2\theta \cosh r + \cos^2\theta \right) - \frac{1}{2} a_3^\dagger (\cosh r - 1) \sin 2\theta) z_2 \\
& + (a_3^\dagger \left(\sin^2\theta + \cos^2\theta \cosh r \right) - \frac{1}{2} a_2^\dagger (\cosh r - 1) \sin 2\theta) z_3] \} \\
& \times \exp[-a_1^\dagger \tanh r (a_2^\dagger \cos\theta + a_3^\dagger \sin\theta)] \left| 000 \right\rangle .
\end{aligned} \tag{34}$$

Substituting Eq.(34) into Eq.(33), noticing that $|000\rangle \langle 000| =: \exp(-a_1^\dagger a_1 - a_2^\dagger a_2 - a_3^\dagger a_3):$, and using the following formula

$$\int \frac{d^2 z}{\pi} \exp\left(\zeta |z|^2 + \xi z + \eta z^* \right) = -\frac{1}{\zeta} e^{-\frac{\xi\eta}{\zeta}}, \quad \mathtt{Re}(\zeta) < 0, \tag{35}$$

as well as the IWOP technique, we can obtain the explicit normally ordered expansion of S_3:

$$\begin{aligned}
S_3 = & \frac{1}{\cosh r} \exp[-a_1^\dagger (a_2^\dagger \cos\theta + a_3^\dagger \sin\theta) \tanh r] \\
& \times : \exp[\frac{1 - \cosh r}{\cosh r} (a_1^\dagger a_1 + a_2^\dagger a_2 \cos^2\theta \\
& + a_3^\dagger a_3 \sin^2\theta + \frac{1}{2} a_2 a_3^\dagger \sin 2\theta + \frac{1}{2} a_2^\dagger a_3 \sin 2\theta)] : \\
& \times \exp[a_1 (a_2 \cos\theta + a_1 \sin\theta) \tanh r].
\end{aligned} \tag{36}$$

5 Wigner function of $\|000\rangle$

Wigner distribution function of quantum states [16, 17, 18] is widely studied in quantum statistics and quantum optics and is very important tool for a global description of nonclassical effect in the quantum system, which can be measured by various means such as photon counting experiment and homodyne tomography. Now we derive the Wigner function of $\|000\rangle$ by using a new method.

Recalling that in Ref. [14] we have introduced the Weyl ordering form of single-mode Wigner operator $\Delta_1(q_1, p_1)$,

$$\Delta_1(q_1, p_1) = \vdots\, \delta(q_1 - Q_1)\, \delta(p_1 - P_1) \,\vdots, \tag{37}$$

where the symbols $\vdots\ \vdots$ denote the Weyl ordering, while its normal ordering form is

$$\Delta_1(q_1, p_1) = \frac{1}{\pi} : \exp\left[-(q_1 - Q_1)^2 - (p_1 - P_1)^2 \right] : . \tag{38}$$

Thus the Wigner function for $|0\rangle$ can be easily expressed as $\langle 0|\,\Delta_1\,(q_1,p_1)\,|0\rangle = \frac{1}{\pi}\exp(-q_1^2-p_1^2)$. Note that the order of Bose operators a_1 and $a_1^\dagger$ within a normally ordered product (or a Weyl ordered product) can be permuted. That is to say, even though $[a_1,a_1^\dagger]=1$, we can have $:a_1a_1^\dagger: \;=: a_1^\dagger a_1:$ and $\vdots\, a_1a_1^\dagger\, \vdots = \vdots\, a_1^\dagger a_1\, \vdots$. The Weyl ordering of operators has a remarkable property, i.e., the Weyl-ordering invariance of operators under similar transformations, which means

$$U \,\vdots\, (\circ\circ\circ) \,\vdots\, U^{-1} = \,\vdots\, U\,(\circ\circ\circ)\,U^{-1} \,\vdots\,, \tag{39}$$

as if the "fence" $\vdots\;\vdots$ did not exist when U operates.

For 3-mode case, the Weyl ordering form of the Wigner operator is

$$\Delta_3\,(\mathbf{q},\mathbf{p}) = \,\vdots\,\delta\,(\mathbf{q}-\mathbf{Q})\,\delta\,(\mathbf{p}-\mathbf{P})\,\vdots\,, \tag{40}$$

where $\mathbf{Q}=(Q_1,Q_2,Q_3)^T$, $\mathbf{P}=(P_1,P_2,P_3)^T$, $\mathbf{q}=(q_1,q_2,q_3)^T$ and $\mathbf{p}=(p_1,p_2,p_3)^T$. Then according to the Weyl ordering invariance under similar transformations and using Eq.(14), we have

$$\begin{aligned}
&S_3^{-1}\Delta_3\,(\mathbf{q},\mathbf{p})\,S_3\\
&= S_3^{-1}\,\vdots\,\delta\,(\mathbf{q}-\mathbf{Q})\,\delta\,(\mathbf{p}-\mathbf{P})\,\vdots\,S_3\\
&= \,\vdots\,\delta\left(q_k-(e^{-\Lambda})_{ki}Q_i\right)\delta\left(p_k-(e^{\Lambda})_{ki}P_i\right)\vdots\\
&= \,\vdots\,\delta\left((e^{\Lambda})_{ki}q_i-Q_k\right)\delta\left((e^{-\Lambda})_{ki}p_i-P_k\right)\vdots\\
&= \,\vdots\,\delta\,(\mathbf{q}'-\mathbf{Q})\,\delta\,(\mathbf{p}'-\mathbf{P})\,\vdots\\
&= \Delta_3\,(\mathbf{q}',\mathbf{p}')\,,
\end{aligned} \tag{41}$$

where $q_k'=(e^{\Lambda})_{ki}q_i$, $p_k'=(e^{-\Lambda})_{ki}p_i$. Thus the Wigner function of $\|000\rangle$ is

$$\begin{aligned}
&\langle 000|\,S_3^{-1}\Delta_3\,(\mathbf{q},\mathbf{p})\,S_3\,|000\rangle\\
&= \langle 000|\,\Delta_3\,(\mathbf{q}',\mathbf{p}')\,|000\rangle\\
&= \frac{1}{\pi^3}\exp\left(-\mathbf{q}^Te^{2\Lambda}\mathbf{q}-\mathbf{p}^Te^{-2\Lambda}\mathbf{p}\right),
\end{aligned} \tag{42}$$

where $e^{2\Lambda}$ and $e^{-2\Lambda}$ are given by e^{Λ} in Eq.(10) and $e^{-\Lambda}$ in Eq.(11), respectively.

In summary, we have shown that the operator $S_3 \equiv \exp[\mu(a_1a_2-a_1^\dagger a_2^\dagger)+\nu(a_1a_3-a_1^\dagger a_3^\dagger)]$ is a new 3-mode squeezed operator by calculating the quantum fluctuation for 3-mode quadratures. We have obtained the new 3-mode squeezed vacuum state and derived the normally ordered expansion of S_3. The IWOP technique brings convenience in our derivation.

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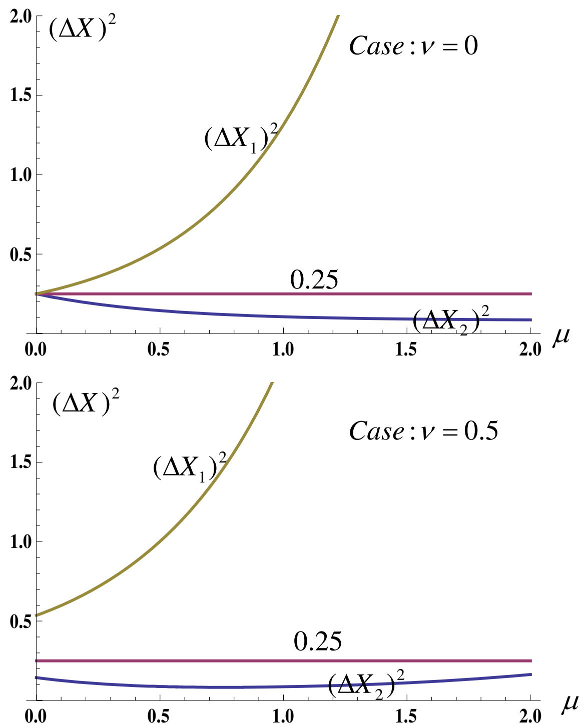

Figure 1: (Colour online) The quantity $(\triangle X_1)^2$ and $(\triangle X_2)^2$ as the function of squeezing parameter μ for different case $\nu = 0$ and $\nu = 0.5$.

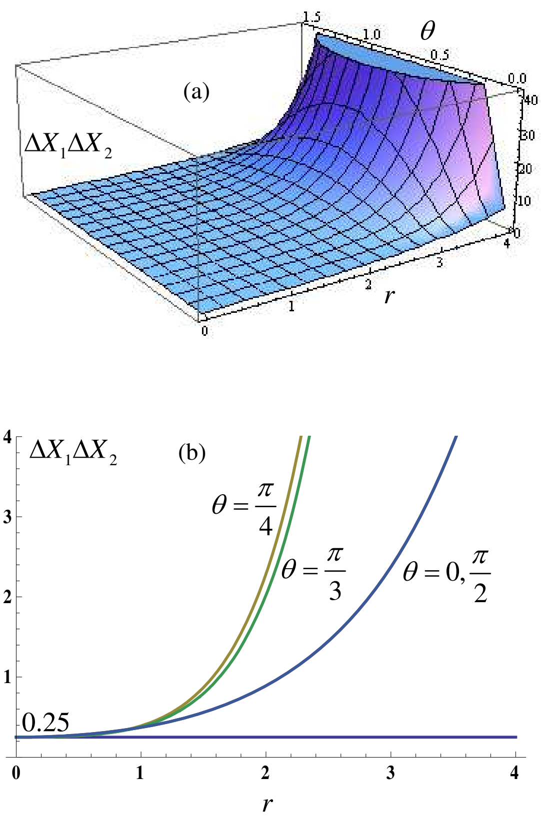

Figure 2: (Colour online) The uncertainty value $(\triangle X_1)(\triangle X_2)$ as the function of r for different θ.