An Experimental Proposal to Test Dynamic Quantum Non-locality with Single-Atom Interferometry

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Quantum non-locality based on the well-known Bell inequality is of kinematic nature. A different type of quantum non-locality, the non-locality of the quantum equation of motion, is recently put forward with connection to the Aharonov-Bohm effect [Nature Phys. 6, 151 (2010)]. Evolution of the displacement operator provides an example to manifest such dynamic quantum non-locality. We propose an experiment using single-atom interferometry to test such dynamic quantum non-locality. We show how to measure evolution of the displacement operator with clod atoms in a spin-dependent optical lattice potential and discuss signature to identify dynamic quantum non-locality under a realistic experimental setting.

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Non-locality dramatically exemplified in the Einstein-Podolsky-Rosen (EPR) paradox [1] is a fundamental concept of quantum mechanics that distinguishes it from classical physics. Quantum non-locality based on the EPR correlation was later formulated into an experimentally testable result known as the Bell inequality [2]. Quantum non-locality based on the EPR correlation and the Bell inequality has been verified in many experiments involving different physical systems [3].

Recently, a different type of quantum non-locality, the non-locality of the quantum equation of motion [4], implied in the famous Aharonov-Bohm effect [5], was put forward by Popescu [6]. Non-locality based on violation of the Bell inequality comes from the Hilbert space structure of quantum mechanics and thus is purely kinematic, while non-locality implied in the Aharonov-Bohm effect is from non-locality of quantum equations of motion in the Heisenberg picture and thus is of dynamic nature [6]. Another significant difference between these two kinds of non-localities is that non-locality from the EPR correlation is assumed to be an exclusive quantum property of two or more well separated but entangled particles, while the dynamic quantum non-locality (DQNL) considered by Aharonov et al. and Popescu can be demonstrated even with evolution of a single particle in a superposition state of two distinct locations. The evolution of the displacement operator provides an explicit example to clearly show the DQNL [6], however, experiment is still lacking in this direction due to the difficulty to measure the displacement operator.

In this paper, we propose a feasible experiment using cold atoms in a spin-dependent optical lattice potential to test the DQNL. We figure out a configuration where the DQNL inherent in the Heisenberg equation leads to a detectable signal qualitatively different from that of the corresponding local (classical) evolution equation, and propose a method to directly measure evolution of the displacement operator in the real experimental system. The required ingredients in this proposed experiment, such as the double well optical lattice and the spin-dependent movement of a particle, have all been realized in previous experiments [7–9], and thus the proposal well fits with the status of the current technology.

Before explaining the proposal, first we briefly recall the concept of DQNL elaborated in Ref. [6]. The Schrodinger equation describing evolution of the wave function of a quantum system is always a local differential equation, however, the wave function by itself is not directly observable. To see the DQNL, one needs to look at the Heisenberg equations which describe evolution of observable physical quantities. The Heisenberg equation for the displacement operator $\hat{D}^Q$ provides an example to explicitly show this kind of DQNL for a single particle [6]. The displacement operator $\hat{D}^Q$ is defined as $\hat{D}^Q \equiv \exp[i\hat{p}L/\hbar]$ with $\hat{p}$ being the momentum operator of the particle. This operator shifts the particle by a finite distance L. For simplicity, we consider a one-dimensional situation where the Hamiltonian of the particle is given by $H = \frac{\hat{p}^2}{2m} + V(x)$ with m being the mass of the particle and $V(x)$ being the potential. For this system, the classical and quantum equations of motion of the displacement operator are quite different [6]. In classical mechanics, we can apply the chain rule for differentiation of a function, and evolution of the quantity $D^C \equiv \exp[ipL/\hbar]$ is given by

$$\frac{dD^C}{dt} = \frac{de^{ipL/\hbar}}{dp}\frac{dp}{dt} = \frac{L}{i\hbar}e^{ipL/\hbar}\frac{dV(x)}{dx}, \qquad (1)$$

which is a local differential equation. However, quantum mechanically, the displacement operator $\hat{D}^Q$ is governed by the Heisenberg equation, which leads to

$$\frac{d\hat{D}^Q}{dt} = \frac{1}{i\hbar}\left[\hat{D}^Q, H\right] = \frac{1}{i\hbar}[V(x+L) - V(x)]\hat{D}^Q, \qquad (2)$$

where we have used $e^{i\hat{p}L/\hbar}V(x) = V(x+L)e^{i\hat{p}L/\hbar}$. This evolution equation is clearly nonlocal as the time derivative of the quantity depends on the potential at two distinct (and possibly remote) locations x and $x+L$.

To demonstrate this kind of DQNL, we need to figure out a configuration where the classical and the quantum evolution equations (1) and (2) for the displacement operator show clear qualitative difference. We also need to find a method to measure the displacement operator in real experimental systems. The evolution operator $\hat{D}^Q$ is non-Hermitian, so it is not directly observable. However, we can look at the real and imaginary parts of $\hat{D}^Q$, and they correspond to observable quantities and still satisfy nonlocal evolution equations in quantum mechanics. To have a configuration that manifests the DQNL represented by Eq. (2), we consider a particle confined in one dimension with a double-well potential, as shown in Fig. 1(a-c). The two potential wells are identical, except that the bottom of one of the wells may be shifted with that of the other well by a constant energy Δ. For classical particles in either of these wells, they see identical force and cannot tell the difference of the wells. The local dynamic equation should be independent of the energy shift Δ. However, for quantum particles in a superposition state, the evolution of the displacement operator can sense this nonlocal constant energy shift Δ. To be explicit, let us assume that the potential $V(x)$ around the two minima $\pm L/2$ can be described by the identical harmonic trap, with $V_1(x) = \left(m\omega^2/2\right)(x+L/2)^2,\ \ V_2(x) = \left(m\omega^2/2\right)(x-L/2)^2 + \Delta$, where ω is the characteristic trap frequency. Let $|\Phi(x)\rangle$ denote an eigenstate of the harmonic trap (for convenience, it can be taken as the ground state). We take the initial state of the particle at time $t=0$ as the following superposition of two localized wave packets,

$$|\Psi(x,0)\rangle = \left[|\Phi(x+L/2)\rangle + |\Phi(x-L/2)\rangle e^{i\theta}\right]/\sqrt{2}, \quad (3)$$

where θ is an arbitrary initial phase difference. The size of the wave packet $|\Phi(x)\rangle$ at each well, estimated by $\sqrt{\hbar/m\omega}$, is assumed to be significantly smaller than L so that the overlap $\int \Phi^*(x+L/2)\Phi(x-L/2)dx \approx 0$. For this state, the quantum Heisenberg equation (2) for the displacement operator directly gives

$$\left\langle \frac{d\hat{D}^Q}{dt} \right\rangle = -i\omega_d \left\langle \hat{D}^Q \right\rangle, \quad (4)$$

where $\omega_d = \Delta/\hbar$. It has the straightforward solution

$$\langle \hat{D}^Q(t) \rangle = \left\langle \hat{D}^Q(0) \right\rangle e^{-i\omega_D t} = (1/2)\, e^{i\theta - i\omega_d t}. \quad (5)$$

So the evolution of the quantum displacement operator is sensitive to the nonlocal constant energy shift Δ. In contrast, for classical particles with the dynamic equation (1), even if they are distributed over the two wells, as long as the distribution function in each well is symmetric with respect to the trap bottom (which is the case for the state shown in Eq. (3)), the average force $\left\langle \frac{dV(x)}{dx} \right\rangle$ is always zero and $\left\langle \frac{dD^C}{dt} \right\rangle = 0$. So, as expected, the classical dynamic equation cannot sense the nonlocal constant energy shift and there is a qualitative difference in the measurement outcomes for the classical and the quantum evolution equations for the displacement operator.

The phenomenon discussed above is closely related to the scalar Aharonov-Bohm effect [10, 11]: in classical physics, the constant energy shift does not lead to any physical difference as long as the force is identical in space. However, for a quantum particle in a superposition state, it can sense the constant energy shift at two remote locations even if the force is strictly zero at any point of the particle's trajectory. This is similar again to the conventional Aharonov-Bohm effect [5], where a quantum charged particle senses a nonzero constant vector potential while the electromagnetic force is zero at any point of the particle's trajectory.

To prepare the particle in a superposition state of a double well potential and to directly measure the displacement operator, we propose to use an optical lattice potential to control cold atoms to fulfill all the requirements. We consider dilute atomic gas in an optical lattice with the average filling number per lattice site much less than 1, so the atomic interaction is negligible and we just have many independent copies of single-particle dynamics. To generate a superposition state over different lattice sites, one may use a double well lattice along the x direction, with the potential $V(x) = -V_1 \sin^2(kx/2+\varphi) - V_2 \sin^2(kx)$ $(V_1, V_2 > 0)$ from two standing wave laser beams with the wave vector $k = 2\pi/\lambda \equiv \pi/L$ [8, 9]. Initially, we set the phase $\varphi = 0$ and $V_2/V_1 = 0$ so that we have only a single lattice V_1 and the atom is in the ground state of this lattice well. By adiabatically tuning up the ratio V_2/V_1 to the region with $V_2 \gg V_1$, each lattice site splits into two as shown in Fig. 1 (a-b), and the atomic state adiabatically follows the ground state configuration and evolves into an equal superposition state of the two wells in the form of Eq. (3) with $\theta = 0$. After we have prepared this initial state, we quickly (within a time scale t_δ) tune φ and V_1 so that $\varphi = \pi/4$ and $V_1 = \Delta/2 \ll V_2$, and look at evolution of the displacement operator under this double well potential with a constant energy shift Δ (Fig. 1c). Under the lattice V_2, the potential well is approximated by a harmonic trap with the trapping frequency $\omega = 2\sqrt{V_2 E_r}/\hbar$, where $E_r = \hbar^2 k^2/2m$ is the atomic recoil energy. We require the time scale t_δ to satisfy the condition $\omega^{-1} \ll t_\delta \ll \hbar/\Delta$ so that on the one hand, we do not generate motional excitations in each well, and on the other hand, t_δ is negligible compared with the evolution time scale (of the order of $\hbar/\Delta$) of the displacement operator.

To demonstrate the nonlocal dynamics of the displacement operator $\hat{D}^Q$ shown in Eq. (5) under this double well lattice, we need to measure $\hat{D}^Q$ after a controllable time delay t. The displacement operator does not correspond to a simple physical quantity, and it is not easy to measure it directly in experiments. To overcome this problem, we make use of the internal (spin) states of the atoms. We show in the following that a Ramsey type of experiment in the internal state, together with a spin-dependent movement of the lattice potential, gives a direct measurement of the expectation value of the displacement operator. The atoms have different hyperfine states, and we use two of them, denoted by effective spin $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively. The atoms are initially assumed to be in the state $|\uparrow\rangle$. To measure the displacement operator after an evolution time t $(t \sim \hbar/\Delta)$, we take the following four steps as illustrated in Fig. 1(d-f): (i) first, we apply a $\pi/2$-pulse within a time much shorter than $\hbar/\Delta$ to the atomic internal state so that the atomic state transfers to $\left[\left(|\uparrow\rangle + |\downarrow\rangle\right)/\sqrt{2}\right]|\psi(t)\rangle$, where $|\psi(t)\rangle$ denotes the atomic motional state at time t. (ii) Second, we apply a spin dependent shift to the lattice potential with the corresponding unitary operation described by $U = |\uparrow\rangle\langle\uparrow|\hat{D}^Q_{L/2} + |\downarrow\rangle\langle\uparrow|\hat{D}^Q_{-L/2}$. This kind of operation has been realized before in experiments to demonstrate controlled atomic collisions [7]. This operation needs to be done in a time scale t_δ which satisfies $\omega^{-1} \ll t_\delta \ll \hbar/\Delta$ so that the shift of the lattice does not generate motional excitations in each well. After this step, the atomic state becomes $\left(|\uparrow\rangle\hat{D}^Q_{L/2} + |\downarrow\rangle\hat{D}^Q_{-L/2}\right)|\psi(t)\rangle/\sqrt{2}$. (iii) After the spin-dependent lattice shift, we apply another $\pi/2$-pulse (within a time negligible compared with $\hbar/\Delta$) to the atomic internal state which transfers the atomic state to $|\psi_f\rangle = \left[\left(|\uparrow\rangle + |\downarrow\rangle\right)\hat{D}^Q_{L/2} + \left(|\downarrow\rangle - |\uparrow\rangle\right)\hat{D}^Q_{-L/2}\right]|\psi(t)\rangle/2$. (iv) Finally, we measure the total atom number $N_\downarrow$ in the spin down state minus the total number $N_\uparrow$ in the spin up state. This number difference is proportional to the probability difference $P_\downarrow - P_\uparrow$ for the state$|\psi_f\rangle$, which is given by

$$P_\downarrow - P_\uparrow = Re\left[\langle\Psi(t)|\hat{D}^Q_L|\Psi(t)\rangle\right]. \quad (6)$$

The imaginary part of the expectation value $\langle\Psi(t)|\hat{D}^Q_L|\Psi(t)\rangle$ can be measured in a similar way. The only difference is that in the step (i) we add a relative phase i to the $\pi/2$-pulse which transfers spin $|\uparrow\rangle$ to the state $\left(|\uparrow\rangle + i|\downarrow\rangle\right)/\sqrt{2}$. The final state is then modified to $|\psi_f\rangle = \left[\left(|\uparrow\rangle + |\downarrow\rangle\right)\hat{D}^Q_{L/2} + i\left(|\downarrow\rangle - |\uparrow\rangle\right)\hat{D}^Q_{-L/2}\right]|\psi(t)\rangle/2$ with the probability difference $P_\downarrow - P_\uparrow = Im\left[\langle\Psi(t)|\hat{D}^Q_L|\Psi(t)\rangle\right]$.

In the above, we have shown how to measure the expectation value of the displacement operator for cold atoms in an optical lattice. The DQNL indicates that the real (imaginary) part of this expectation value oscillates with the evolution time t as $\cos(\omega_d t)$ (-$\sin(\omega_d t)$) as predicted by Eq. (5), which is sensitive to the nonlocal constant energy shift $\Delta = \hbar\omega_d$. This signal distinguishes it from the corresponding classical case where $\langle D^C\rangle$ is independent of Δ and shows no oscillation with time t. For real experiments in an optical lattice, however, there is inevitably a global harmonic trap potential (taking the form of $V_t = m\omega_t^2 x^2/2$ in the x direction) which could complicate the situation [8, 9]. The measured atom number difference $N_\downarrow - N_\uparrow$ involves average of the probability difference $P_\downarrow - P_\uparrow$ over all the independent double-well potentials. Due to the global trap V_t, the probability difference $P_\downarrow - P_\uparrow$ in each double well potential oscillates with slightly different frequencies, and one needs to check whether an average over the whole lattice will wash out the oscillation signal. For this purpose, we simulate in Fig. (2) the averaged signal for a typical experimental configuration. The averaged probability difference is given by

$$\langle P_\downarrow - P_\uparrow\rangle = \frac{1}{2N_l}\sum_j \cos(\Delta_j t/\hbar)e^{-\gamma t}, \quad (7)$$

where $\Delta_j = \Delta + \delta_j$ with $\delta_j \approx m\omega_t^2 L x_j$ ($L = \lambda/2$, and x_j denotes the coordinate of the center of the double wells). The summation of j is over all the occupied double wells (with number N_l) in the global harmonic trap. To better model the experimental situation, we also add a phenomenological decay $e^{-\gamma t}$ to each oscillation term which corresponds to a nonzero dephasing rate γ inevitable in reality. Under typical experimental parameters we have damped oscillations as shown in Fig. 2. The signal is still clearly observable in this case. In the frequency domain, the spectrum centers at the energy shift $\Delta/\hbar$ that is independent of the experimental imperfection discussed above.

Before ending the paper, we briefly discuss the requirements for the relevant experimental parameters. To assure locality, we assume the wave packet overlap between different wells is negligible. This overlap is estimated by e^{-L/l_0}, where $l_0 = \sqrt{\hbar/m\omega}$ is the size of the wave packet in each well and $L = \lambda/2$ is the distance between the wells. For Rb^{87} atoms in an optical lattice with $\lambda = 800\ nm$, $\omega = 2\sqrt{V_2 E_r} \sim 2\pi \times 42$ kHz and $l_0 \sim 52\ nm$ for a typical lattice barrier $V_2 = 35$ E_r, the condition $e^{-L/l_0} \sim e^{-7.6} \ll 1$ is well satisfied. During the state preparation and the detection of the displacement operator, we require the operation time t_δ to satisfy $\omega^{-1} \ll t_\delta \ll \hbar/\Delta$. If we take $\Delta \sim 0.3E_r \sim 0.025\omega$ and

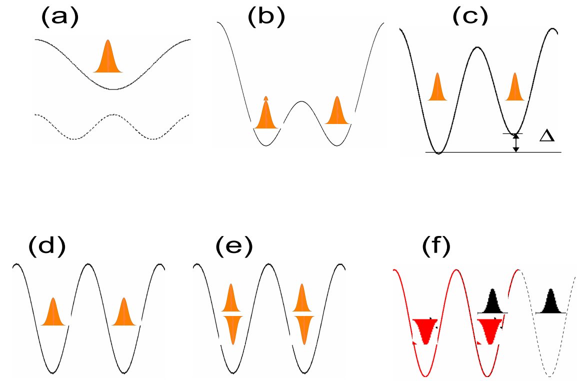

FIG. 1: (Color online) Illustration of the experimental steps for initial state preparation and for detection of the displacement operator. Figs. (a)-(c) show the steps to prepare a non-local superposition state (given by Eq. (3)) in a double-well lattice. After adiabatic preparation of this superposition state, a bias potential Δ is tuned on within a time scale t_δ (specified in the text) to start evolution of the displacement operator. Figs. (d-f) show the steps to measure the displacement operator after a certain evolution time t. Right before the measurement, the bias potential Δ is turned off, so we have a regular optical lattice (d). The atom is then transfer to an equal superposition of two spin components with a $\pi/2$-pulse (e). After a spin-dependent shift of the optical lattice (f), followed by another $\pi/2$-pulse, we measure the population difference in these two spin components, and this difference gives directly the expectation value of the displacement operator.

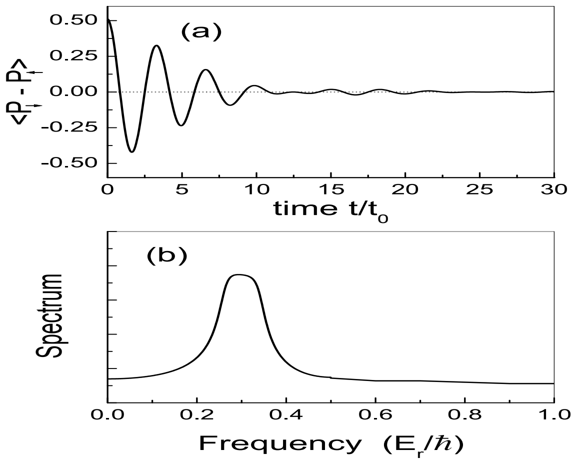

FIG. 2: Simulated experiential signal in an inhomogeneous optical lattice with a global harmonic trap. (a): Averaged population difference of the two spin components as a function of evolution time (in the unit of $t_0 = \hbar/E_r$) of the displacement operator. The average is taken over 40 occupied double wells in an global harmonic trap with the trap frequency $\omega_t = 2\pi \times 50$ Hz. Other parameters include $L = \lambda/2 = 400$nm, the atomic mass $m = 1.45 \times 10^{-25}$ kg for ^{87}Rb atoms, and the bias potential $\Delta = 0.3E_r$. A dephasing rate $\gamma = 0.1E_r/\hbar$ is assumed (see Eq. (7). (b) Fourier transform of the signal in Fig. (a). Instead of a sharp line at the bias potential ($\Delta/\hbar$,) the curve has a broad peak due to the broadening from average in the inhomogeneous global trap and the nonzero dephasing rate. However, the peak is still centered at the bias potential $\Delta/\hbar$.

$t_\delta \sim 8\omega^{-1} \sim 30\ \mu s$, the motional excitations estimated by the Landau-Zener formula is small and all the requirements seem to be reasonable with the current experimental technology.

In summary, we have proposed a feasible experiment using cold atoms in an optical lattice to test the DQNL associated with evolution of the displacement operator. The DQNL is different from and complementary to the kinetic quantum non-locality represented by the Bell inequalities. Similar to tests of the Bell inequalities, an experimental test of the DQNL could shed new light on our understanding of fundamentals of quantum mechanics.

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