# Cooperative Distributed Sequential Spectrum Sensing


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***Abstract*—We consider cooperative spectrum sensing for cognitive radios. We develop an energy efficient detector with low detection delay using sequential hypothesis testing. Sequential Probability Ratio Test (SPRT) is used at both the local nodes and the fusion center. We also analyse the performance of this algorithm and compare with the simulations. Modelling uncertainties in the distribution parameters are considered. Slow fading with and without perfect channel state information at the cognitive radios is taken into account.**

*Keywords- Cognitive Radio, Spectrum Sensing, Cooperative Distributed Algorithm, SPRT.*

## I. Introduction

Cognitive Radio has evolved as a working solution for the scarcity of spectrum due to the proliferation of wireless services. Cognitive Radios (CRs) access the spectrum licensed to other service providers opportunistically without interference to the existing communication services. For this the Cognitive users sense the spectrum to detect the usage of the channel by the primary (licensed) users. However due to the inherent transmission impairments of wireless channels and strict spectrum sensing requirements for Cognitive Radios [17] spectrum sensing has become one of the main challenges faced by them.

Cooperative spectrum sensing ([20], [23]) in which different cognitive radios interact with each other, is proposed as an answer to the problems caused by multipath fading, shadowing and hidden node problem in single node spectrum sensing methods. Also it improves the probability of false alarm and the probability of miss-detection. These are achieved via the exploitation of spatial diversity among the Cognitive users.

Cooperative spectrum sensing can be either centralized or distributed [23]. In the centralized algorithm a central unit gathers sensing data from the Cognitive Radios and identifies the spectrum usage ([23], [15]). On the other hand, in the distributed case each secondary user collects observations, makes a local decision and sends to a fusion node to make the final decision. The information that is exchanged between the secondary users and the fusion node can be a soft decision (summary statistic) or a hard decision [15]. Soft decisions can give better gains at the fusion center but also consume higher bandwidth at the control channels (used for sharing information among secondary users). However hard decisions provide as good a performance as soft decisions when the number of cooperative users increases [5].

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Spectrum sensing algorithms used at a node can use a fixed sample size (one shot) or sequential detection ([7], [10], [16], [23]). In case of fixed sample size detectors with the complete knowledge of primary signal, matched filter is the optimal detector [8] that maximises the SNR. When the only known apriori information is the noise power, the energy detector is optimal [8]. Sequential detection can provide better performance [12]. In the sequential approach one can consider detecting when a primary turns ON (or OFF) (change detection) or just the hypothesis testing whether the primary is ON or OFF. Sequential change detection is well studied in ([2], [10], [12]). In sequential hypothesis testing ([6], [9], [16]) one considers the case where the status of the primary channel is known to change very slowly, e.g., detecting occupancy of a TV transmission. Usage of idle TV bands by the Cognitive network is being targeted as the first application for cognitive radio. In this setup Walds'Sequential Probability Ratio Test (SPRT) provides the optimal performance for a single node ([14], [22]). [23] has an extensive survey of spectrum sensing methods. Other spectrum sensing schemes include methods based on higher order statistics [13], wavelet transforms [18] and compressed sensing [19].

We use the sequential hypothesis testing framework in the cooperative setup. We use SPRT at each local node and again at the fusion center. This has been motivated by our previous algorithm, DualCUSUM used for distributed change detection. Thus we will call this algorithm DualSPRT. However this has been studied in ([9] and [16]) as well. But unlike ([9], [16]) we also provide theoretical analysis of this algorithm and consider the effect of fading in the channel between the primary and secondary nodes. We also model the receiver noise at the fusion node and use physical layer fusion to reduce the transmission time of the decisions by the local nodes to the fusion node.

This paper is organised as follows. Section II describes the model. Section III starts with the DualSPRT algorithm. Simulation results and analysis are also provided in Section III. Then we consider the case where the SNRs are different at different Cognitive Radios. The received SNR may or may not be known to the CR nodes. In Section IV we introduce fading at the channel between the primary transmitter and the Cognitive Radios. The channel gains may not be available to the local secondary nodes. Section V concludes the paper.

## II. SYSTEM MODEL

We consider a Cognitive Radio system with one primary transmitter and $L$ secondary users. The $L$ nodes sense the channel to detect the spectral holes. The decisions made by the secondary users are transmitted to a fusion node via a Multiple Access Channel (MAC) for it to make a final decision.

Let $X_{k,l}$ be the observation made at secondary user $l$ at time $k$. The $\{X_{k,l},\ k \geq 1\}$ are independent and identically distributed (iid). It is assumed that the observations are independent across Cognitive Radios. Based on $\{X_{n,l},\ n \leq k\}$ the secondary user $l$ transmits $Y_{k,l}$ to the fusion node. It is assumed that the secondary nodes are synchronised so that the fusion node receives $Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_k$, where $\{Z_k\}$ is iid receiver noise. The fusion center uses $\{Y_k\}$ and makes a decision. The observations $\{X_{k,l}\}$ depend on whether the primary is transmitting (Hypothesis $H_1$) or not (Hypothesis $H_0$) as

$$X_{k,l} = \begin{cases} Z_{k,l}, & k = 1, 2, \ldots, \ \textit{under}\ H_0 \\ h_l S_k + Z_{k,l}, & k = 1, 2, \ldots, \ \textit{under}\ H_1 \end{cases} \quad (1)$$

where $h_l$ is the channel gain of the $l^{th}$ user, $S_k$ is the primary signal and $Z_{k,l}$ is the observation noise at the $l^{th}$ user at time $k$. We assume $\{Z_{k,l}, k \geq 1\}$ are iid. Let $N$ be the time to decide on the hypothesis by the fusion node. We assume that $N$ is much less than the coherence time of the channel so that the slow fading assumption is valid. This means that $h_l$ is random but remains constant during the spectrum sensing duration.

The general problem is to develop a distributed algorithm in the above setup which solves the problem:

$$\min E_{DD} \stackrel{\triangle}{=} E[N|H_i]\ , \quad (2)$$
$$\text{subject to } P_{FA} \leq \alpha$$

where $H_i$ is the true hypothesis, $i = \{0, 1\}$ and $P_{FA}$ is the probability of false alarm, i.e., probability of making a wrong decision. We will separately consider $E[N|H_1]$ and $E[N|H_0]$. It is well known that for a single node case ($L = 1$) Wald's SPRT performs optimally in terms of reducing $E[N|H_1]$ and $E[N|H_0]$ for a given $P_{FA}$. Motivated by the good performance of DualCUSUM in ([1], [7]) and the optimality of SPRT for a single node, we propose using DualSPRT in the next section and study its performance.

## III. DUALSPRT ALGORITHM

To explain the setup and analysis we start with the simple case, where the channel gains, $h_l$=1 for all $l's$. We will consider fading in the next section. DualSPRT is as follows:

1) Secondary node, $l$, runs SPRT algorithm,

$$W_{0,l} = 0$$
$$W_{k,l} = W_{k-1,l} + \log\left[f_{1,l}\left(X_{k,l}\right)/f_{0,l}\left(X_{k,l}\right)\right], k \geq 1 \quad (3)$$

where $f_{1,l}$ is the density of $X_{k,l}$ under $H_1$ and $f_{0,l}$ is the density of $X_{k,l}$ under $H_0$.

2) Secondary node $l$ transmits a constant $b_1$ at time $k$ if $W_{k,l} \geq \gamma_1$ or transmits $b_0$ when $W_{k,l} \leq \gamma_0$, i.e., $Y_{k,l} = b_1 1_{\{W_{k,l} \geq \gamma_1\}} + b_0 1_{\{W_{k,l} \leq \gamma_0\}}$ where $\gamma_0 < 0 < \gamma_1$ and $1_A$ denotes the indicator function of set A. Parameters $b_1, b_0, \gamma_1, \gamma_0$ are chosen appropriately.

3) Physical layer fusion is used at the Fusion Centre, i.e. , $Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_k$, where $Z_k$ is the iid noise at the fusion node.

4) Finally, Fusion center runs SPRT:

$$F_k = F_{k-1} + \log\left[g_1\left(Y_k\right)/g_0\left(Y_k\right)\right], \quad F_0 = 0, \quad (4)$$

where $g_0$ is the density of $Z_k + \mu_0$, the MAC noise at the fusion node, and $g_1$ is the density of $Z_k + \mu_1$, $\mu_0$ and $\mu_1$ being design parameters.

5) The fusion center decides about the hypothesis at time $N$ where

$$N = \inf\{k : F_k \geq \beta_1 \ or \ F_k \leq \beta_0\}$$

and $\beta_0 < 0 < \beta_1$. The decision at time $N$ is $H_1$ if $F_N \geq \beta_1$, otherwise $H_0$.

In order to have equal $P_{FA}$ under both hypothesis, we choose

$$\gamma_1 = -\gamma_0 = \gamma \ and \ \beta_1 = -\beta_0 = \beta.$$

Of course $P_{FA}$ can be taken different under $H_0$ or $H_1$ by appropriately choosing $\gamma_1,\ \gamma_0,\ \beta_1,\ \beta_0$. Any prior information available about $H_0$ or $H_1$ can be used to decide constants. Performance of this algorithm depends on $(\gamma_1, \gamma_0, \beta_1, \beta_0, b_1, b_0, \mu_1, \mu_0)$. Also we choose these parameters such that the probability of false alarm, $P_{fa}$ at local nodes is much lower than $P_{FA}$. A good set of parameters for given SNR values can be obtained from known results of SPRT.

Deciding at local nodes and transmitting them to the fusion node reduces the transmission rate and transmit energy used by the local nodes in communication with the fusion node. Also, physical layer fusion in Step 3 reduces transmission time, but requires synchronisation of different local nodes. If synchronisation is not possible, then some other algorithm, e.g., TDMA can be used.

DualSPRT (without physical layer synchronization and fusion receiver noise) has been shown to perform well in ([9], [16]). In the rest of the following we analyse the performance under our setup.

### A. Performance Analysis

We first provide the analysis for $E_{DD}$ and then for $P_{FA}$.The analysis for $E_{DD}$ is similar to that of DualCUSUM in [7]. For simplicity, in the following we will take $\gamma_1 = -\gamma_0 = \gamma$, $\beta_1 = -\beta_0 = \beta$, $\mu_1 = -\mu_0 = \mu$ and $b_1 = -b_0 = 1$. Then $P_{FA}$ under the two hypothesis is same.

*$E_{DD}$ Analysis:* At the fusion node $F_k$ crosses $\beta$ under $H_1$ when a sufficient number of local nodes transmit $b_1$. The dominant event occurs when the number of local nodes transmitting are such that the mean drift of the random walk $F_k$ will just have turned positive. In the following we find the mean time to this event and then the time to cross $\beta$ after this. The $E_{DD}$ analysis is same under hypothesis $H_0$ and $H_1$. Hence we provide the analysis for $H_1$.

At secondary node $l$ SPRT $\{W_{k,l}, k \geq 0\}$ is a random walk. Let $\delta = E_{H_1}[\log(f_1(X_{k,l})/f_0(X_{k,l}))]$ , $\sigma^2 = Var[\log[f_1(X_{k,l})/f_0(X_{k,l})]]$. We know $\delta > 0$. The time $\tau_\gamma$ for $W_k$ at each local node to cross the threshold $\gamma$ satisfies $E[\tau_\gamma] \sim \gamma/\delta$ for large values of $\gamma$ (needed for small $P_{FA}$. Then by central limit theorem we can show that at each node

$$\tau_\gamma \sim \mathcal{N}(\frac{\gamma}{\delta}, \frac{\sigma^2\gamma}{\delta^3}) \ . \quad (5)$$

Now, as in [7], we can show that,

$$E_{DD} \approx E[t_j] + \frac{\beta - \bar{F}_j}{\delta_j} \quad (6)$$

where $\delta_j$ is the drift of the fusion center SPRT, $F_k$ when $j$ local nodes are transmitting, $t_j$ is the point at which the drift of $F_k$ changes from $\delta_{j-1}$ to $\delta_j$, $\bar{F}_j = E[F_{t_j-1}]$, the mean value of $F_k$ just before transition epoch $t_i$ and

$$j = min\{i : \delta_i > 0 \ and \ \frac{\beta - \bar{F}_i}{\delta_i} < E[t_{i+1}] - E[t_i]\}.$$

An iterative method is proposed [2] to calculate $E[t_i]$ and $\bar{F}_j$ in an efficient manner.

For the above analysis for $E_{DD}$ we followed the analysis of DualCUSUM in [7]. However there are some difference in the SPRT at the fusion center here from the DualCUSUM in [7]. But comparison with simulations show that we will get an acceptable approximations.

*$P_{FA}$ Analysis:* It can be easily verified that $t_k$, defined earlier is the $k^{th}$ order statistics of $L$ iid random variables, $\tau_{\gamma,l}$ (first passage time to threshold $\gamma$ by the $l^{th}$ node,whose probability density function is given in (5)) . Then $P_{FA}$ when $H_1$ is the true hypothesis is given by,

$$\begin{aligned} P_{H_1}(False\ alarm) &= P_{H_1}(False\ alarm\ before\ t_1) \quad (7)\\ &+P_{H_1}(False\ alarm\ between\ t_1 and\ t_2)\\ &+P_{H_1}(False\ alarm\ between\ t_2\ and\ t_3)+.... \end{aligned}$$

One expects that the first term in (7) should be the dominant term. This is because $P_{fa}$ is much smaller than $P_{FA}$ and hence after $t_1$, the drift of $F_k$ will be more positive. Therefore the probability of false alarm goes down. We have verified this from simulations also. Hence we focus on the first term.

Let $S_k = \log[g_1(Y_k)/g_0(Y_k)]$ and $\theta = \beta/2\mu$.

Therefore $F_k = S_1 + S_2 + ... + S_k$. Every $S_i, 1 \leq i \leq k$ has a common term $2\mu$ (in case of Gaussian $g_1$ and $g_0$) , thus changing the threshold to $\theta = \beta/2\mu$. Then

$$\begin{aligned} &P_{H_1}(FA\ before\ t_1)\\ &= \sum_{k=1}^{\infty} P\Big[\{F_k < -\theta\} \cap_{n=1}^{k-1} \{F_n > -\theta\}\Big| t_1 > k\Big] P[t_1 > k]\\ &= \sum_{k=1}^{\infty}\Big(P[F_k < -\theta| \cap_{n=1}^{k-1}\{F_n > -\theta\}]\ P[\cap_{n=1}^{k-1}\{F_n > -\theta\}]\Big)\\ &\quad \Big(1 - \Phi_{t_1}(k)\Big)\\ &\overset{(A)}{=} \sum_{k=1}^{\infty}\Big(P[F_k < -\theta | F_{k-1} > -\theta]\ P[\inf_{1\leq n\leq k-1} F_n > -\theta]\Big)\\ &\quad \Big(1 - \Phi_{t_1}(k)\Big)\\ &\overset{(B)}{\geq} \sum_{k=1}^{\infty}\Big(\int_{c=0}^{2\theta} P[S_k < -c] f_{F_{k-1}}\{-\theta + c\} dc\Big)\\ &\quad \Big(1 - 2P[F_{k-1} < -\theta]\Big)\Big(1 - \Phi_{t_1}(k)\Big) \end{aligned}$$

where $\Phi_{t_1}$ is the Cumulative Distribution Function of $t_1$. As we are considering only $\{F_k, k \leq t_1\}$, we remove the dependencies on $t_1$. (A) is because of the Markov property of the random walk. (B) is due to the inequality,

$$P[\sup_{k\leq n} F_k \geq \theta] \leq 2P[F_n \geq \theta]$$

for the Gaussian random walk $F_k$ [4]. Similary we can write an upper bound by replacing $P[\cap_{n=1}^{k-1}\{F_n > -\theta\}]$ with $P[F_{k-1} > -\theta]$. In Table I we compare the lower bound on $P_{FA}$ with the simulation results. We can make this lower bound tighter if we do the same set of analysis for the Gaussian random walk between $t_1$ and $t_2$ with appropriate changes and add to the results we already obtained.

### *B. Example*

We apply the DualSPRT on the following example and compare the $E_{DD}$ and $P_{FA}$ via analysis provided above with the simulation results. We assume that the pre-change distribution $f_0$ and the post-change distribution $f_1$ are Gaussian with different means. This model is relevant when the noise and interference are log-normally distributed [20]. This is a useful model when $X_{k,l}$ is the sum of energy of a large number of observations at the secondary node at low SNR.

Parameters used for simulation are as follows: There are 5 secondary nodes, ($L = 5$), $f_0 \sim \mathcal{N}(0, 1)$ and $f_1 \sim \mathcal{N}(1, 1)$, where $\mathcal{N}(a, b)$ denote Gaussian distribution with mean $a$ and variance $b$. Also $f_0 = f_{0,l}$ and $f_1 = f_{1,l}$ for $1 \leq l \leq L$, $\gamma_1 = -\gamma_0 = \gamma$, $\beta_1 = -\beta_0 = \beta$, $\mu_1 = -\mu_0 = \mu$ and $b_1 = -b_0 = 1$. The $P_{FA}$ and the corresponding $E_{DD}$ are provided in Table I. The parameters are chosen to provide good performance for the given $P_{FA}$. The table also provides the results obtained via analysis.

### *C. Analysis for different SNRs*

The above analysis is for the case when $X_{k,l}$ have the same distribution for different $l$ under the hypothesis $H_0$ and

| $hyp$ | $P_{FA} Sim.$ | $P_{FA} Anal.$ | $E_{DD} Sim.$ | $E_{DD} Anal.$ |
|---|---|---|---|---|
| H1 | 0.00125 | 0.0012 | 15.6716 | 16.4216 |
| H1 | 0.01610 | 0.0129 | 13.928 | 12.6913 |
| H0 | 0.0613 | 0.0497 | 11.803 | 10.583 |
| H0 | 0.0031 | 0.0027 | 15.1766 | 14.830 |

TABLE I
DUALSPRT: COMPARISON OF $E_{DD}$ AND $P_{FA}$ OBTAINED VIA ANALYSIS (LOWER BOUND ON THE DOMINATING TERM) AND SIMULATION

$H_1$. However in practice the $X_{k,l}$ for different local nodes $l$ will often be different because their receiver noise can have different variances and / or the path losses from the primary transmitter to the secondary nodes can be different. The above analysis for this case needs slight changes for $E_{DD}$ as well as $P_{FA}$.

For the analysis of $E_{DD}$ one difference is that $\tau_{\gamma,l}$, $l = 1, \ldots, L$ are no longer iid. Now the iterative scheme used in Section III A to calculate $E_{t_j}$ and $\bar{F}_j$ does not work. Thus, knowing the minimum number of local nodes needed to make the mean drift of $F_k$ positive (say it is $i^*$), we compute the mean of the $i^*$ order statistics of the independent random variable $\tau_{\gamma,l}, l = 1, \ldots, L$ via [3]. Then we approximate the $E_{DD}$ by

$$E[t_{i^*}] + \frac{\beta - \left(\frac{E[t_{i^*}] - E[t_{i^*-1}]}{\delta_{i^*-1}}\right)}{\delta_{i^*}} \ . \tag{8}$$

For $P_{FA}$ analysis we need the distribution of the first order statistics $t_1$ for $\tau_{\gamma,l}, l = 1 \ldots, L$ and then use the method proposed in Section III A.

We provide an example to verify the accuracy of the performance analysis provided above.

### D. Example

There are five secondary nodes with primary to secondary channel gain being 0, -1.5, -2.5, -4 and -6 dB respectively (corresponding post change means are 1, 0.84, 0.75, 0.63, 0.5). $f_0 \sim \mathcal{N}(0, 1), f_0 = f_{0,l}$ for $1 \leq l \leq L$. Table II provides the $E_{DD}$ and $P_{FA}$ via analysis and simulations. We see a good match.

| $P_{FA} Sim.$ | $P_{FA} Anal.$ | $E_{DD} Sim.$ | $E_{DD} Anal.$ |
|---|---|---|---|
| $26.68e-4$ | $27.51e-4$ | 36.028 | 34.634 |
| $18.78e-4$ | $19.85e-4$ | 44.319 | 43.290 |
| $36.30e-4$ | $35.16e-4$ | 27.770 | 25.977 |

TABLE II
DUALSPRT FOR DIFFERENT SNR'S BETWEEN THE PRIMARY AND THE SECONDARY USERS: COMPARISON OF $E_{DD}$ AND $P_{FA}$ OBTAINED VIA ANALYSIS AND SIMULATION.

### E. Different and unknown SNRs

Next we consider the case where the received signal power is fixed but not known to the local Cognitive Radio nodes. This can happen if the transmit power of the primary is not known and / or there is unknown shadowing. Now we limit ourselves to the energy detector where the observations $X_{k,l}$ are a summation of energy of $N$ samples received by the $l^{th}$ Cognitive Radio node. Then for somewhat large $N$, the pre and post change distributions of $X_{k,l}$ can be approximated by Gaussian distributions: $f_{0,l} \sim \mathcal{N}(\sigma^2, 2\sigma^4/N)$ and $f_{1,l} \sim \mathcal{N}(P_l + \sigma^2, 2(P_l + \sigma^2)^2/N)$, where $P_l$ is the received power at the $l^{th}$ CR node and noise $Z_{k,l} \sim \mathcal{N}(0, \sigma^2)$. Under low SNR conditions $\left(P_l + \sigma^2\right)^2 \approx \sigma^4$ and hence $X_{k,l}$ are Gaussian distributed with mean change under $H_0$ and $H_1$. Now taking $X_{k,l} - \sigma^2$ as the data for the detection algorithm at the $l^{th}$ node, since $P_l$ is unknown we can formulate this problem as a sequential hypothesis testing problem with

$$H_0 : \theta = 0 \ ; \ H_1 : \theta \geq \theta_1 \ . \tag{9}$$

where $\theta$ is $P_l$ and $\theta_1$ is appropriately chosen.

The problem

$$H_0 : \theta \leq \theta_0 \ ; \ H_1 : \theta \geq \theta_1 \ , \tag{10}$$

subject to the error constraints

$$P_\theta\{rejectH_0\} \leq \alpha \ for \ \theta \leq \theta_0 \tag{11}$$

$$P_\theta\{rejectH_1\} \leq \beta \ for \ \theta \geq \theta_1$$

for exponential family of distributions is well studied in ([11], [12]). The following algorithm of Lai is asymptotically Bayes optimal [11] and hence we use it at the local nodes instead of SPRT. Let $\theta \in A = [a_1, a_2]$. Define

$$W_{n,l} = max \left[ \sum_{k=1}^{n} \log \frac{f_{\hat{\theta_n}}(X_k)}{f_{\theta_0}(X_k)}, \sum_{k=1}^{n} \log \frac{f_{\hat{\theta_n}}(X_k)}{f_{\theta_1}(X_k)} \right] \ , \tag{12}$$

$$N(g, c) = \inf\{n : W_{n,l} \geq g(nc)\} \ , \tag{13}$$

where $g()$ is a time varying threshold. Its approximate expression is given in [11]. At time $N(g, c)$ decide upon $H_0$ or $H_1$ according as

$$\hat{\theta}_{N(g,c)} \leq \theta^* \ or \ \hat{\theta}_{N(g,c)} \geq \theta^* \ ,$$

where $\theta^*$ is obtained by solving $I(\theta^*, \theta_0) = I(\theta^*, \theta_1)$, and $I(\theta, \lambda)$ is the Kullback-Leibler information number. Also for Gaussian $f_0$ and $f_1$, $\hat{\theta}_n = max\{a_1, min[S_n/n, a_2]\}$.

The choice of $\theta_1$ in (9) affects the performance of $E[N|H_0]$ and $E[N|H_1]$ for the algorithm (12)-(13), where $N = N(g, c)$. For our case where $H_0 : \theta = 0$, unlike in (10) where $H_0 : \theta \leq 0$, $E[N|H_0]$ largely depends upon the value $\theta_1$. As $\theta_1$ increases, $E[N|H_0]$ decreases and $E[N|H_1]$ increases. If $P_l \in [\underline{P}, \overline{P}]$ for all $l$ then a good choice of $\theta_1$, is $(\overline{P} - \underline{P})/2$.

In the distributed setup with received power at the local nodes unknown, the local nodes will use the Lai's algorithm mentioned above while the fusion node runs the SPRT. All other details remain same. We call this algorithm GLR-SPRT.

The performance of GLR-SPRT is compared with Dual-SPRT (where the received powers are assumed known at the local nodes) for Example III D in Table III. Interestingly $E[N|H_1]$ for GLR-SPRT is actually lower than for DualSPRT , but $E[N|H_0]$ is higher.

| $hyp$ | $E_{DD}$ | $P_{FA} = 0.1$ | $P_{FA} = 0.05$ | $P_{FA} = 0.01$ |
|---|---|---|---|---|
| H1 | DualSPRT | 2.06 | 3.177 | 5.264 |
| H1 | GLRSPRT | 1.425 | 2.522 | 4.857 |
| H0 | DualSPRT | 1.921 | 3.074 | 5.184 |
| H0 | GLRSPRT | 2.745 | 3.852 | 6.115 |

TABLE III
COMPARISON BETWEEN GLRSPRT AND DUALSPRT FOR DIFFERENT SNR'S BETWEEN THE PRIMARY AND THE SECONDARY USERS.

## IV. Channel with Fading

In this section we consider the system where the channels from the primary transmitter to the secondary nodes have fading $(h_l \neq 1)$. We assume slow fading, i.e., the channel coherence time is longer than the hypothesis testing time. We consider two cases, Case 1: the fading gain is known to the CR nodes. Case 2: the fading gain is not known to the CR nodes.

When the fading gain $h_l$ is known to the $l^{th}$ secondary node then this case can be considered as the different SNR case studied in Section III C. Thus we only consider Case 2 where the channel gain $h_l$ is not known to the $l^{th}$ node.

We consider the energy detector setup of Section III E. However, $P_l$, the received signal power at the local node $l$ is random. If the fading is Rayleigh distributed then $P_l$ has exponential distribution. The hypothesis testing problem becomes

$$H_0 : f_{0,l} \sim \mathcal{N}(0, \sigma^2); H_1 : f_{1,l} \sim \mathcal{N}(\theta, \sigma^2) \qquad (14)$$

where $\theta$ is random with exponential distribution and $\sigma^2$ is the variance of noise. We are not aware of this problem being handled via sequential hypothesis testing. However we use Lai's algorithm in Section III E where we take $\theta_1$ to be the median of the distribution of $\theta$, such that $P(\theta \geq \theta_1) = 1/2$. This seems a good choice for $\theta_1$ to compromise between $E[N|H_0]$ and $E[N|H_1]$.

We use this algorithm on an example where $\sigma^2 = 1, \theta = exp(1)$, Var($Z_k$) = 1, and $L = 5$. The performance of this algorithm is compared with that of DualSPRT (with perfect channel state information) in Table IV (under $H_0$) and Table V (under $H_1$). The $E_{DD}$ and $P_{FA}$ were computed by simulations each case by 100000 times and taking the average. We observe that under $H_1$, for high $P_{FA}$ this algorithm works better than DualSPRT with channel state information, but as $P_{FA}$ decreases DualSPRT becomes better and the difference increases. For $H_0$, GLRSPRT is always worse and the difference is almost constant.

| $E_{DD}$ | $P_{FA} = 0.1$ | $P_{FA} = 0.05$ | $P_{FA} = 0.01$ |
|---|---|---|---|
| DualSPRT | 1.669 | 2.497 | 4.753 |
| GLRSPRT | 3.191 | 4.418 | 7.294 |

TABLE IV
COMPARISON BETWEEN GLRSPRT AND DUALSPRT WITH SLOW-FADING BETWEEN PRIMARY AND SECONDARY USER UNDER H0.ENERGY DETECTION STATISTIC IS USED AT THE SECONDARY NODES

| $E_{DD}$ | $P_{FA} = 0.1$ | $P_{FA} = 0.08$ | $P_{FA} = 0.06$ |
|---|---|---|---|
| DualSPRT | 1.74 | 1.854 | 2.417 |
| GLRSPRT | 1.62 | 3.065 | 5.42 |

TABLE V
COMPARISON BETWEEN GLRSPRT AND DUALSPRT WITH SLOW-FADING BETWEEN PRIMARY AND SECONDARY USER UNDER H1.ENERGY DETECTION STATISTIC IS USED AT THE SECONDARY NODES

## V. Conclusions and Future Work

We have proposed an energy efficient, distributed cooperative spectrum sensing technique, DualSPRT which uses SPRT at the cognitive radios as well as at the fusion center. We also provide analysis of DualSPRT. Next we modify the algorithm so as to be able to detect when the received SNR is not known and when there is slow fading channels between the primary and the secondary nodes. Future work should consider analysis of the GLR algorithms and optimising over the current setup.

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