Particle Ratio Fluctuations and Isobaric Ensemble for Chemical Freeze-out Scenario

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In this work, we study the behavior of event-by-event distribution of particle ratios in Gran Canonical and Isobaric ensemble.We show that the experimental value of the width in the K/π ratio distribution is much wider than that given by a unique grand canonical ensemble or Isobaric ensembe and maybe a mixture is required.

1 Introduction

The main purpose of the relativistic heavy ion program at CERN and RHIC is to study the properties of matter in hadronic interactions at very high energy density. Among many novel phenomena expected, the observation of the quark gluon plasma (QGP) predicted by the QCD is one of the main goals. For these studies, a crucial factor is to know whether or not the system reaches thermal equilibrium at some stage of the collision. In this respect, thermal models are extensively used to analyze the final hadronic abundances [1]. In these models, it is assumed that there exists a stage where the chemical abundances of hadrons (including resonances) are suddenly frozen out, keeping the exact memory of the last instant of the chemical equilibrium. Such stage is referred to as "chemical freeze-out" and the observed hadronic abundances are completely determined by the temperature T and baryonic chemical potential μ_B of the system at this point.

The above picture is equivalent to say that the hadronic gas at the chemical freeze-out is described by a grand canonical ensemble (GC). The fact is that the observed particle ratios are remarkably well reproduced adjusting just two parameters, T and μ_B. Such fits have been done for 4π NA49 data and for RHIC Au+Au data in a rapidity window of rapidity. Although the resulting fits are impressive[1], it is very difficult to imagine that final hadrons are in a global chemical equilibrium (rapidity distributions of particles, especially those of hyperons, are far from constant). It may well be possible that to have a nice fit only for the chemical abundances does not imply that T and μ_B obtained correspond to those for a *real* chemical equilibrium. They could be just some effective parameters to characterize certain average of statistical ensembles corresponding to different physical conditions for chemical freeze-out.

In order to clarify the above mentioned aspect, we should go one step further than the mean values of chemical abundances, and study the behavior of less inclusive quantities such as event-by-event fluctuations and correlations of chemical abundances among different hadrons. In this paper, we analyze the event-by-event fluctuations of K/π ratio in the thermal model. The distribution of K/π ratio is measured for $NA49$ experiment. We show that the pure statistical fluctuation given by a unique GC ensemble of resonance gas is much smaller than the experimental data. This imply that the observed particle abundances can not be reproduced in terms of one single GC ensemble, but we need surely a mixture of many statistical ensembles. To explain the observed width of the distribution of K/π ratio[2], we have to fluctuate, for example, the chemical potential μ_B. For nucleus-nucleus collisions, the mixture of different μ_B is rather natural because of the baryons in the incident nuclei and fluctuating initial conditions[3].

When we consider the fluctuation of the ratio K/π, it is important to see if there exists any correlation among K and π. For a GC ensemble of ideal hadronic gas, there exists no correlation for the production of K and π. For an infinite system these assumptions really do not matter, but when one consider a domain of particle production with finite volume (fireball) it is not clear at all whether a GC ensemble represents the best scenario. In particular for kaons, the strangeness conservation imposes some effects. Analysis based on the canonical ensemble has been done by Koch [4]. Another possible approach is the use of isobaric ensemble (ISO). This ensemble describe a system with fixed pressure allowing volume fluctuations.

When we deal only with the average values of chemical abundances, there will be no difference between the GC and ISO ensembles, especially if the total number of the particles in the system is large enough. However, as we discuss later, the multiplicity distribution and correlations among particles are completely different between them. To understand more clearly, let us consider the K/π ratio. For ISO ensemble, the pressure is fixed so that for a fixed event large multiplicity of pions corresponds to a large volume event. Therefore kaons are also abundant in such events and vice-versa. That is, K and π multiplicities have positive correlations in ISO ensembles. For GC ensembles, if there exists no excluded volume correlation, the K and π multiplicities have no correlation among them. Furthermore, since the chemical freeze-out should be a local process, it seems natural that its mechanism is governed by the intensive parameters. Statistically speaking, the observed chemical freeze-out state may correspond to the state of maximum entropy under a fixed *pressure*, and not under a fixed *volume*. It is worthwhile to investigate if there exist differences between these two scenarios in the particle abundances, especially in the distribution of K/π ratio.

2 Grand Canonical Ensemble

Let us consider first the case of GC ensemble of hadronic resonances. For simplicity, we consider ideal Boltzmann gas of hadronic resonances, neglecting the quantum statistics and the effect of excluded volume. Except for pions, for temperatures we are interested, the Boltzmann gas approximation is quite well. Furthermore, since we consider just the ratio of particle multiplicities, the effect of excluded volume is not very important. Under these approximations, the probability $P(N_1, ...N_h)$ of having N_1 particles of the hadron 1, N_2 particles of the hadron 2, and so on, till h-th hadron species is given as just the product of Poisson distributions for each hadronic species,

$$\begin{aligned} P(N_1, ...N_h) &= P_{GC}(N_1, ...N_h; T, \mu_B) = \\ &= \prod_{i=1}^{h} \frac{1}{N_i!} \left[\phi_i V\right]^{N_j} e^{-V\phi_i}, \end{aligned} \quad (1)$$

where i refers to the hadron species, V is the volume and

$$\phi_i = \frac{g}{2\pi^2} m_i^3 \frac{K_2(\beta m_i)}{\beta m_i} e^{\mu_i \beta} \quad (2)$$

with $\beta = 1/T$ and μ_i is the chemical potential of the $i - th$ hadron. It is given by

$$\mu_i = B_i \mu_B + S_i \mu_S + T_i^{(3)} \mu_3 \quad (3)$$

with B_i, S_i and $T_i^{(3)}$ are baryon number, strangeness and 3-rd component of isospin of the $i - th$ hadron. The strangeness and isospin chemical potentials μ_S and μ_3 are determined using the charge and strangeness neutrality. m_i and g_i are the mass and statistical factor of the hadron, and K_2 is the modified Bessel function.

The observed particle abundances are not directly given by the thermal multiplicities. We should take into account the feeding stream from the resonance decays. The multiplicity of a stable (observable) hadron species, say α, in a single collision event is then given by

$$N_\alpha^{obs} = \sum_i N_{i \to \alpha} N_i, \quad (4)$$

where $N_{i \to \alpha}$ is the number of α hadron produced by the decay chain of the parent hadron i. If we consider the decay chain has no correlation with the thermal production mechanism, we may safely substitute $N_{i \to \alpha}$ (integers) in terms of their average values $F_{i \to \alpha}$ (not necessarily integers). We constructed the table for all $F_{i \to \alpha}\acute{s}$ following the decay chain of the $i - th$ hadron using the values given in the particle data table [5]. The distribution of the particle ratio, $r_{\alpha\beta}$ is then calculated to be

$$P\left(r_{\alpha/\beta}\right) = \left\langle \sum_{\{N_i\}} P(N_1, ...N_h)\, \delta\left(r_{\alpha\beta} - \frac{\bar{N}_\alpha}{\bar{N}_\beta}\right) \right\rangle, \quad (5)$$

where $\langle O \rangle$ denotes the average over collisional events of an observable O. In practice, we introduce a finite bin size Δ for the variable $r_{\alpha/\beta}$ and calculate the probability of finding an event within a given bin specified by $[r_{\alpha\beta} - \Delta/2, r_{\alpha\beta} + \Delta/2]$. The average number of $a - th$ hadron is

$$\langle \bar{N}_\alpha \rangle = \sum_i \langle N \rangle_i F_{i \to \alpha} \quad (6)$$

where in the case of GC, we have

$$\langle N \rangle_i = \phi_i V.$$

In Fig. 1, we show the result of Monte Carlo evaluation of $P\left(r_{K/\pi}\right)$ for the Pb+Pb central collision.

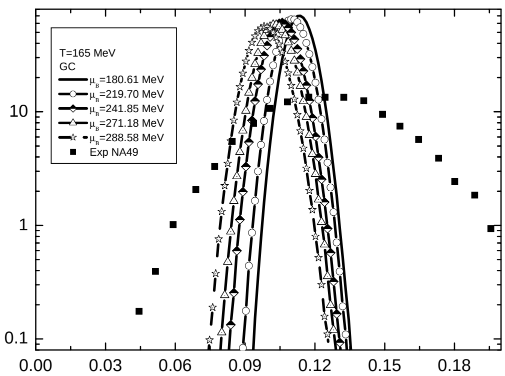

Figure 1. Grand canonical distributions of K/π ratio. Different curves correspond to different chemical potential indicated. Black circles are experimental data.

Thermal parameters $\{T, \mu_B\}$ are determined to give the best χ^2-fit to the particle ratios of stable hadrons as done in Ref. [1]. The value of χ^2 is around 20, which is comparable to that of [1]. This means that the average particle ratios are very well reproduced. However, as we see, the calculated distribution is very much narrower than the experimental value. The data [2] refers to a narrow rapidity window, and the corrections from detector acceptance are not included we may not consider the value shown here as definitive but the difference between the value from a single GC ensemble seems to be too large. It may be possible that the contamination of kaons from the neighboring rapidity windows causes such a broadening of the distribution [4]. Therefore the observed large width may be, is due to the mixture of many GC with different thermodynamical parameters, $\{T, \mu_B\}$. For Pb+Pb collisions at SPS, the baryonic chemical potential μ_B varies more than $200 MeV$ from the central to fragmentation region. In Fig. 1, we also show how the distribution of K/π

ratio changes with the chemical potential for fixed temperature. This results indicates that, the observed width of the K/π ratio distribution requires a mixture of GC ensemble of appreciably different parameters.

3 Isobaric Ensemble

As mentioned in Introduction, it maybe possible that the chemical freeze-out mechanism is governed by a pressure value than a fixed volume as constraint. The isobaric partition function is given as

$$Z_P(\beta,\mu,P)=\int_0^\infty dV e^{-\beta PV} Z_{GC}(\beta,\mu,V) \quad (7)$$

where $Z_{GC}(\beta,\mu,V)$ is the partition function of the GC ensemble. For a Boltzmann ideal gas, we can perform the integral easily to get

$$Z_P(\beta,\mu,P)=\frac{1}{\beta P-\sum_i \phi_i e^{-Pv_0\beta}}, \quad (8)$$

where we have introduced the excluded volume v_0 here as in [6].

It can be shown that the probability function corresponding to Eq.(1) becomes

$$P_{ISO}(N_1...N_h;T,P)=(1-\sum_i \eta_i)\ N!\ \prod_j \frac{{\eta_j}^{N_j}}{N_j!}, \quad (9)$$

where

$$\eta_i=\frac{\phi_i e^{-\beta P v_0}}{\beta P}. \quad (10)$$

The single particle multiplicity distribution is then given by

$$\begin{aligned} P_{ISO}(N_j) &= \sum_{N_1\neq N_j}...\sum_{N_h\neq N_j} P_{ISO}(N_1...N_h)= \\ &= (1-\zeta_j)\zeta_j^{N_j} \end{aligned} \quad (11)$$

where:

$$\zeta_j=\frac{\eta_j}{1-\sum\limits_{i\neq j}\eta_i}.$$

which is a geometric distribution.

Different from the GC case, the convolution of geometric distributions is not a geometric distribution, but so-called negative binomial distribution. Suppose that the final state of nuclear collision is formed of k independent 'fireballs', described by the same isobaric ensemble specified T and P. Let $N_i^{(q)}$ be the number of $i-th$ hadron contained in the $q-th$ "fireball". Then the total number of $i-th$ hadron is

$$\sum_q N_i^{(q)}=N_i. \quad (12)$$

In this case, the final probability distribution of having a set of hadrons, $\{N_1,...,N_h\}$, is given as the convolution of Eq.(9),

$$P_{ISO}^{(k)}(N_1,...,N_\alpha)=\sum_{\left\{\sum_q N_i^{(q)}=N_i\right\}}\cdots\sum\left[\prod_{q=1}^{k}P(N_1^q,...,N_h^q).\right], \quad (13)$$

where the summations are taken over all $(N_1^q,...,N_h^q)$ satisfying the constraints Eq.(12). Correspondingly, the single particle distribution Eq.(11) is modified as[7]

$$P_{ISO}^{(k)}(N_\alpha)=(1-\zeta_\alpha)^k\ \zeta_\alpha^{N_\alpha}\ \frac{(N_\alpha+k-1)!}{N_\alpha!(k-1)!}. \quad (14)$$

The average multiplicity of $i-th$ hadron is then

$$\langle N_i\rangle=k\frac{\zeta_i}{1-\sum_i\zeta_i}. \quad (15)$$

The particle ratio and its distribution are calculated by evaluating Eq.5) by the Monte Carlo method, generating $\{N_i\}$ which obey the probability distribution Eq.(13). For this purpose, use of the conditional probabilities for the ISO ensemble is found to be efficient. The probability of having N_i particles for $i-th$ hadron after having already N_1 particles for the hadron 1, N_2 particles for the hadron 2, and so on till N_{i-1} is

$$P\left(N_i||N_1,N_2,..,N_{i-1}\right)=(1-\bar{\eta}_i)^{Z_i+1}\frac{(Z_i+N_i)!}{Z_i!N_i!}\bar{\eta}_i^{N_i}, \quad (16)$$

where $Z_i=\sum_{j=1}^{i-1}N_j$ and $\bar{\eta}_i=\eta_i/\left(1-\sum_{j>i}\eta_j\right)$. From this, we can evaluate Eq.(5) very efficiently. In Fig.2, we show the calculated distribution of K/π ratio for the ISO ensemble for several clusters. The ISO ensemble gives a larger width compared to the GC ensemble, especially for smaller numbers of clusters. This can be understood in terms of the positive correlation among K and π multiplicities in the ISO ensemble, as mentioned before. On the other hand, for large number of clusters, the convoluted distribution Eq. (13) tends to the GC ensemble. This is a natural consequence of the statistical central limit theorem. However, different from the GC ensemble, for a given set of T an P, the value of chemical potential changes to keep the baryon number conservation so that the average values of K/π value changes for different number of clusters in the system.

4 Discussion and Perspectives

In this work, we studied the behavior of event-by-event distribution of particle ratios in different statistical ensembles. We have shown that the experimental value of the width in the K/π ratio distribution is much wider than that given by a unique grand canonical ensemble. If the observed width is due to the mixture of GC ensembles corresponding to different values of chemical potentials, a very large variation of chemical potential is required to reproduce the data. On

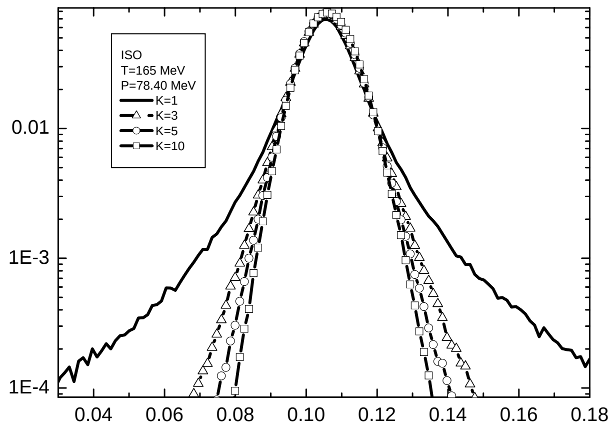

Figure 2. Isobaric distributions of K/π ratio. Different curves correspond to different number of clusters as indicated.

the other hand, the isobaric ensemble seems to be more adequate for the description of the chemical freeze-out scenario. It already furnishes a wider distribution of K/π ratio due to the correlations in particle multiplicities. On the other hand, if the final state of the system is composed of many clusters, convolution of the ISO distributions from these clusters tends to that of the GC distribution. However, in this case, the fluctuation of number of clusters with fixed (T, P) can also contribute to the width of the K/π distribution. A further investigation in this line is in progress.

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