Engineering two-mode squeezed states of cold atomic clouds with a superconducting stripline resonator

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Abstract

A scheme is proposed for engineering two-mode squeezed states of two separated cold atomic clouds positioned near the surface of a superconducting stripline resonator. Based on the coherent magnetic coupling between the atomic spins and a stripline resonator mode, the desired two-mode squeezed state can be produced via precisely control on the dynamics of the system. This scheme may be used to realize scalable on-chip quantum networks with cold atoms coupling to stripline resonators.

Key words: Two-mode squeezed state, circuit QED, cold atoms
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Two-mode squeezed states, which exhibit Einstein-Podolsky-Rosen entanglement, play a significant role in realizing quantum information protocols and investigating quantum nonlocality with continuous-variables [1]. Since entangled distant atomic ensembles are the building blocks for scalable quantum networks [2], several proposals have been given to produce two-mode squeezed states of distant atomic ensembles with cavity QED system [3,4]. Recently, hybrid systems consisting of ensembles of atomic or molecular system and superconducting transmission line resonators have been intensively investigated [5-21]. Solid-state superconducting quantum circuits are believed to have the advantages of integration and scaling on a chip [22]. Ultracold atoms are very attractive candidates for storing and processing quantum information, in view of their long coherence times and the well-developed techniques for detecting and manipulating the ground electronic (hyperfine) states. To realize scalable

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on-chip quantum networks or other quantum information applications with hybrid systems of cold atoms and stripline resonators, it is desirable to design a scheme that can generate two-mode squeezed states of separated cold atomic clouds in a controllable way.

In this work, utilizing the strong magnetic coupling of cold atoms to a superconducting transmission line resonator [8], we propose a hybrid scheme to engineer two-mode squeezed states of two separated cold atomic clouds. We show that, under certain conditions the coupled system of two atomic spin modes and the cavity mode can behave as three coupled harmonic oscillators with controllable coefficients. Through coherent control on the dynamics of the system, at some instants the cavity mode is decoupled from the atomic modes, leaving the latter in a two-mode squeezed state. Compared to other proposals based on electric-dipole coupling between atoms and fields in cavity QED [3,4], this scheme, utilizing strong magnetic coupling of atomic spins to cavity mode [8], has the advantage of being immune to charge noise and could constitute qubits with much longer coherence times. Moreover, this protocol, which is robust against the effect of thermal photons, is scalable and much easier to be integrated on a chip. With the technology of atomchip and circuit QED, our study may open promising perspectives for the implementation of on-chip quantum networks.

The system we consider consists of two separated atomic clouds trapped above the surface of a superconducting transmission line resonator, as sketched in Fig. 1. Each atomic cloud contains $N_i(i = 1, 2)$ atoms. The transmission line
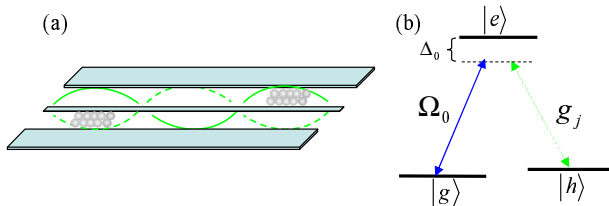

Fig. 1. (a) Two clouds of ultracold atoms positioned above the surface of a superconducting transmission line resonator. (b) Atomic level structure with couplings to the cavity mode and driving field. The atoms in ensemble 1 and 2 are initially cooled in the ground states $|\mathbf{0}\rangle_1$ and $|\mathbf{0}\rangle_2$, respectively.

resonator consists of three conducting stripes: the central conductor plus two ground planes. The electromagnetic field of the resonator is confined near the gaps between the conductor and the ground planes. Employing a variety of macroscopic electrostatic traps or other atom chip technology, clouds of cold ^{87}Rb atoms can be trapped close to the resonator [23]. Each atom has the level structure of three-level Λ configuration, with $|g\rangle$ and $|h\rangle$ the ground states, and $|e\rangle$ the excited state. The dominant interactions with a microwave field for Rb atoms cooled in the ground state are the magnetic dipole transitions

between the atomic hyperfine states of the $5S_{1/2}$ ground state. We choose $|F = 1, m_F = -1\rangle = |g\rangle$, $|F = 1, m_F = 1\rangle = |h\rangle$, and $|F = 2, m_F = 0\rangle = |e\rangle$ [23]. Assume that initially the atoms in the first ensemble are cooled in the ground state $|g\rangle$, and those in the second ensemble $|h\rangle$. The classical field (frequency ω_0) drives the transition $|g\rangle \leftrightarrow |e\rangle$ with Rabi frequency Ω_0, while the cavity mode (frequency ν_0) couples the transition $|h\rangle \leftrightarrow |e\rangle$ with the coupling constant g_j in the jth ensemble. To cancel the additional ac-Stark shifts to the states $|g\rangle$ and $|h\rangle$, we need two virtually excited states $|r\rangle$ and $|s\rangle$ [24]. The state $|r\rangle$ is virtually excited from the ground state $|g\rangle$ by another largely detuned classical field (frequency ω_1) with Rabi frequency Ω_1, while the cavity mode virtually couples the states $|h\rangle$ and $|s\rangle$ with coupling constant λ_j. The corresponding detunings for the related transitions are $\Delta_0 = \omega_{eg} - \omega_0 = \omega_{eh} - \nu_0$, $\Delta_r = \omega_{rg} - \omega_1$, and $\Delta_s = \omega_{sh} - \nu_0$, where $\omega_{eg}, \omega_{eh}, \omega_{rg}, \omega_{sh}$ are the transition frequencies for the atoms. In the interaction picture, the Hamiltonian describing the atom-field interaction is (let $\hbar = 1$)

$$
\begin{aligned}
H_I = \sum_{j=1,2} \{ \sum_{n=1}^{N_j} [\Omega_0 |e_n^j\rangle\langle g_n^j| e^{i\Delta_0 t} + g_j \hat{a} |e_n^j\rangle\langle h_n^j| e^{i\Delta_0 t} \\
+ \Omega_1 |r_n^j\rangle\langle g_n^j| e^{i\Delta_r t} + \lambda_j \hat{a} |s_n^j\rangle\langle h_n^j| e^{i\Delta_s t}] \} + \text{H.c.,}
\end{aligned}
\tag{1}
$$

where $\hat{a}$ is the annihilation operator for the cavity mode. We consider dispersive detuning $|\Delta_0|, |\Delta_r|, |\Delta_s| \gg |\Omega_0|, |\Omega_1|, |g_j|, |\lambda_j|$. Therefore, the excited states can be adiabatically eliminated. Then the interaction Hamiltonian describing the coupling of the cold atoms to the cavity mode is

$$
\begin{aligned}
H_I = \sum_{j=1,2} \left[\left(\frac{\Omega_0^2}{\Delta_0} + \frac{\Omega_1^2}{\Delta_r} \right) \sigma_{gg}^j + \left(\frac{g_j^2}{\Delta_0} + \frac{\lambda_j^2}{\Delta_s} \right) \sigma_{hh}^j \hat{a}^\dagger \hat{a} \right] \\
+ (\beta_2 \hat{c}_2 + \beta_1 \hat{c}_1^\dagger) \hat{a}^\dagger + \text{H.c.,}
\end{aligned}
\tag{2}
$$

where $\sigma_{gg}^j = \sum_{n=1}^{N_j} |g_n^j\rangle\langle g_n^j|$, $\sigma_{hh}^j = \sum_{n=1}^{N_j} |h_n^j\rangle\langle h_n^j|$, $\beta_i = \frac{\sqrt{N_i}\Omega_0 g_i}{\Delta_0}$, $\hat{c}_1^\dagger = 1/\sqrt{N_1} \sum_{n=1}^{N_1} |h_n^1\rangle\langle g_n^1|$, and $\hat{c}_2^\dagger = 1/\sqrt{N_2} \sum_{n=1}^{N_2} |g_n^2\rangle\langle h_n^2|$. To appropriately choose the parameters, we assume that $\frac{\Omega_0^2}{\Delta_0} + \frac{\Omega_1^2}{\Delta_r} = 0$ and $\frac{g_j^2}{\Delta_0} + \frac{\lambda_j^2}{\Delta_s} = 0$. For weak excitations of the atoms, the operator $\hat{c}_i$ obeys approximate harmonic oscillator commutation relations $[\hat{c}_i, \hat{c}_i^\dagger] \simeq 1$. In such a case, we can describe the atomic excitations in the ith ensemble as a set of harmonic oscillator states $|\mathbf{0}\rangle_1 = |g_1 g_2 ... g_N\rangle_1$, $|\mathbf{0}\rangle_2 = |h_1 h_2 ... h_N\rangle_2$, $|\mathbf{1}\rangle_i = \hat{c}_i^\dagger |\mathbf{0}\rangle_i$, etc. Assume $\beta_1 = i\xi_1, \beta_2 = i\xi_2, |\xi_2| > |\xi_1|$, and introduce $\Theta = \sqrt{|\xi_2|^2 - |\xi_1|^2}$. Then we obtain the effective Hamiltonian

$$
\begin{aligned}
H_I &= H_1 + H_2 \\
&= i\xi_1 \hat{c}_1^\dagger \hat{a}^\dagger - i\xi_1^* \hat{c}_1 \hat{a} + i\xi_2 \hat{c}_2^\dagger \hat{a} - i\xi_2^* \hat{c}_2 \hat{a}^\dagger.
\end{aligned}
\tag{3}
$$

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The above Hamiltonian shows that the ensemble excitations and the cavity mode represent a system of three coupled harmonic oscillators with controllable coefficients. Hamiltonian H_1 describes simultaneous creation or annihilation of an atomic spin in ensemble 1 and a photon, while H_2 describes the exchange of excitation quanta between the collective atomic spin-waves and cavity mode.

The Hamiltonian H_I commutates with the constant of motion $\hat{N} = \hat{c}_2^\dagger \hat{c}_2 - \hat{c}_1^\dagger \hat{c}_1 + \hat{a}^\dagger \hat{a}$. So if the system starts from the state $|0\rangle_c |\mathbf{0}\rangle_1 |\mathbf{0}\rangle_2$, with $|n\rangle_c$ is the Fock state for the cavity mode, then we have $\hat{N} = 0$ at any time during the evolution. The time evolution of the operators in the Heisenberg representation is given by [25]

$$\hat{c}_1(t) = \frac{\xi_1}{\Theta} \hat{a}^\dagger(0) \sin \Theta t - \frac{\xi_1 \xi_2}{\Theta^2} [1 - \cos \Theta t] \hat{c}_2^\dagger(0) + \frac{1}{\Theta^2} [|\xi_2|^2$$
$$- |\xi_1|^2 \cos \Theta t] \hat{c}_1(0), \tag{4a}$$

$$\hat{c}_2(t) = \frac{\xi_2}{\Theta} \hat{a}(0) \sin \Theta t + \frac{\xi_1 \xi_2}{\Theta^2} [1 - \cos \Theta t] \hat{c}_1^\dagger(0) - \frac{1}{\Theta^2} [|\xi_1|^2$$
$$- |\xi_2|^2 \cos \Theta t] \hat{c}_2(0), \tag{4b}$$

$$\hat{a}(t) = \hat{a}(0) \cos \Theta t + \frac{1}{\Theta} [-\xi_2^* \hat{c}_2(0) + \xi_1 \hat{c}_1^\dagger(0)] \sin \Theta t. \tag{4c}$$

Generally these solutions describe tripartite entanglement between cavity mode and collective spin excitations. One can find that after half a period $T_\pi = \pi/\Theta$, the cavity mode is decoupled from the collective atomic spin excitations. Moreover, the two atomic spin modes exhibit quantum correlations, namely

$$\hat{c}_1(T_\pi) = \frac{|\xi_1|^2 + |\xi_2|^2}{\Theta^2} \hat{c}_1(0) - \frac{2\xi_1 \xi_2}{\Theta^2} \hat{c}_2^\dagger(0), \tag{5a}$$

$$\hat{c}_2(T_\pi) = \frac{2\xi_1 \xi_2}{\Theta^2} \hat{c}_1^\dagger(0) - \frac{|\xi_1|^2 + |\xi_2|^2}{\Theta^2} \hat{c}_2(0). \tag{5b}$$

This result shows that if at $t = 0$ the state of the system is $|\Psi(0)\rangle = |\mathbf{0}\rangle_1 |\mathbf{0}\rangle_2 |0\rangle_c$, then at $t = T_\pi$, the two atomic modes are in the two-mode squeezed state

$$|\phi\rangle_a = \left(\frac{1 - r^2}{1 + r^2} \right) \sum_{n=0}^\infty [\frac{2r}{1 + r^2}]^n |\mathbf{n}\rangle_1 |\mathbf{n}\rangle_2, \tag{6}$$

while the cavity mode has been decoupled from two atomic modes. The squeezing parameter is $\epsilon = \tanh^{-1}(\frac{2r}{1+r^2})$, which is determined by $r = |\xi_2/\xi_1|$, thus by the ratio $|\beta_2/\beta_1|$. When the squeezed state is generated at the time of T_π, we switch off the lasers and decouple the atomic clouds to the cavity. Then the squeezed state can be preserved.

It is known that the two-mode squeezed state exhibits EPR entanglement.

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To further quantify the squeezing property of the two atomic modes, we employ the two-mode relative number squeezing parameter [26] $\zeta_{12} = \sigma^2(\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_2^\dagger \hat{c}_2)/(\langle \hat{c}_1^\dagger \hat{c}_1 \rangle + \langle \hat{c}_2^\dagger \hat{c}_2 \rangle)$, where $\sigma^2(X) = \langle X^2 \rangle - \langle X \rangle^2$. ζ_{12} taking the value of 0 signifies two-mode squeezing of the atomic spin modes, while $\zeta_{12} = 1$ means the two atomic modes are in independent states. To calculate the parameter ζ_{12} under the effect of cavity losses, we employ the quantum master equation approach. The evolution of the system is governed by the following master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_I, \rho] + \kappa \hat{a}\rho\hat{a}^\dagger - \kappa\{\hat{a}^\dagger a, \rho\}, \tag{7}$$

where κ stands for the cavity photon decay rate. In Fig. 2, we display the nu-
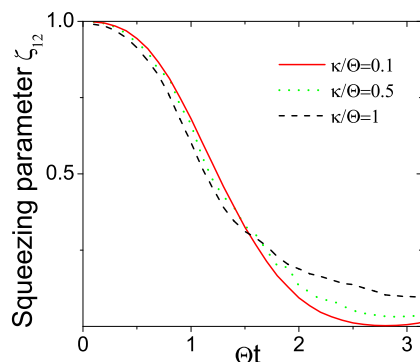
Fig. 2. Time evolution of the parameter ζ_{12} under $\kappa/\Theta = 0.1, 0.5, 1$. Other parameters are chosen such that $|\beta_2/\beta_1| = 1.5$.

merical results for the time evolution of the two-mode relative number squeezing parameter for several values of the parameter κ through the Monte Carlo wave function method. We see that nearly ideal squeezed states between the two atomic modes can occur at the instant $T_\pi = \pi/\Theta$ in the strong coupling regime $\Theta \gg \kappa$. In fact, even in the relatively weak coupling regime $\Theta \sim \kappa$, one can still obtain two-mode squeezing between the two atomic modes at the instant $T_\pi = \pi/\Theta$.

We now analyze the experimental feasibility of this proposal. As mentioned above, a promising candidate for the atoms is ^{87}Rb coupled to a stripline resonator. With the help of atom-chip technology, ensembles of cold Rb atoms and a superconducting transmission line resonator could be integrated in a hybrid device on a single superconducting atomchip. The preparation of the initial atomic state can be accomplished using the well-developed optical pumping and adiabatic population transfer techniques. We assume modest atomic densities, so that atomic interactions can safely be neglected when they are in the ground state. In such a case, mechanical interaction between atoms can be avoided, and the internal degree of freedom of atoms can be decoupled

from atomic motion. A typical ensemble of cold Rb atoms confined in an elongated trap on the atomchip possesses a transverse extension of about 1 μm and a length of up to several millimeters. In this situation, the variation of the microwave field of the resonator is neglectable on these length scales. If the atomic clouds are fixed parallel to the transmission line resonator, we can neglect the change in the microwave field over each atomic cloud and assume uniform coupling constants for all the atoms in each ensemble. For a transmission line cavity with the strip-line length L and effective dielectric constant ϵ_r, the frequency of the mth standing-wave mode of the cavity is $\omega_c = \pi m c / L \sqrt{\epsilon_r}$. The coupling between the atoms and the cavity mode can be estimated as $g \sim D \sqrt{\frac{2\hbar\omega_c}{\pi^2 h^2 L}}$, where D represents the dipole matrix element and h is the height for the atoms above the cavity surface. For an atomic ensemble of $N \sim 10^6 - 10^8$ ^{87}Rb atoms trapped several μm above the surface of a stripline cavity, a collective coupling strength of $\sqrt{N}g/2\pi \sim 400$ kHZ can be obtained, which dominates the cavity decay $\kappa/2\pi \sim 7$ kHZ. We choose the coupling strength for the photon-atom interaction in each ensemble as $g_1 \sim g$, $g_2 \sim 1.5g$, and the detuning $\Delta_0 \sim 10g$. For the hyperfine transition between $|F = 1, m_F\rangle$ and $|F = 2, m_F\rangle$ states at a frequency of $\nu_a = \omega_a/2\pi = 6.83$ GHz, the frequencies of the classical Raman field and the cavity mode can be $\omega_0 \sim \nu_a - \Delta_0$, $\nu_0 \sim \nu_a - \Delta_0$. With the given parameters, one can obtain the angular frequency $\Theta/2\pi \sim 90$ kHZ, and the time to prepare the squeezed state $T_\pi \sim 5\mu$s (less than $1/\kappa \sim 25\mu$s). The readout this squeezed state can be accomplished by matter-light state mapping techniques. Employing another electronic excited state to establish two distinct Raman transition channels via optical fields between the two ground states, the states of ensembles 1 and 2 can be transferred to propagating light outputs, which can be detected via the standard homodyne detection.

It is worth emphasizing that, though the cavity mode plays a fundamental role in establishing the entanglement between the atomic modes, its initial state does not affect the efficiency of the process. Therefore, if the cavity mode is in a thermal distribution initially, the above discussion still holds. So this scheme is robust against the effect of thermal photons, which is a distinct feature compared to the scheme proposed in Ref.[3]. Compared to other protocols for producing two-mode squeezed states of two distant atomic clouds based on electric-dipole coupling between atoms and fields in cavity QED, this scheme, utilizing strong magnetic coupling of atomic spins to cavity mode, has the advantage of being immune to charge noise and could constitute qubits with much longer coherence times. Moreover, this proposal can be extended to implement scalable on-chip quantum networks, as entangled distant atomic ensembles are the building blocks for quantum networks.

To conclude, we have proposed an efficient scheme to generate two-mode squeezed states of two separated cold atomic clouds coupled to a superconducting transmission line resonator. It is based on the strong magnetic coupling of

the cold atoms to the cavity mode, and has the advantage of scalability and integration.

References

[1] S. L. Braunstein, P. van Loock, Rev. Mod. Phys. 77 (2005) 513.

[2] H. J. Kimble, Nature (London) 453 (2008) 1023.

[3] A. S. Parkins, E. Solano, J. I. Cirac, Phys. Rev. Lett. 96 (2006) 053602.

[4] Shi-Biao Zheng, Zhen-Biao Yang, Yan Xia, Phys. Rev. A 81 (2010) 015804.

[5] For a review, see M.Wallquist, K. Hammerer, P. Rabl, M. Lukin, P. Zoller, Phys. Scr., T 137 (2009) 014001.

[6] A. S. Sorensen, C. H. van der Wal, L. I. Childress, M. D. Lukin, Phys. Rev. Lett. 92 (2004) 063601.

[7] L. Tian, P. Rabl, R. Blatt, P. Zoller, Phys. Rev. Lett. 92 (2004) 247902.

[8] J. Verdu, H. Zoubi, C. Koller, J. Majer, H. Ritsch, J. Schmiedmayer, Phys. Rev. Lett. 103 (2009) 043603.

[9] D. Petrosyan, M. Fleischhauer, Phys. Rev. Lett. 100 (2008) 170501.

[10] A. Imamoglu, Phys. Rev. Lett. 102 (2009) 083602.

[11] J. H. Wesenberg, A. Ardavan, G. A. D. Briggs, J. J. L. Morton, R. J. Schoelkopf, D. I. Schuster, K. Molmer, Phys. Rev. Lett. 103 (2009) 070502.

[12] A. Andre, D. DeMille, J.M. Doyle,M. D. Lukin, S. E. Maxwell, P. Rabl, R. J. Schoelkopf, P. Zoller, Nat. Phys. 2 (2006) 636.

[13] P. Rabl, D. DeMille, J.M. Doyle,M. D. Lukin, R. J. Schoelkopf, P. Zoller, Phys. Rev. Lett. 97 (2006) 033003.

[14] K. Tordrup, A. Negretti, K. Molmer, Phys. Rev. Lett. 101 (2008) 040501.

[15] Jens Siewert, Tobias Brandes, Giuseppe Falci, Opt. Commun. 264 (2006) 435.

[16] Peng-Bo Li, Fu-Li Li, Phys. Rev. A 81 (2010) 035802.

[17] David Petrosyan, Guy Bensky, Gershon Kurizki, Igor Mazets, Johannes Majer, Jorg Schmiedmayer, Phys. Rev. A 79 (2009) 040304.

[18] Karl Tordrup, Klaus Molmer, Phys. Rev. A 77 (2008) 020301.

[19] P. Rabl, P. Zoller, Phys. Rev. A 76 (2007) 042308.

[20] J. Twamley, S. D. Barrett, Phys. Rev. B 81 (2010) 241202.

[21] Guido Burkard, Atac Imamoglu, Phys. Rev. B 74 (2006) 041307.

[22] R. J. Schoelkopf, S. M. Girvin, Nature (London) 451 (2008) 664.

[23] P. Treutlein, P. Hommelhoff, T. Steinmetz, T. W. Hansch, J. Reichel, Phys. Rev. Lett. 92 (2004) 203005.

[24] Peng-Bo Li, Y. Gu, Q. H. Gong, Guang-can Guo, Chin. Phys. Lett. 26 (2009) 030301.

[25] S. Pirandola, S. Mancini, D. Vitali, P. Tombesi, Phys. Rev. A 68 (2003) 062317.

[26] T. Gasenzer, D. C. Roberts, K. Burnett, Phys. Rev. A 65 (2002) 021605(R).