Disentanglement and Decoherence without dissipation at non-zero temperatures

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Abstract

Decoherence is well understood, in contrast to disentanglement. According to common lore, irreversible coupling to a dissipative environment is the mechanism for loss of entanglement. Here, we show that, on the contrary, disentanglement can in fact occur at large enough temperatures T even for vanishingly small dissipation (as we have shown previously for decoherence). However, whereas the effect of T on decoherence increases exponentially with time, the effect of T on disentanglement is constant for all times, reflecting a fundamental difference between the two phenomena. Also, the possibility of disentanglement at a particular T increases with decreasing initial entanglement.

Entanglement, which describes correlations between two or more particles or subsystems, is an essential characteristic of quantum mechanics and plays a key role in all applications related to information science [1–4]. But entanglement is poorly understood, so here we attempt to learn more about it by carrying out an exact calculation for the simplest non-trivial system and we will contrast our results with those from an analogous calculation which we already carried out for decoherence. In common with decoherence (which can occur for just a single particle in a superposition state), entanglement can be destroyed by interaction with a dissipative heat bath. But, motivated by the fact that we have previously shown that decoherence can actually occur at non-zero temperatures T for vanishingly small dissipation [5, 6], our purpose here is to show how disentanglement is affected by T.

In a previous communication [7], which was concerned with comparison of entanglement measures, we considered an entangled system in the absence of a heat bath and at zero temperature. We now extend our analysis of this model to incorporate non-zero temperatures and we present an exact calculation showing that disentanglement can in fact occur in the absence of dissipation. As we emphasized previously [5, 6], the situation is like that for an ideal gas in that collisions (dissipation) are necessary to bring about an approach to equalibrium but weak enough so that they do not appear in the equation of state nor in the velocity distribution.

Before proceeding, we should perhaps remark that our method contrasts with the usual master equation approaches where, in general, one starts with an initially uncoupled quantum state, a free particle, say. Thus, the free particle is essentially at zero temperature with no cognizance of even the zero-point oscillations of the electromagnetic field. In addition, the initial state of the heat bath is in equilibrium at some temperature T but not coupled to the free particle. Next, the free particle and heat bath are brought into contact and, as we have shown explicitly [11], the free particle receives an initial impulse with the result that the center of the wave packet drifts to the origin. But, since for a free particle the origin cannot be a special point, we see that the translational invariance of the problem is broken by the assumption that the initial state corresponds to an uncoupled system. This problem exists in so-called "exact" master equation formulations, which are exactly only in the sense that they incorporate time-dependent coefficients but they suffer from the same defects as the more conventional master equations; in fact, the same results arise more easily from the use of the initial value Langevin equation which enabled us to obtain solutions of these "exact" master equations in a much more simplified form than one finds in the literature [11].

As in [7], we consider two free particles, each of mass m, at positions x_1 and x_2, in an initially entangled Gaussian state, but we extend our previous analysis to allow both particles 1 and 2 to have velocities v_1 and v_2, respectively, which we will eventually take to be the random velocities associated with a bath at temperature T. Thus, we are dealing with a system with continuous degrees of freedom applicable to particle position or momenta or to the field modes of light (of interest in connection with linear optical quantum computing).

The most general initial Gaussian wave function that is symmetric in the two particles has the form

$$\psi(x_1, x_2; 0) = \frac{(a_{11}^2 - a_{12}^2)^{1/4}}{\sqrt{2\pi}} \exp\left\{ -\frac{a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{11}x_2^2}{4} + i\frac{m}{\hbar}\left(v_1x_1 + v_2x_2\right) \right\}. \quad (1)$$

In order that this state be square-integrable we must of course assume that a_{11} is positive and that $a_{11}^2 - a_{12}^2 > 0$. With this wave function we find the following expressions for the initial correlations

$$\begin{aligned}
\left\langle x_1^2(0)\right\rangle &= \left\langle x_2^2(0)\right\rangle = \frac{a_{11}}{a_{11}^2 - a_{12}^2}, \\
\left\langle x_1(0)\, x_2(0)\right\rangle &= -\frac{a_{12}}{a_{11}^2 - a_{12}^2}, \\
\left\langle p_1^2(0)\right\rangle &= m^2 v_1^2 + \frac{\hbar^2}{4} a_{11}, \\
\left\langle p_2^2(0)\right\rangle &= m^2 v_2^2 + \frac{\hbar^2}{4} a_{11}, \\
\left\langle p_1(0)\, p_2(0)\right\rangle &= \frac{\hbar^2}{4} a_{12}, \\
\frac{\left\langle x_1(0)\, p_1(0) + p_1(0)\, x_1(0)\right\rangle}{2} &= \frac{\left\langle x_2(0)\, p_2(0) + p_2(0)\, x_2(0)\right\rangle}{2} = 0, \\
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\end{aligned}$$

These results are standard quantum mechanics. Next, we consider an ensemble of particles in thermal equilibrium at a temperature T, and so we regard v_1 and v_2 as random velocities generated by thermal motion. Also, we consider that the particles are so weakly coupled to a heat bath that we can neglect dissipation in the equation of motion. In order to ensure that a normalizable thermal state exists for our field - free Hamiltonian, we first put the two particles in an oscillator potential and later take the limit of negligibly small oscillator frequency. Noting that the initial correlations have no linear terms in the velocities but simply have quadratic terms, we thus obtain the corresponding expressions by averaging over our thermal distribution of initial velocities such that

$$v_1^2 \to \frac{kT}{m}, \quad v_2^2 \to \frac{kT}{m}. \tag{3}$$

With this in the expressions (2) we have

$$\left\langle p_1^2(0)\right\rangle = \left\langle p_2^2(0)\right\rangle = mkT + \frac{\hbar^2}{4}a_{11}. \tag{4}$$

To obtain the time correlations at time t, we introduce the time-dependent (Heisenberg) operators:

$$\begin{aligned} x_1(t) &= x_1(0) + \frac{p_1(0)}{m}t, \quad p_1(t) = p_1(0), \\ x_2(t) &= x_2(0) + \frac{p_2(0)}{m}t, \quad p_2(t) = p_2(0). \end{aligned} \tag{5}$$

With this it is a simple matter to construct the correlations

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Next, we address the question of entanglement. Since we are dealing with a Gaussian state, we can use the necessary and sufficient condition of Duan et al. [9]. A zero-mean Gaussian state is fully characterized by its second moments which, for the symmetric case, can be represented by the following variance (correlation) matrix [9, 10]

$$\mathbf{M} = \begin{pmatrix} \mathbf{G} & \mathbf{C} \\ \mathbf{C} & \mathbf{G} \end{pmatrix}, \tag{7}$$

where

$$\mathbf{G} = \begin{pmatrix} \frac{\left\langle x_1^2(t)\right\rangle}{L^2} & \frac{\langle x_1(t)p_1(t)+p_1(t)x_1(t)\rangle}{2\hbar} \\ \frac{\langle x_1(t)p_1(t)+p_1(t)x_1(t)\rangle}{2\hbar} & \frac{L^2\left\langle p_1^2(t)\right\rangle}{\hbar^2} \end{pmatrix},$$

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and L is a constant of dimension length introduced to make the matrix elements dimensionless.

In order to discuss entanglement, Duan et al. perform a sequence of rotations and squeezes to bring $\mathbf{M}$ to a form in which

$$\mathbf{G} = \begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} c & 0 \\ 0 & c' \end{pmatrix}. \tag{9}$$

Since the determinants are invariant under these transformations, we have the following simple relations for determining the quantities g, c and c' in terms of these invariants.

$$\det\mathbf{G} = g^2, \quad \det\mathbf{C} = cc', \quad \det\mathbf{M} = \left(g^2 - c^2\right)\left(g^2 - c'^2\right). \tag{10}$$

With the expressions (6) for the correlations we find

$$\det\mathbf{G} = \frac{\left(a_{11} + \frac{4mkT}{\hbar^2}\right)a_{11}}{4\left(a_{11}^2 - a_{12}^2\right)},$$

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Putting these in (10) and solving, we find

$$g = \frac{1}{2}\sqrt{\frac{\left(a_{11} + \frac{4mkT}{\hbar^2}\right)a_{11}}{\left(a_{11}^2 - a_{12}^2\right)}},$$

$$c = \frac{|a_{12}|}{2}\sqrt{\frac{a_{11} + \frac{4mkT}{\hbar^2}}{\left(a_{11}^2 - a_{12}^2\right)a_{11}}},$$

$$c' = -\frac{a_{11}\,|a_{12}|}{2\sqrt{\left(a_{11}^2 - a_{12}^2\right)\left(a_{11} + \frac{4mkT}{\hbar^2}\right)a_{11}}}. \tag{12}$$

Duan et al have obtained a necessary and sufficient condition that a Gaussian state is separable. In terms of these quantities their condition is equivalent to the inequality

$$\left(g - c\right)\left(g - c'\right) \geq \frac{1}{4}. \tag{13}$$

With the expressions (12) this becomes

$$\frac{a_{11} - |a_{12}| + \frac{4mkT}{\hbar^2}}{a_{11} + |a_{12}|} \geq 1, \tag{14}$$

so that

$$|a_{12}| \leq \frac{2mkT}{\hbar^2}. \tag{15}$$

It should be emphasized that this condition for distanglement is independent of time. This is in stark contrast with the corresponding result for decoherence where the temperature effect increases exponentially to the power of t^2 [5]. Moreover, if $|a_{12}| > \frac{2mkT}{\hbar^2}$, the system remains entangled for all time.

Our conclusion is that whereas decoherence and disentanglement always occur for the same system in the presence of dissipation, this is not so for neglible dissipation at temperatures such that $|a_{12}| > 2mkT/\hbar^2$, in which case decoherence still occurs but disentanglement does not. It is clear that they are very different phenomena but, whereas decoherence is well understood, the opposite is true for disentanglement.

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