International Journal of Modern Physics D


# Undecidability as solution to the problem of measurement: fundamental criterion for the production of events

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Received August 4 2010
Revised Day Month Year
Communicated by Managing Editor

In recent papers we put forth a new interpretation of quantum mechanics, colloquially known as “the Montevideo interpretation”. This interpretation is based on taking into account fundamental limits that gravity imposes on the measurement process. As a consequence one has that situations develop where a reduction process is undecidable from an evolution operator. When such a situation is achieved, an event has taken place. In this paper we sharpen the definition of when and how events occur, more precisely we give sufficient conditions for the occurrence of events. We probe the new definition in an example. In particular we show that the concept of undecidability used is not “FAPP” (for all practical purposes), but fundamental.

## 1. Introduction

The problem of measurement in quantum mechanics arises in standard treatments as the requirement of an external process called the reduction process when a measurement takes place. Such process is not contained within the unitary evolution of the quantum theory but has to be postulated externally and is not unitary. It is usually justified through the interaction with a large, classical measuring device and an environment with many degrees of freedom. Attempts to formulate a coherent framework with purely quantum rules taking into account the environment have however failed to provide a consistent picture of quantum mechanics and the measurement process. Objections have been levied onto two aspects of the solution of the problem of measurement through decoherence. First of all, although a quantum system interacting with an environment with many degrees of freedom will very likely give the appearance that the initial quantum coherence of the system is lost, since the evolution of the system plus environment is unitary, that coherence could potentially be regained. This phenomenon is called “revival”, and although in practice it may take a very long time to arise, it exists as an issue of principle. It is always in principle possible to recover the information lost during the measurement process carrying out a measurement that includes the environment. The fact that such measurements are hard to carry out in practice does not prevent the issue from existing as a conceptual problem. The second criticism has to do with the fact that in a picture where evolution is unitary "nothing ever occurs". That is, one may have a way of characterizing the interaction with the environment that yields a density matrix with zero off diagonal elements, but one is left with a description of a set of coexistent options and not with a definite assignment of probabilities to alternative options, as one has after a measurement has taken place.

We have recently proposed a solution to the problem of measurement, leading to a new interpretation of quantum mechanics commonly known as "the Montevideo interpretation" [1]. The idea is that gravity fundamentally limits how accurate our measurements of space and time can be. This requires reformulating quantum mechanics in terms of real clocks and rods, that have errors in their measurements. The resulting picture of quantum theory is one where there is fundamental loss of quantum coherence: pure states evolve into mixed states. This eliminates the problem of revivals, just waiting longer does not improve things as more quantum coherence is lost. One also has an operational definition for when an event takes place: when the fundamental loss of coherence is such that one cannot distinguish the unitary evolution from a reduction, an event has taken place. We call this situation "undecidability" between reduction and unitary evolution.

A criticism that could be levied against our proposal is that there was no clear criterion given for when undecidability takes place. In particular, is the criterion supplied fundamental or is it "FAPP" (for all practical purposes). To analyze this in detail we will consider a model where the quantum system, the measuring apparatus and the environment are under control. This is the case of an example that we have already considered in this context and is a variation of a model proposed by Zurek in the context of decoherence. In section 2 we describe the model briefly, in section 3 we discuss undecidability and in section 4 we discuss a fundamental limit in the measurement of spins we will need to discuss the example. In section 5 we outline the sharp criterion for a production of an event and illustrate it with an example. We end with a discussion.

## 2. The model

In a previous paper[2] we introduced a model of decoherence in order to study the appearance of undecidability and its implications for the measurement problem. The model is a variation of a model presented by Zurek[3]. Here we outline some of the results in order to make this paper self contained.

The model consists of a spin $S$ located in the center of a chamber with a magnetic field $B$ pointed in the $z$ direction. Into the chamber flow, one by one, a set of $N$ "environmental" spins. The interaction Hamiltonian for the $k$-th spin of the environment is,

$$\hat{H}_k = \hat{H}_k^B + \hat{H}_k^{\text{int}}, \tag{1}$$

with,

$$\hat{H}_k^B = \gamma_1 B \hat{S}_z \otimes \hat{I}_k + \gamma_2 B \hat{I} \otimes S_z^k, \tag{2}$$

and

$$\hat{H}_k^{\text{int}} = f_k \left( \hat{S}_x \hat{S}_x^k + \hat{S}_y \hat{S}_y^k + \hat{S}_z \hat{S}_z^k \right), \tag{3}$$

where $\gamma_1$ y $\gamma_2$ are the magnetic moments of the central and environment spins respectively and the $\hat{S}$ are spin operators.

Starting with the initial state,

$$|\Psi\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \bigotimes_k^N (\alpha_k|\uparrow\rangle_k + \beta_k|\downarrow\rangle_k) \tag{4}$$

one can see that in the limit of weak couplings ($f_k \ll B\gamma_1, B\gamma_2$) one has decoherence in the $z$ basis, yielding a state after the passage of $N$ spins,

$$\begin{aligned} |\Psi(t)\rangle = a|\uparrow\rangle \prod_{k=1}^N \otimes \left[ \alpha_k \exp\left( i\int_0^\tau dt f_k \right) |\uparrow\rangle_k + \beta_k \exp\left( -i\int_0^\tau dt f_k \right) |\downarrow\rangle_k \right] \\ +b|\downarrow\rangle \prod_{k=1}^N \otimes \left[ \alpha_k \exp\left( -i\int_0^\tau dt f_k \right) |\uparrow\rangle_k + \beta_k \exp\left( i\int_0^\tau dt f_k \right) |\downarrow\rangle_k \right]. \end{aligned} \tag{5}$$

where $\tau$ is the time of flight of the environment spins passing through the chamber. The reduced density matrix becomes,

$$\hat{\rho}_S = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \underset{N\gg 1}{\longrightarrow} \begin{pmatrix} |a|^2 & \sim 0 \\ \sim 0 & |b|^2 \end{pmatrix}. \tag{6}$$

This implies that from the point of view of local observables the system behaves *almost* as if it were in one of the possible states and no local experiment allows to check if it is in a quantum superposition or not. With those types of measurements it therefore becomes increasingly more difficult to check if evolution was unitary or a collapse of the wavefunction has taken place. This fact is sometimes used to argue that this effect provides a solution to the measurement problem. However, as noted in [2], there exists observables of global nature, for instance one proposed by d'Espagnat [4], whose expectation value is different depending on if collapse has or has not taken place. It is given by,

$$\hat{M} \equiv \hat{S}_x \otimes \prod_k^N \hat{S}_x^k. \tag{7}$$

One has that $\langle\hat{M}\rangle_{collapse} = 0$ whereas,

$$\langle\psi|M|\psi\rangle = ab^* \prod_k^N \left[\alpha_k\beta_k^* + \alpha_k^*\beta_k\right] e^{-2i\Omega_k\tau} + a^* b \prod_k^N \left[\alpha_k\beta_k^* + \alpha_k^*\beta_k\right] e^{2i\Omega_k\tau} \neq 0, \quad (8)$$

with $\Omega_k \equiv \sqrt{4f_k^2 + B^2(\gamma_1 - \gamma_2)^2}$ and $\tau$ is the time of flight of the environmental spins through the chamber. One therefore has the possibility of determining experimentally if a collapse has taken place or if the system remains in a superposition that behaves classically only when probed with local observables.

## 3. Undecidability and the FAPP problem

As we discussed in the introduction, the notion of undecidability arises due to the fundamental limitations in the measurement of times and lengths that gravity imposes on us. When one takes into account that the Newtonian time $t$ that appears in the Schrödinger equation is really not observable (it is an ideally classical quantity whereas all real clocks are quantum systems and the best one can do it to associate the eigenvalue $T$ of some quantum operators $\hat{T}$ to the measurement of time), one notes that the resulting evolution loses quantum coherence[5]. The rate at which coherence is lost depends on how good or bad the clock is. Using heuristic estimates[6] for the fundamental uncertainty of a clock one notes that off diagonal elements of the density matrices die off at a rate, $\exp\left(-\frac{2}{3}\omega_{nm}^2 T_{\rm P}^{4/3} T^{2/3}\right)$, where $\omega_{nm}$ is the Bohr frequency between levels $n$ and $m$ of an energy eigenbasis, $T$ is the time measured by the clock and $T_{\rm P} \sim 10^{-44}$s is Planck's time.

Taking this effect into account the expectation value of the observable $\hat{M}$ is,

$$\langle\hat{M}\rangle = ab^* e^{-i2N\Omega T} e^{-4NB^2(\gamma_1-\gamma_2)^2\theta} \prod_k^N \left[\alpha_k\beta_k^* e^{-16B^2\gamma_1\gamma_2\theta} + \alpha_k^*\beta_k\right] \quad (9)$$

$$+ba^* e^{i2N\Omega T} e^{-4NB^2(\gamma_1-\gamma_2)^2\theta} \prod_k^N \left[\alpha_k\beta_k^* + \alpha_k^*\beta_k e^{-16B^2\gamma_1\gamma_2\theta}\right] \quad (10)$$

where $\Omega \equiv B(\gamma_1 - \gamma_2)$, $\theta \equiv \frac{3}{2} T_{\rm P}^{4/3} \tau^{2/3}$, $\tau$ is the time of flight of the environment spins within the chamber and $T$ is the length of the experiment.

There exist a series of conditions for the experiment to be feasible that imply certain inequalities,

$$a) \qquad 1 < f\tau = \frac{\mu\gamma_1\gamma_2}{\hbar}\frac{\tau}{d^3}, \quad (11)$$

$$b) \qquad \Delta x \sim \sqrt{\frac{\hbar T}{m}}, \quad (12)$$

$$c) \qquad f \ll |B(\gamma_1 - \gamma_2)|, \quad (13)$$

$$d) \qquad \langle\hat{M}\rangle \sim \exp\left(-6NB^2(\gamma_1-\gamma_2)^2 T_{\rm Planck}^{4/3}\tau^{2/3}\right), \quad (14)$$

with $f$ the interaction energy between spins which was assumed constant through the cell, $\mu$ the permeability of the vacuum, $d$ the impact parameter of the spins of the environment, $m$ their mass, and $\Delta x$ the spatial extent of the environment particles.

Condition a) stems from ensuring that the coupling between spins is not too weak, in order for decoherence to occur; b) is to prevent the particles of the environment from dispersing too much and therefore making us unable to find them within the detectors at the end of the experiment; c) is the condition for decoherence to be in the $z$ basis, as we mentioned; d) is an estimation of the the expectation value of the observable when the effect of the real clock is taken into account. For details of the derivation of these conditions see our previous paper[2].

As can be seen, there is an exponentially decreasing factor that makes the expectation value of the observable tend to that of the case in which collapse occurs as $\tau$ and $N$ increase. One has that,

$$\langle \hat{M} \rangle \approx \langle \hat{M} \rangle_{collapse}. \tag{15}$$

¿From the previous discussion one can gather that as one considers environments with a larger number of degrees of freedom and as longer time measurements are considered, distinguishing between collapse and unitary evolution becomes harder. But can this be considered a fundamental claim? Could one not, repeating the experiment many times, distinguish one case from the other? Is such a construction only a solution "for all practical purposes" (FAPP)? A similar criticism could be levied in interpretations based on environmental decoherence, even ignoring the problem of "revivals" or of potential global observables that distinguish both cases. Since environmental decoherence effects make the off-diagonal elements of the density matrix small but non-zero, one could make the coherences apparent by repeating the experiment in question a large number of times.

In the next sections we will show that there exist fundamental uncertainties in the measurement of quantum observables that prevent one from distinguishing the presence of small values in the density matrix with the elements vanishing.

## 4. Fundamental limit on the measurement of spins

Following Brukner and Kofler[7], let us consider a device for measuring the spin in a given direction (for instance a Stern–Gerlach setup). If $L$ is the angular momentum of the device and $\theta$ the angle that indicates the direction it is measuring, the uncertainty principle implies that[a],

$$\Delta L \Delta \theta \geq \frac{\hbar}{2}. \tag{16}$$

The commutator between the angle and angular momentum operator is,

$$[\theta, L] = i\hbar \tag{17}$$

[a]For small angular uncertainties.

whereas the Hamiltonian of the measuring device may be modeled by a rigid rotator,

$$H = \frac{L^2}{2I}, \tag{18}$$

where $I \approx MR^2$ is its moment of inertia, $M$ its mass and $R$ its characteristic length.

The evolution equation for the operator $\theta(t)$ in Heisenberg's representation is

$$\frac{d\theta}{dt} = -i\frac{[\theta, H]}{\hbar} = \frac{L}{I}, \tag{19}$$

and,

$$\theta(\tau) = \theta(0) + \frac{L\tau}{I}. \tag{20}$$

Recalling that for two observables $A$ and $B$ we have that $\Delta A \Delta B \geq 1/2|\langle[A, B]\rangle|$, we obtain,

$$\Delta\theta(0)\Delta\theta(\tau) \geq \frac{\hbar\tau}{2I} \approx \frac{\hbar\tau}{MR^2}. \tag{21}$$

Therefore at least one of the observables $\theta(0)$ and $\theta(\tau)$ has a dispersion such that,

$$\Delta\theta \gtrsim \frac{1}{R}\sqrt{\frac{\hbar\tau}{M}}. \tag{22}$$

The parameters of the measuring device cannot take any given value. Special relativity bounds the characteristic size $R$, which cannot exceed the length light would travel in the time it takes the measurement, therefore $R \leq c\tau$, from where

$$\Delta\theta \gtrsim \sqrt{\frac{\hbar}{cMR}}. \tag{23}$$

General relativity adds another constraint, $R \geq 2GM/c^2$, since $R$ must be bigger than the Schwarzschild radius associated with the mass $M$. With this, we get,

$$\Delta\theta \gtrsim \frac{l_P}{R}, \tag{24}$$

where $l_P \equiv \sqrt{\hbar G/c^3} \approx 10^{-35}m$. If we take the radius of the observable universe as a characteristic length, $R \approx 10^{27}m$, we reach a fundamental bound on the measurement of the angle,

$$\Delta\theta \geq 10^{-62}. \tag{25}$$

So we see that from a very general quantum mechanical analysis together with bounds from special and general relativity we have a fundamental uncertainty in the measurements of angles. Let us see what consequences follow if we wish to measure the expectation value of the spin in the $z$ direction. If the spin's state is $\Phi = a|\uparrow\rangle + b|\downarrow\rangle$, then,

$$\langle S_z\rangle = |a|^2 - |b|^2. \tag{26}$$

So instead of measuring $S_z$ we really are measuring $S_z^{\Delta\theta} = S.\hat{n}$, con $\hat{n} = (\sin(\Delta\theta), 0, \cos(\Delta\theta))$. Then,

$$\langle S_z^{\Delta\theta}\rangle = \sin(\Delta\theta)\langle S_x\rangle + \cos(\Delta\theta)\langle S_z\rangle \approx \langle S_z\rangle + \Delta\theta(ab^* + a^*b) - \frac{(\Delta\theta)^2}{2}\langle S_z\rangle. \quad (27)$$

We therefore see that one has an error of order[b] $\Delta\theta$ (or $(\Delta\theta)^2$ depending on the initial state). It is worthwhile pondering why these angular errors do not average to zero since when one measures $\langle S_z\rangle$ one carries out the experiment several times. The point to emphasize is that the uncertainty we are talking about is different from that stemming from a procedure where there are random measurement errors. In such a case one would have a certain probability that the device measures a direction different from the desired one. In our case the experimental device has an intrinsic error $\Delta\theta$ which implies that one *cannot know* in which direction the measurement took place. An analogy would be the random errors and the perception errors in a measurement. Random errors can be minimized measuring many times. Perception errors cannot.

## 5. Solving the FAPP problem

Let us apply the result of the previous section to the observable $M$ we discussed in sections 2 and 3. We saw that its expectation value was, (14),

$$\langle\hat{M}\rangle \sim \exp\left(-6NB^2(\gamma_1 - \gamma_2)^2 T_{\rm Planck}^{4/3}\tau^{2/3}\right) \equiv e^{-K}. \quad (28)$$

Let us recall that to distinguish if there is collapse or not, one needs to distinguish $\langle\hat{M}\rangle$ from 0. However, according to the result of the previous section the observable will have an error that depends on $\Delta\theta$. If this error is larger than $\langle\hat{M}\rangle$, there would be no way of distinguishing collapse from a unitary evolution.

To compute the error let us recall the expression for the observable and lets add the uncertainty in the direction that is measured,

$$\hat{M}^{\Delta\theta} \equiv \hat{S}_x^{\Delta\theta} \otimes \prod_k^N \hat{S}_x^{k,\Delta\theta}, \quad (29)$$

and,

$$\hat{S}_x^{\Delta\theta} \approx \hat{S}_x + \Delta\theta\hat{S}_z. \quad (30)$$

Therefore, the observable we really measure will have a term $\hat{S}_x \otimes \prod_k^N \hat{S}_x^k$, a term $(\Delta\theta)^{N+1}\hat{S}_z \otimes \prod_k^N \hat{S}_z^k$, and cross terms of the form $(\Delta\theta)^n \otimes \prod_i^{N-n} \hat{S}_x^i \otimes \prod_j^n \hat{S}_z^j$. Each of the new terms containing powers of $\Delta\theta$ will add noise to the measurement of the observable. Let us define $E(\Delta\theta)$ as all the terms except $\hat{S}_x \otimes \prod_k^N \hat{S}_x^k$ and

[b]Up to now we are ignoring errors associated with the preparation of the initial state.

$(\Delta\theta)^{N+1}\hat{S}_z \otimes \prod_k^N \hat{S}_z^k$. We then get

$$\hat{M}^{\Delta\theta} \approx \hat{S}_x \otimes \prod_k^N \hat{S}_x^k + (\Delta\theta)^{N+1}\hat{S}_z \otimes \prod_k^N \hat{S}_z^k + E(\Delta\theta) = \hat{M} + (\Delta\theta)^{N+1}\hat{S}_z \otimes \prod_k^N \hat{S}_z^k + E(\Delta\theta). \tag{31}$$

Using (4) as initial state we have,

$$\langle \hat{M}^{\Delta\theta} \rangle \sim e^{-K} + (\Delta\theta)^N (|a|^2 - |b|^2) \prod_k^N (|\alpha_k|^2 - |\beta_k|^2) + \langle E(\Delta\theta) \rangle. \tag{32}$$

To demonstrate the occurrence of undecidability it is enough to focus on the error that goes like $(\Delta\theta)^N$, the terms in $\langle E(\Delta\theta)\rangle$ will only add further noise to the measurement. As can be seen, it could still happen, depending on the initial state, that the error we are concentrating on vanishes. In fact, in our previous paper[2] we saw that the state that is convenient for the experiment would be such that $|\alpha_k|^2 = |\beta_k|^2$. However, given the errors in measuring the components of the spin, it is also impossible to prepare perfect initial states. For instance instead of preparing $\alpha_k|\uparrow\rangle_k + \beta_k|\downarrow\rangle_k$ one would prepare $\alpha_k|\uparrow\rangle_k^{\Delta\theta} + \beta_k|\downarrow\rangle_k^{\Delta\theta}$, with $\left(|\uparrow\rangle_k^{\Delta\theta}, |\downarrow\rangle_k^{\Delta\theta}\right)$ eigenstates of the observable $S_z + \Delta\theta S_x$ (to first order in $\Delta\theta$). These are given by,

$$|\uparrow\rangle_k^{\Delta\theta} = |\uparrow\rangle_k + \frac{\Delta\theta}{2}|\downarrow\rangle_k, \qquad |\downarrow\rangle_k^{\Delta\theta} = |\downarrow\rangle_k - \frac{\Delta\theta}{2}|\uparrow\rangle_k \tag{33}$$

Therefore the prepared state is,

$$\alpha_k|\uparrow\rangle_k^{\Delta\theta} + \beta_k|\downarrow\rangle_k^{\Delta\theta} = \left(\alpha_k - \frac{\Delta\theta}{2}\beta_k\right)|\uparrow\rangle_k + \left(\beta_k + \frac{\Delta\theta}{2}\alpha_k\right)|\downarrow\rangle_k. \tag{34}$$

As can be seen the probability amplitudes are not exactly the wanted ones. Using these in (32) one gets,

$$\begin{aligned} \langle \hat{M}^{\Delta\theta} \rangle &\sim e^{-K} + \langle E(\Delta\theta) \rangle \qquad (35) \\ &+ (\Delta\theta)^N \Big(|a|^2 - |b|^2 + \Delta\theta(ab^* + a^*b)\Big) \prod_k^N \Big(|\alpha_k|^2 - |\beta_k|^2 + \Delta\theta(\alpha_k\beta_k^* + \alpha_k^*\beta_k)\Big). \end{aligned}$$

Even if one wishes to impose the optimal condition $|\alpha_k|^2 = |\beta_k|^2$, the preparation errors will lead to the observable $\hat{M}$ having an associated error of the order of $(\Delta\theta)^{2N}$,

$$\langle \hat{M}^{\Delta\theta} \rangle \sim e^{-K} \pm (\Delta\theta)^{2N} + \langle E(\Delta\theta) \rangle. \tag{36}$$

Using the conditions (11)-(14) one finds out that $K$ satisfies,

$$K \gg \frac{N^5 T_{\text{Planck}}^{4/3} \hbar^{20/3}}{m^4 (\gamma_1\gamma_2)^{8/3} \mu^{8/3}}. \tag{37}$$

So we have

$$e^{-K} \ll \exp\left[-\frac{N^5 T_{\text{Planck}}^{4/3} \hbar^{20/3}}{m^4 (\gamma_1\gamma_2)^{8/3} \mu^{8/3}}\right] \tag{38}$$

and given the strong dependence of $e^{-K}$ on $N$, it follows that[c]

$$e^{-K} \leq (\Delta\theta)^{2N} + \langle E(\Delta\theta)\rangle, \tag{39}$$

with which the result that one obtains with or without collapse differ less than the observable $\hat{M}$ error and therefore it is impossible to distinguish both cases experimentally.

Let us finally see how these considerations also make it impossible to check if the system is or not in a quantum superposition through local observables (as we discussed in section 3 the fundamental decoherence effect does not eliminate entirely the interference terms). From equation (5) for the final state of the system, we see that the decoherence factor that multiplies the interference term in the reduced density matrix is,

$$z \equiv \prod_k^N \left[\cos\left(2\int_0^\tau dt f_k\right) + i\left(|\alpha_k|^2 - |\beta_k|^2\right)\sin\left(2\int_0^\tau dt f_k\right)\right]. \tag{40}$$

Let us consider the ideal situation for the experiment where $|\alpha_k|^2 - |\beta_k|^2 = 0$. We have,

$$z \sim \cos\left(2\int_0^\tau dt f_k\right)^N. \tag{41}$$

This factor $z$ would appear on measurements of the observable $S_x$, as,

$$\langle S_x\rangle = z(ab^* + a^*b). \tag{42}$$

Now, due to the angular uncertainty in measurement, and the error in probabilities due to the preparation procedure, equations (27) and (34), we have:

$$\begin{aligned}\langle S_x^{\Delta\theta}\rangle &\approx \langle S_x\rangle + \Delta\theta\langle S_z\rangle \\ &\approx z\left(\left(a - \frac{\Delta\theta}{2}b\right)\left(b + \frac{\Delta\theta}{2}a\right)^* + CC\right) + \Delta\theta\left(|a|^2 - |b|^2 + \Delta\theta(ab^* + a^*b)\right).\end{aligned} \tag{43}$$

As can be seen, there is a factor proportional to $z$, which decreases with the size of the environment, and then an error of order $(\Delta\theta)^2$. Given the exponential dependence on $N$ of the coherences, they will clearly be smaller than the error for environments large enough. Therefore, we see that collapse cannot be distinguished from unitary evolution with local observables either.

## 6. Conclusions

We have shown that fundamental quantum noise in the preparation procedure and in the observables being measured prevents one from distinguishing between a collapsed state and an evolved state. This is done by noticing that even if one takes

[c]The behavior of $K$ as $N^5$ was reached by using inequalities (11)-(13) taking into consideration all aspects of this particular model, so it need not be the same for other models.

the measuring apparatus to occupy the whole universe, which would decrease its errors to a minimum, quantum uncertainties will completely blur the different outcomes. We show that undecidability ultimately occurs, even though one cannot define sharply when it occurs.

The proof consists of two parts. First, completing our previous results to show that global observables cannot be used to check whether collapse has occurred or not, from a fundamental point of view rather than with FAPP arguments. Second, we see that similar arguments also show that local observables cannot be used either, also for fundamental reasons.

The fact that undecidability can be established in a sharp way and not "for all practical purposes" only, allows to construct a realist interpretation of quantum mechanics based on the definition of event introduced. This offers further support for the "Montevideo" interpretation of quantum mechanics we outlined[1] in a previous paper.

## Acknowledgments

We wish to thank Časlav Brukner and Johannes Kofler for sharing their results prior to publication. This work was supported in part by grant NSF-PHY-0650715, funds of the Hearne Institute for Theoretical Physics, FQXi, CCT-LSU, Pedeciba and ANII PDT63/076.

## References

1. R. Gambini, L. P. Garcia-Pintos and J. Pullin, "Complete quantum mechanics: an axiomatic formulation of the Montevideo interpretation," arXiv:1002.4209 [quant-ph]; R. Gambini and J. Pullin, J. Phys. Conf. Ser. **174**, 012003 (2009) [arXiv:0905.4402 [quant-ph]]; R. Gambini and J. Pullin, "Free will, undecidability, and the problem of time in quantum gravity," arXiv:0903.1859 [quant-ph].
2. R. Gambini, L. P. G. Pintos and J. Pullin, Found. Phys. **40**, 93 (2010) [arXiv:0905.4222 [quant-ph]].
3. W. Zurek, Phys. Rev. **D26**, 1862 (1982).
4. B. d'Espagnat "Veiled reality", Addison Wesley, New York (1995).
5. R. Gambini, R. Porto and J. Pullin, Gen. Rel. Grav. **39**, 1143 (2007).
6. F. Károlhyázy, A. Frenkel, B. Lukács in "Quantum concepts in space and time" R. Penrose and C. Isham, editors, Oxford University Press, Oxford (1986); Y. J. Ng and H. van Dam, Annals N. Y. Acad. Sci. **755**, 579 (1995) [arXiv:hep-th/9406110]; Mod. Phys. Lett. A **9**, 335 (1994); G. Amelino-Camelia, Measurability Of Space-Time Distances In The Semiclassical 3415 (1994) [arXiv:gr-qc/9603014]; S. Lloyd, J. Ng, Scientific American, November (2004).
7. C. Brukner, J. Kofler, "Are there fundamental limits for observing quantum phenomena from within quantum theory?", arXiv:1009.2654 [quan-ph]