

Inequality reversal: effects of the savings propensity and correlated returns

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Abstract

In the last decade, a large body of literature has been developed to explain the universal features of inequality in terms of income and wealth. By now, it is established that the distributions of income and wealth in various economies show a number of statistical regularities. There are several models to explain such static features of inequality in an unifying framework and the kinetic exchange models, in particular, provide one such framework. Here we focus on the dynamic features of inequality. In the process of development and growth, inequality in an economy in terms of income and wealth follows a particular pattern of rising in the initial stage followed by an eventual fall. This inverted U-shaped curve is known as the Kuznets Curve. We examine the possibilities of such behavior of an economy in the context of a generalized kinetic exchange model. It is shown that under some specific conditions, our model economy indeed shows inequality reversal.

1 Introduction

The distributions of income and wealth have long been found to possess some robust and stable features independent of the specific economic, social and political conditions of the economies. Traditionally, the economists have preferred to model the left tail and the mode of the distributions of the workers'

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incomes with a log-normal distribution and the heavier right tail with a Pareto distribution. For incomes from assets, other functional forms have been used as models, e.g., the family of functions introduced by Camilo Dagum [1]. For a detailed survey of the distributions used to fit the income and wealth data see Ref. [2]. However, there have been several studies recently that argue that the left tail and the mode of the distribution fit well with the gamma distribution and the right tail of the distribution follows a power law [3]. It has been argued that this feature might be considered to be a natural law for economics [4]. There is yet another well known observation, represented by the so-called Kuznets Curve, that over time a growing economy shows a rise in inequality followed by the eventual fall [5].

The standard explanation of the Kuznets Curve [6,7] goes like the following. Suppose, there is an economically underdeveloped area A , one in which people live from pre-industrial farming, perhaps not even involved in much of a monetized market economy. They generate little wealth and less money. Then a small part of A , say B , undergoes initial industrialization generating more wealth and creating a small monetized market economy. Hence, A 's inequality as a whole increases because those who are in B , enjoy a larger amount of wealth and the rest still remain the same. However, as more and more industrialization takes place, a larger fraction of A 's people come into B and hence, inequality falls. It is clear that this explanation rests on the effects of migration and growth of an economy during the process of industrialization.

However, in this paper, we examine the possibility of such inequality reversals in a conservative kinetic exchange model of market economy where no growth or migration takes place. Angle *et al* [8] made one of the first attempts to explain the Kuznets Curve by a kinetic exchange model. They assumed two populations characterized by two different gamma distributions and by incorporating the effects of 'education' and 'purchasing power of money', they had shown the possibility of Kuznets Curve in such an economy. Here, we consider only a single population of constant size. A very simple binary trading process based on a micro-economic framework, is modeled. Later, we show that the same process essentially captures the idea of the market returns being correlated. Next, it is shown that this model is very general as it reproduces a number of basic ideal gas like market models in certain limits of its parameters and the model becomes useful as it allows us to examine the effects of the variations in the savings propensity and the correlation in market returns explicitly.

We then study the dynamic behavior of the steady state distributions of asset with changes in the savings propensity and the correlation parameter. We show that in this model economy, the incorporation of the above two phenomena can produce the 'inequality reversal' in terms of asset-holding.

This paper is organized as follows. In section 2, we propose a particular kind of binary trading process in a competitive market. In the next section, we propose a model which enables us to examine the effects of correlation in market returns when the agents trade in several markets simultaneously. It is shown that same binary trading process can be used to capture the effects of correlation in market returns as well. In section 4, we study the derived kinetic exchange model and in the next section, we derive the Kuznets Curve. Then follows a summary.

2 The Asset Exchange Equations

Borrowing the framework studied in Ref. [9], we consider an N -agent exchange economy. Each of the agents produce a single non-storable good and each of the goods is different from another. Money is treated as another commodity. However, the model is conservative in that money is neither created nor destroyed. No agent dies and none is born. Money facilitates transactions in this model economy. Time is discrete. The agents care for their future consumptions and hence they care about their savings in the current period as well. Each of these agents are endowed with an initial amount of money which we assume to be unity for every agent for simplicity. At each time step, two agents are chosen randomly and they carry out transactions according to their utility maximization principle in a general equilibrium set up i.e., prices are so determined that the demands of the goods are exactly matched by the supplies, clearing the markets. We assume that the parameters of the utility function can vary over time [9,10] reflecting the fact that the agents' preferences alter with time. Ref. [10] presented a similar framework in a different context.

Suppose agent 1 produces Q_1 amount of commodity 1 only and agent 2 produces Q_2 amount of commodity 2 only and the amounts of money in their possession at time t are $m_1(t)$ and $m_2(t)$ respectively. In this scenario, both of them would be willing to trade and buy the other good by selling a fraction of their own productions as well as with the money that they hold. Hence, at each time step there would be a net transfer of money from one agent to the other due to such trade. For notational convenience, we denote $m_i(t+1)$ as m_i and $m_i(t)$ as M_i (for $i = 1, 2$). We define the utility functions as follows. For agent 1, $U_1(x_1, x_2, m_1) = x_1^{\alpha_1} x_2^{\alpha_2} m_1^\lambda$ and for agent 2, $U_2(y_1, y_2, m_2) = y_1^{\beta_1} y_2^{\beta_2} m_2^\lambda$. The arguments in both of the utility functions are consumption of the first (i.e., x_1 and y_1) and second good (i.e., x_2 and y_2) and amount of money in their possession respectively. For simplicity, we normalize the sum of the powers to 1 i.e., $\alpha_1 + \alpha_2 + \lambda = 1$ and $\beta_1 + \beta_2 + \lambda = 1$. Let the market clearing prices be denoted by p_1 and p_2 . Note that money acts as the *numeraire* good. Hence, its price is unity. Now, we can define the budget constraints as follows. For

agent 1 the budget constraint is $p_1x_1 + p_2x_2 + m_1 \leq M_1 + p_1Q_1$ and similarly, for agent 2 the constraint is $p_1y_1 + p_2y_2 + m_2 \leq M_2 + p_2Q_2$.

Proposition: *In such a competitive market with binary trading between agents characterized by the above-mentioned Cobb-Dauglas type utility function, the asset exchange equations will contain two correlated random variables.*

Proof: Formally, agent 1's problem is to maximize $U_1(x_1, x_2, m_1) = x_1^{\alpha_1} x_2^{\alpha_2} m_1^\lambda$ subject to the budget constraint $p_1x_1 + p_2x_2 + m_1 = M_1 + p_1Q_1$ and for agent 2, the problem is to maximize $U_2(y_1, y_2, m_2) = y_1^{\beta_1} y_2^{\beta_2} m_2^\lambda$ subject to the constraint $p_1y_1 + p_2y_2 + m_2 = M_2 + p_2Q_2$.

Let us solve the problem for the first agent by using Lagrange multiplier technique.

$$\mathcal{L} = x_1^{\alpha_1} x_2^{\alpha_2} m_1^\lambda - \mu(p_1x_1 + p_2x_2 + m_1 - M_1 + p_1Q_1) \quad (1)$$

Equating the first derivatives (with respect to x_1 , x_2 , m_1 and μ) with zero, one can derive the demand functions of the first agent as the following.

$$x_1^* = \alpha_1 \frac{(M_1 + p_1Q_1)}{p_1}, \quad x_2^* = \alpha_2 \frac{(M_1 + p_1Q_1)}{p_2},$$

$$m_1^* = \lambda(M_1 + p_1Q_1).$$

Similarly for agent 2, the demand functions are

$$y_1^* = \beta_1 \frac{(M_2 + p_2Q_2)}{p_1}, \quad y_2^* = \beta_2 \frac{(M_2 + p_2Q_2)}{p_2},$$

$$m_2^* = \lambda(M_2 + p_2Q_2).$$

The market clearing conditions are $x_1^* + y_1^* = Q_1$ and $x_2^* + y_2^* = Q_2$ (i.e., demand is exactly matched by supply in both the markets). By substituting the values of x_1^* , x_2^* , y_1^* and y_2^* and by solving these two equations we get market clearing prices (\hat{p}_1, \hat{p}_2) where

$$\hat{p}_1 = \frac{(\lambda\alpha_1 + \beta_1(1 - \lambda)) M_1 + \beta_1 M_2}{\lambda Q_1(1 - \alpha_1 + \beta_1)}$$

and

$$\hat{p}_2 = \frac{\alpha_2 M_1 + ((1 - \lambda)\alpha_2 + \lambda\beta_2) M_2}{\lambda Q_1(1 - \alpha_1 + \beta_1)}.$$

By substituting (\hat{p}_1, \hat{p}_2) in the money demand equations, we get

$$m_1^* = \lambda M_1 + \frac{\lambda\alpha_1 + (1 - \lambda)\beta_1}{1 - \alpha_1 + \beta_1} M_1 + \frac{\beta_1}{1 - \alpha_1 + \beta_1} M_2$$

$$m_2^* = \lambda M_2(t) + \frac{\alpha_2}{1 - \alpha_1 + \beta_1} M_1 + \frac{\lambda \beta_2 + (1 - \lambda) \alpha_2}{1 - \alpha_1 + \beta_1} M_2. \quad (2)$$

Now, we denote m_i as $m_i(t + 1)$ and M_i as $m_i(t)$ (for $i = 1, 2$).

Note that by assuming $\alpha_i = \beta_i$ (for $i=1, 2$) one can derive the CC model [14] (see Ref. [9] for the derivation of the CC model). The above set of equations can be rewritten as

$$\begin{aligned} m_1(t + 1) &= \lambda m_1(t) + \theta_{11} m_1(t) + \theta_{12} m_2(t) \\ m_2(t + 1) &= \lambda m_2(t) + \theta_{21} m_1(t) + \theta_{22} m_2(t) \end{aligned} \quad (3)$$

by appropriately defining θ_{ij} s (for $i, j=1, 2$). One can very easily verify that the total amount of money remains conserved at each trading i.e.,

$$m_1(t + 1) + m_2(t + 1) = m_1(t) + m_2(t).$$

The presence of a positive savings propensity is evident in Eqn. 3. Assuming that α_i and β_i (for $i=1, 2$) are random variables (because of the time dependence of preference ordering), we see that the θ_{ij} s are correlated (for $i, j=1, 2$). Hence, the money transfer equations consists of two correlated random terms. \square

For simplicity we assume the θ_{ij} s to be correlated in the following form.

$$\begin{aligned} m_i(t + 1) &= \lambda m_i(t) + \omega_1(1 - \lambda)m_i(t) + (\alpha\omega_1 + (1 - \alpha)\omega_2) \\ &\quad (1 - \lambda)m_j(t) \\ m_j(t + 1) &= \lambda m_j(t) + (1 - \omega_1)(1 - \lambda)m_i(t) + (1 - \alpha\omega_1 - (1 - \alpha)\omega_2) \\ &\quad (1 - \lambda)m_j(t) \end{aligned} \quad (4)$$

where $\omega_1, \omega_2 \sim \text{uniform}[0, 1]$ and independent. Note that in the above formulation ω_1 and $(\alpha\omega_1 + (1 - \alpha)\omega_2)$ are the two correlated random terms. Later, in section 5, we shall see that the distributional assumptions of ω_1 and ω_2 will help us to calculate the moments of the resultant steady state distributions of money, very easily. The savings propensity and the degree of correlation between the stochastic terms are denoted by λ and α respectively and both can vary between 0 and 1. Technically, α is not the *correlation coefficient*. It is a term that helps us to tune the correlation between the two random terms.

While deriving the above model, we have assumed that the agents are producing only one commodity at every time-point. However, we can present the same model by incorporating the idea of risk-aversion explicitly where each agent

produces a vector of commodities. Below we discuss the notion of correlation in the returns from trading of several commodities simultaneously.

3 Correlated Markets

Let us begin with a simple calculation. Suppose, an agent invests a certain amount of money in K number of assets where the returns are stochastic. More precisely, let us assume that the returns (ϵ_k) are i.i.d. variables with finite mean (μ) and variance (σ^2) . The problem of the agent is to decide what fractions (f_k) of his money holding he would invest in each asset k for $k = 1, 2, \dots, K$. Assuming risk aversion, the problem is to minimize the variance of his portfolio $(\sum_k \epsilon_k f_k)$ or formally, the problem is to minimize

$$\sigma^2(\epsilon) (f_1^2 + f_2^2 + \dots + f_K^2)$$

subject to the condition that

$$f_1 + f_2 + \dots + f_K = 1.$$

Clearly, the solution would be $f_k^* = 1/K$ for all k . Now, consider the following set of stochastic difference equations representing how the money-holding changes among the agents over time.

$$\begin{aligned} m_i(t+1) &= \epsilon[m_i(t) + m_j(t)] \\ m_j(t+1) &= (1-\epsilon)[m_i(t) + m_j(t)] \end{aligned} \tag{5}$$

This framework had been borrowed from the statistical mechanics of the scattering process and numerous variations of Eqn. 5 have been studied in the literature, in the context of income and wealth distributions (see e.g., Ref. [4,11]). However, Ref. [9] presents a microeconomic model in which the same set of equations is obtained by the market-clearing trading process between the agents. There it had been assumed that each of the agents produced a single non-storable commodity and money acted as an asset that helps to make transactions (the same assumptions have been made in the last section while deriving the binary trading equations). However, we can generalize the situation assuming that each of the agents produce a vector of commodities and engage in trading with each other, then it is perfectly possible for a risk averse agent to diversify his money holding at time t following the above calculation, instead of putting all his money in trading of a single commodity. The market has the following structure. Each agent produces K ($K \geq 1$) number of commodities and each of these commodities is different from the

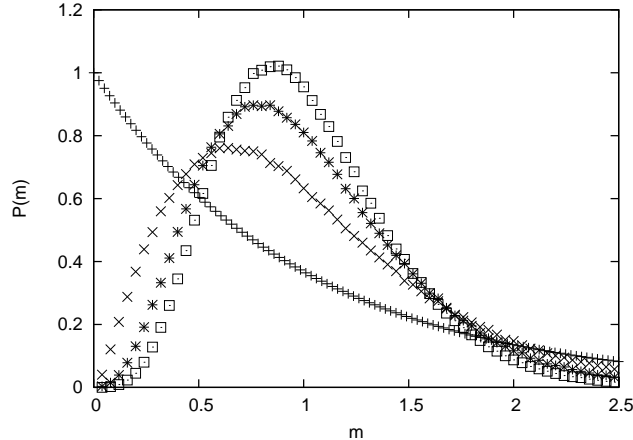


Fig. 1.

Money distribution among risk-averse agents. Four cases are shown above, viz., $K = 1$ (+), $K = 2$ (×), $K = 3$ (*), $K = 4$ (□). All simulations are done for $O(10^6)$ time steps with 100 agents and averaged over $O(10^3)$ time steps.

other. Hence, the agents would be willing to trade with each other. Ref. [9] deals with the case where $K = 1$ i.e., each agent produces a single commodity and it shows that Eqn. 5 captures the basic process of money exchange in such an economy. Here, we consider the case where $K \geq 1$. Clearly, the risk averse agents would diversify their money holding in order to minimize the risk from trading. The mode of trading is such that at each instant, two randomly chosen agents engage in trading each producing K number of different goods so that, in total, $2K$ number of goods are traded at each instant. For trading the k -th pair of goods ($k = 1, 2, \dots, K$), the i -th and the j -th agent uses m_i/K and m_j/K amounts of money respectively because we have already shown that the variance (risk) minimizing choice is to diversify equally among all assets. So the money transfer equations become the generalisation of Eqn. 5, viz.,

$$\begin{aligned}
 m_i(t+1) &= \frac{\sum_k \epsilon_k}{K} [m_i(t) + m_j(t)] \\
 m_j(t+1) &= \left(1 - \frac{\sum_k \epsilon_k}{K}\right) [m_i(t) + m_j(t)]
 \end{aligned}
 \tag{6}$$

for all possible integer values of K .

Corollary: If K is 1, then we get back Eqn. 5 which implies that the steady state distribution of money would be exponential. In the other extreme for $\lim K \rightarrow \infty$, by applying Lindeberg-Levy central limit theorem, we have

$$\left(\frac{\sum_k \epsilon_k}{\sqrt{K}} \right) \sim N(\mu, \sigma^2)$$

where μ and σ^2 are finite for uniformly distributed variables ϵ_k . This in turn

implies that

$$\frac{\sum_k \epsilon_k}{K} \sim N\left(\mu, \frac{\sigma^2}{K}\right).$$

Hence, the distribution is a Δ function at μ for large K . The resulting distribution of money would also be a Δ function i.e., perfect equality will be achieved. For finite values of K greater than unity, the distribution of money would resemble a gamma probability density function (see Fig. 1). \square

It may be noted that the assumption of the returns having the same mean and variance alongwith independence among themselves, is not very realistic. We relax the assumption of independence below but the assumption of the means and the variances being the same, is maintained throughout the paper as it helps to avoid unnecessary complications. Also note that the shifts in the distribution are discrete since the distribution alters only if the number of pairs of the different commodities traded (K) alters. This is because we have made a strong assumption that the returns viz., $\epsilon_1, \epsilon_2, \dots, \epsilon_K$ etc. are i.i.d. variables as has been mentioned above.

By allowing the returns to be correlated we can generate continuous shifts in the distributions instead of discrete jumps for $K = 1, 2, 3, \dots$ etc. For example, consider two specific cases where in the first case, there is only one pair of commodities and in the other, two pairs of commodities to be traded (more precisely, we assume $K = 1$ and 2 respectively in Eqn. 6). In the first case, the steady state distribution of money would be a pure exponential whereas in the second case, the distribution resembles a gamma pdf (see Fig. 1). By assuming zero correlation we can derive these two limits only. However, if we assume that the returns may be correlated then by varying the degree of correlation we can get continuous shifts. One noteworthy feature of this case, is that for a risk-averse agent the risk-minimizing choice would be to diversify equally even if there is any correlation among the random terms.

4 A Generalized Kinetic Exchange Model

However, it becomes very difficult to work with the Eqn. 6 since there are two parameters viz., K (the number of commodities traded) and the correlation among the returns. An even more troublesome issue is that as K changes, the equilibrium distribution jumps discontinuously. Hence, we simplify Eqn. 6 by assuming that there are only two simultaneous trading processes going on at each instant (that is $K = 2$, which essentially implies that there are only two random terms; recall the proposition stated earlier). We modify the model to incorporate the savings propensity and the correlation parameter explicitly in the trading equations in the following fashion.

$$\begin{aligned}
m_i(t+1) &= \lambda m_i(t) + \omega_1(1-\lambda)m_i(t) + (\alpha\omega_1 + (1-\alpha)\omega_2) \\
&\quad (1-\lambda)m_j(t) \\
m_j(t+1) &= \lambda m_j(t) + (1-\omega_1)(1-\lambda)m_i(t) + (1-\alpha\omega_1 - (1-\alpha)\omega_2) \\
&\quad (1-\lambda)m_j(t)
\end{aligned} \tag{7}$$

which is identical to Eqn. 4. Several points are to be noted.

- (a) If $\lambda = 0$ and $\alpha = 1$, then we have the very basic framework of ideal gas which gives rise to a purely exponential distribution (Gibbs distribution: $p(m) \sim e^{-m/T}$ with $T = 1$ in this case). See Ref. [4,12].
- (b) If $\lambda = 0$ and $\alpha = 0$, then we have a model with two uncorrelated stochastic terms. This model has been studied and solved in Ref. [13]. This model gives rise to a probability distribution characterized by a gamma probability density function of the form $p(m) \sim 4me^{-2m}$.
- (c) If $\lim \lambda \rightarrow 1$, then the distribution would be a delta function.
- (d) If only $\alpha = 1$, the above model reduces to the so-called CC model [14] which gives rise to gamma-function like behavior.
- (e) If only $\alpha = 0$, then we have a new model which has savings propensity (CC model) and two uncorrelated random terms (see point (b) above).

5 Inequality Reversal

Inequality can be measured by a number in indices. However, the most useful one in this case is simply the coefficient of variation which is basically the standard deviation of the distribution normalized by the expectation. The economy is modelled in such a way that the expectation is always set equal to unity (recall that all agents are initially endowed with unit amount of money and the economy under study is a conserved one i.e., money is neither created nor annihilated in this economy). We can calculate the moments recursively (see Ref. [15] for more on finding the moments). We consider the i -th agents money evolution equation only.

$$m_i(t+1) = [\lambda + \omega_1(1-\lambda)]m_i(t) + [\alpha\omega_1 + (1-\alpha)\omega_2](1-\lambda)m_j(t) \tag{8}$$

Lemma 1: $\langle m \rangle = 1$.

Proof: By taking expectation over both sides of Eqn. 8, we get

$$\langle m_i(t+1) \rangle = [\lambda + \frac{1}{2}(1-\lambda)]\langle m_i(t) \rangle + \frac{1}{2}(1-\lambda)\langle m_j(t) \rangle.$$

Note that $\langle m_j(t) \rangle = \sum_{j=1}^N m_j(t)/N = 1$. Since the expected money-holding is free of the time index, the result readily follows. \square

Lemma 2:

$$\Delta m = \frac{2(1-\lambda) \left[\frac{\alpha(1-\lambda)}{3} + \frac{\lambda}{2} + \frac{(1-\lambda)(1-\alpha)}{4} \right]}{1-z} - 1$$

where $z = (1-\lambda)^2 \left(\frac{1}{3} + \frac{\lambda}{(1-\lambda)^2} + \frac{\alpha^2 + (1-\alpha)^2}{3} + \frac{\alpha(1-\alpha)}{2} \right)$.

Proof: It follows from the definition of variance that

$$\Delta m_i = \langle m_i^2 \rangle - (\langle m_i \rangle)^2$$

where m_i is given by Eqn. 8 and Δ stands for the second central moment (variance). By substituting m_i in the *l.h.s.* of the expression of variance and noting that $\langle m \rangle = 1$, we get

$$\Delta m_i(t+1) = \langle [(\lambda + \omega_1(1-\lambda))m_i(t) + (\alpha\omega_1 + (1-\alpha)\omega_2)(1-\lambda)m_j(t)]^2 \rangle - 1.$$

Here, we use the fact that in the steady state, the variance of the distribution should be free of the time and the agent indices. Also, since $\omega_i \sim$ uniform $[0,1]$, $\langle \omega_i \rangle = 1/2$ and $\Delta \omega_i = 1/12$ (for $i=1, 2$). On simplification, we get

$$\Delta m = z(\Delta m + 1) + 2(1-\lambda)\langle (\lambda + \omega_1(1-\lambda))(\alpha\omega_1 + (1-\alpha)\omega_2) \rangle - 1$$

where $z = (1-\lambda)^2 \left(\frac{1}{3} + \frac{\lambda}{(1-\lambda)^2} + \frac{\alpha^2 + (1-\alpha)^2}{3} + \frac{\alpha(1-\alpha)}{2} \right)$. On further simplification, we get

$$\Delta m = \frac{2(1-\lambda) \left[\frac{\alpha(1-\lambda)}{3} + \frac{\lambda}{2} + \frac{(1-\lambda)(1-\alpha)}{4} \right]}{1-z} - 1 \quad (9)$$

where z is defined as above. \square

Clearly the variance is a function of λ and α only. Now, we make use of two observations. First, for a sustainable growth the savings propensity has to increase (see Ref. [16] for more on this topic). The second observation is that the modern markets are characterized by correlated returns with fluctuations [17] in the most efficient state [18]. The implications are that both λ and α increases over time unidirectionally. By plugging different values of λ and α in the expression of variance, one can find how inequality changes over time with increases in the parameters. Note that since the parameters are ranging between 0 and 1, the parameter space is a square with unit length (Fig. 3 shows the relevant region). Now, we assume that the path followed by the

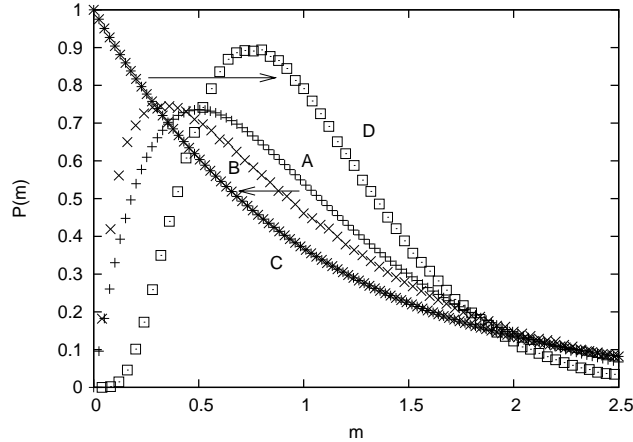


Fig. 2.

Steady state distributions of money for different values of the parameters. Curve A: $\lambda = 0$ and $\alpha = 0$. Curve B: $\lambda = 0$ and $\alpha = 0.7$. Curve C: $\lambda = 0$ and $\alpha = 1$. Curve D: $\lambda = 0.5$ and $\alpha = 1$. As the correlation goes up the distribution becomes more skewed to the left (from A to B to C; see the arrow). Then as the savings propensity goes up, it moves in the opposite direction (from C to D; see the arrow). All simulations are done for $O(10^6)$ time steps with 100 agents and averaged over $O(10^3)$ time steps.

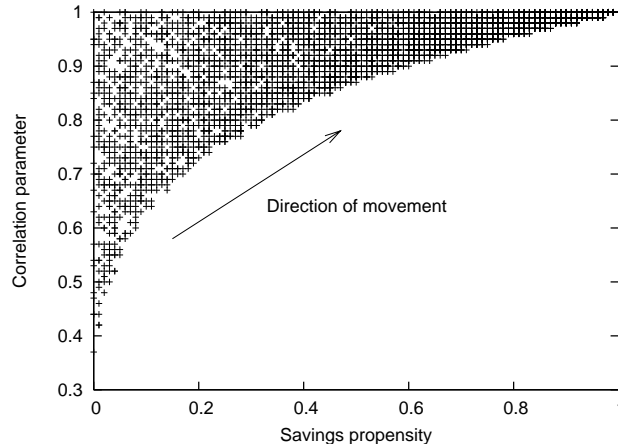


Fig. 3.

The (λ, α) parameter space. If the economy moves through the shaded region (the region above the curve $\alpha = \lambda^{1/5}$) then it shows inequality reversal.

economy starts from the origin ($\lambda = 0, \alpha = 0$) and ends at ($\lambda = 1, \alpha = 1$). The simplest functional relationship between λ and α satisfying the above assumption is of the form

$$\alpha = \lambda^{\frac{1}{\tau}} \quad (10)$$

where τ is a positive number. It is numerically seen that for $\tau \geq 5$, the economy shows a very prominent inequality reversal (see Fig. 4). It should be noted,

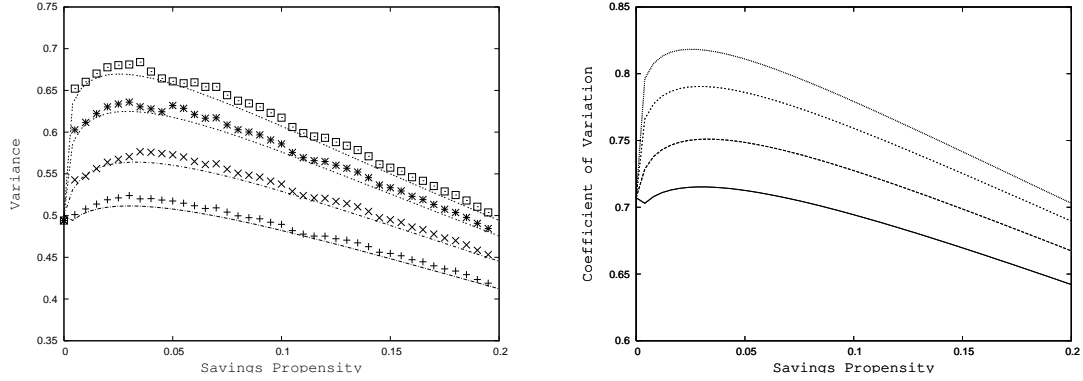


Fig. 4.

Left panel: The changes in the variance of the distribution (Δm) is shown with changes in the savings propensity (λ) (from below, $\tau=5, 7, 10$ and 13). The Monte Carlo simulation results agree with the theoretical curves (dotted lines) obtained from the Eqn. 9 and 10. *Right panel:* The Kuznets Curve in terms of the Coefficient of Variation i.e., $\sqrt{\Delta m}$ (from below, $\tau=5, 7, 10$ and 13): inequality increases and then falls.

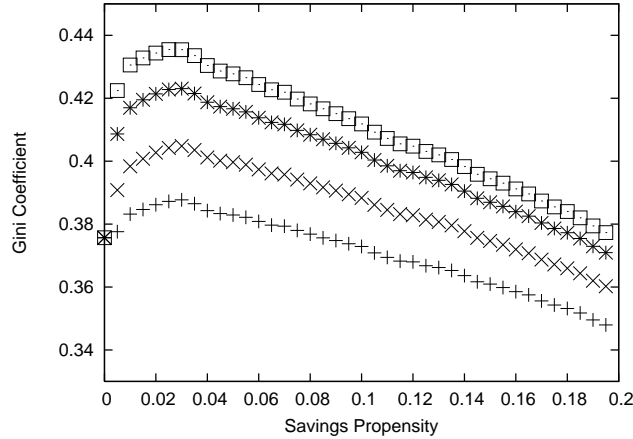


Fig. 5.

The Kuznets Curve in terms of the Gini concentration ratio (from below, $\tau=5, 7, 10$ and 13): inequality increases and then falls. All Monte Carlo simulations are done for $O(10^5)$ time steps with 100 agents and averaged over $O(10^2)$ time steps.

however, that if the economy follows some other paths in the parameter space, then it may show other types of behavior as well.

For the sake of completeness, we also provide Monte Carlo simulation results of the Kuznets Curves where the inequality is measured in terms of the Gini concentration ratio. The definition of the measure G is the following [2].

$$G \equiv \frac{\sum_{i=1}^N \sum_{j=1}^N |m_i - m_j|}{2\mu N(N-1)} \quad (11)$$

where N is the number of agents (which is set to 100), μ is the money per agent (which is set to unity) and m_i is the money holding of the i -th agent. Fig. 5 shows the rise and the subsequent fall in the Gini concentration ratio.

In its original formulation, the Kuznets Curve is a plot of income distribution with the changes in income per capita. However, the trading process that we have considered is a conservative one, implying that the average income in this model remains fixed over time. Instead of average income, we consider the changes in the savings propensity and the correlation among the markets and we trace the corresponding changes in the inequality in money-holding. In such cases, it is clearly seen that the economy shows Kuznets-type behavior.

6 Summary

The presence of inequality is a persistent phenomena in any economy. The static nature of inequality has been under investigation for more than a decade [3,4]. Here in this paper, we examine the dynamic aspects of inequality. It is seen that as an economy grows over time, its inequality first increases and then falls with further growth. This particular dynamic feature was first pointed out by Kuznets [6,7]. This observation has attracted considerable interests in Economics. In this paper, we have tried to explain the origin of such a phenomena in an appropriately modified kinetic exchange model (see Ref. [3,11] for recent reviews on kinetic exchange models).

First, following Ref. [9] we propose a basic binary trading process and we derive the corresponding money transfer equations as the competitive solution the trading action. Then we propose another model to show the effects of correlated returns on the equilibrium distributions of money. Here, we consider the case where each agent produces a number of commodities and hence, would diversify their money holding in trading of different commodities to minimize the risk (Sec. 3). Then we formulate a very general kinetic market model incorporating the effects of savings propensity and correlated returns (Sec. 4) explicitly. In different limits, this model gives rise to a number of kinetic exchange models viz., the Dragulescu-Yakovenko model [12], Chakraborti-Chakrabarti model [14] and yet another model proposed in Ref. [13]. It is observed that with the growth of an economy, the savings propensity of its people increases [16] and the efficient markets are characterized by correlated returns from trade [17,18]. Using these observations, we show that if the economy moves through a certain region in the parameter space then it shows inequality reversal as was found by Kuznets [6,7] (Sec. 5). The Monte Carlo simulation results are also explained by finding out the expression of the inequality index as well (Eqn. 9).

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