Terminal Value Parameters: A Short Note[#]

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Some textbooks and a just a couple of articles address the problems of evaluation of so called terminal value. The term terminal value describes the value of certain set of company's capital at the point in the time, from which the company is assumed to be stable in the terms of profit growth and investments or to converge by a stable rate to the "stable state". Much of the attention is concentrated rather on the estimation of cost of capital (see Vélez-Pareja – Burbano-Pérez, 2005). The "terminal value" is used in many two- (or more-) stage valuation formulas based on discounting an income nowadays and is of extreme importance in discounted dividend models discounted cash flow models and can significantly influence results of residual income (RIM) models (Courteau – Kao – Richardson, 2000). In this short note I address the problem of estimation of other terminal value parameters, mainly the income, investments and earnings.

Gordon's formula

For valuation of a project or a company the models based on discounting cash flows or earnings are quite commonly used. The most frequent forms are the one-stage and two-stage models. These can be written for example in the form of so-called Gordon's formula (one-stage models)[1]:

$$V_0 = f_1 \cdot \int_0^\infty e^{-t\cdot(r-g)} dt = \frac{f_1}{r-g}, \tag{1}$$

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[1] For the two-stage models (1) forms a part of value, called terminal value

where r = required return rate (interest rate); $\alpha \in \Re^{+}$,
f = flow of income; $f \in \Re$,
g = growth rate; $g \in \Re$,
t = time; $t \in \Re^{+}$.

The assumption behind (1) is that the income or cash flow grows by constant rate in the infinite time horizon. We want to analyze further only the part of discounted income formula, which deals with the infinite (growing) income flow. This part of project (or business) value called the terminal value usually accounts for more than 50 % of the whole value of the project. Taking into account the possibility that equation (1) is on the level of enterprise, so debt would be deducted, the leverage can also have a role magnifying the importance of proper terminal value estimation.

Properties of Income for the Terminal Value

One could write down long list of less or more consistent methods, used for estimation of the investments for so called "stabilized year income" (either cash flow or EBIT, EAT, etc.).[2] Practitioners describe the "stabilized year" as the last year of financial plan, i. e. the last year of explicit forecast of cash flow and as a basic point for estimation of income for the terminal value. The stabilization is achieved through adjustments of parts of income (earnings or cash flows) so it's structure is set consistently with assumption of the infinite constant rage growth. The main two approaches could be pointed at, which show signs of consistency in the process of estimation of the investments into fixed assets and working capital. The first one is based on a relationship between the growth rate of cash flow in the infinite time horizon and the other originates from the need for the constant growth of all the parts of income. We will shortly introduce both of them and describe their properties.

[2] Depends upon which method and it's variation we would like to use. In English-speaking countries, DCF methods are most commonly used.

The constantly growing parts of the income

The composition of income can be written down the following way:

$$f = (\alpha \cdot f) + (1-\alpha) \cdot f \,, \tag{2}$$

where α = multiplier; $\alpha \in \Re$.

This statement of cash flows composition, allows us to analyze impacts of assumptions about the properties of any parts of cash flows without loss of generality. Thus we can decompose cash flow into the analyzed part (e.g. investments or depreciation) and the rest of the cash flow.

Proposition I: *If income f is assumed to grow each period with constant rate, then for every $t_{1,}t_2 \in \Re; t_1 \neq t_2$*

$$f_2 = f_1 \cdot e^{g \cdot (t_2 - t_1)} \,, \tag{3}$$

so that $f_2 \neq f_1$. This holds if and only if both parts (2) of the income grow the same rate as the whole income.

Proof: Suppose that the right parts of (2) in the brackets grow each by different rate, mark the differences $h, j \in \Re$. Using (1) and (2) and substituting $\tau = t_1 - t_2$ we get

$$f \cdot e^{g \times \tau} = \alpha \cdot f \cdot e^{(g+h) \cdot \tau} + (1-\alpha) \cdot f \cdot e^{(g+j) \cdot \tau} \,, \tag{4}$$

which is nothing else than

$$1 = \alpha \cdot e^{h \cdot \tau} + (1-\alpha) \cdot e^{j \cdot \tau} \,. \tag{5}$$

Solving (5) for $\tau \in \{1,2\}$[3] we get

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[3] Each period τ can be divided into at least two equally long subsequent periods. If Proposition 1 would not hold for (1), then it should not hold for each and every two subsequent periods. We can shorten or prolong interest rate period and interest rate adequately to every possible case for which the equation is to be solved, so the whole-number parameters can be used without loss of generality.

$$1 = \alpha \cdot e^{h} + (1-\alpha) \cdot e^{j} \tag{6}$$

and

$$1 = \alpha \cdot e^{2h} + (1-\alpha) \cdot e^{2j} . \tag{7}$$

Applying natural logarithm to (6) and (7) and substituting (6) into (7) instead of h we get

$$2\ln\left[1-(1-\alpha)\cdot e^{j}\right] - 2\ln\alpha = \ln\left[1-(1-\alpha)\cdot e^{2j}\right] - \ln\alpha , \tag{8}$$

which after few rearrangements yields

$$2e^{j} - 1 = e^{2j} . \tag{9}$$

This holds only for $j = 0$. After substituting this result into (4), we get also $h = 0$, Q.E.D.

The Parametric Formula

So-called parametric formula is used to estimate the terminal value in discounted cash flow methods, however it could be used (in a modified form) for some other income methods of business valuation, either discounted dividends, or methods based on the long-term viable amount of income that can be paid out without endangering the going-concern assumption.[4]

Parametric formula is equation based on the following assumptions:

- The income is divided into two parts: investments and the rest of income (earnings), so that we can write down the composition of income also by equation (2). However, we need to specify, which part of (2) represents investments and which one represents earnings. Let us assume that α represents investments and $(1-\alpha)$ represents earnings.

[4] The income is appropriately adjusted so that necessary investments are deducted, extraordinary items are excluded, the salary of sole proprietor is deducted, etc.

- The assets used in the business $A \in \Re^{+}$ depreciate (their value is lower) by rate $\lambda \in \Re^{+}$, so that investments comprise of the investments replenishing the depreciated assets and part, determining whether investments are lower or higher than needed:

$$\alpha \cdot f = \lambda \cdot \alpha \cdot f + (1-\lambda) \cdot \alpha \cdot f, \tag{10}$$

so the net investments representing the intertemporal change in the amount of assets employed in the evaluated business are $\Delta A = \alpha \cdot f$.

- We are able to estimate the future growth of earnings

$$g = \frac{f_2 \cdot (1-\alpha)}{f_1 \cdot (1-\alpha)} - 1 = \frac{\Delta f \cdot (1-\alpha)}{f_1 \cdot (1-\alpha)}. \tag{11}$$

This specific way of estimation of the investments in the infinite time horizon is used for it's explicit relationship between return on investments

$$-r_\alpha = \frac{\Delta f \cdot (1-\alpha)}{\alpha \cdot f} \tag{12}$$

on one side and the growth rate of earnings on the other side. Then (2) can be rewritten in the following way

$$f = f \cdot (1-\alpha) \cdot \left(1 + \frac{\alpha}{1-\alpha}\right) = f \cdot (1-\alpha) \cdot \left(1 + \frac{g}{-r_\alpha}\right).^{5} \tag{13}$$

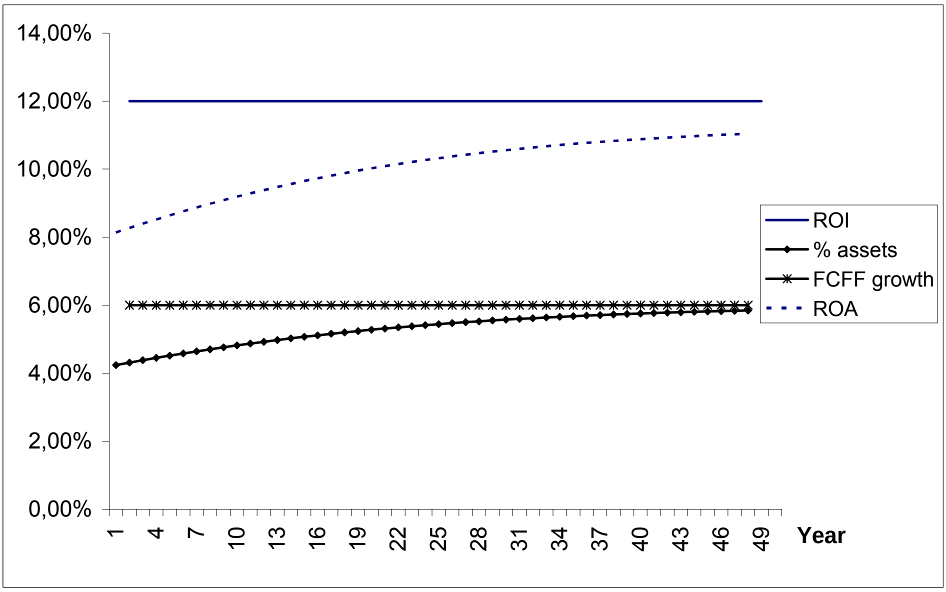
The above derived conclusion (13) holds if and only if $\Delta A = g \cdot A_1$, so that return on assets in any of the subsequent years in the infinite time horizon, for which the terminal value is computed are equal $r_{A,n} = r_{A,n+1}$.

[5] This derivation appears directly from widely used in textbooks on corporate finance and investments (e.g. Bodie – Kane – Marcus, 1996) formula $g = ROE \cdot b$, where ROE is return on equity and b is so called retention ratio, representing the part of earnings after taxes, which is retained in the company to finance investments. Extending this solution to the whole space of possible profit levels and applying dynamics in the definition of return on equity (assets), we get the general expression applicable for estimation of the investments share on earnings: $b = \Delta\alpha / \Delta(1-\alpha) \cdot g$, which is nothing else than $b = g / r_\alpha$ (compare to (13)).

However, compared to literature (Mařík, 1998) dealing with this problem we assumed constant share of investments on the income through the time horizon for which the terminal value is computed. If we did not use that assumption, (13) could be derived without the additional condition stated in this paragraph.

This is shown on the Fig. 1, which represents results of simulation of the course of growth of assets, earnings and cash flow in the first 50 years of period used for computation of terminal value, using the parametric formula. The parameters were $r_\alpha = 10\,\%$, $g = 6\,\%$, $r_A = 8\,\%$. Thus the return on assets is lower than the return on investments and adjustment proceeds through the lower growth rate of sum of assets, so the return on assets converges to the return on investments gradually (asymptotically).

Fig. 1: An example of result of parametric formula



Source: author’s calculation

Under the assumptions of perfect (efficient) capital market and also long-run efficient physical asset markets the return on investments should not differ significantly from the cost of capital, thus $r_\alpha = r$ (cp. Kendall, 1953).

Using (1) as expression of the terminal value and (13) as expression of the income being discounted, the result is then very simple formulae for the terminal value. We have to remind that investments are deducted from the income (12). Then the terminal value becomes a ratio between earnings and the cost of capital (required rate of return).

$$V_0 = \frac{f_1 \cdot (1-\alpha) \cdot \left(1 - \frac{g}{r}\right)}{r-g} = \frac{f_1 \cdot (1-\alpha)}{r}. \tag{14}$$

At the first glance the parametric formula seems to be the panacea for the problems with setting all the parameters of earnings and income for the purposes of terminal value computation. However, a pitfall comes around when the parametric formula is used in practice.

The problem with parametric formula is that it has no corrective power to the errors made in the course of estimation of investments and earnings before the beginning of the terminal value period. Equation (1) is usually used as an expression of value of a business or project after a time horizon for which the components of the discounted income are reasonably and accurately predictable. If during this phase of explicit financial planning wrong estimation is made, with respect to return on investments, the amount of fixed assets at the end of this so called "first phase" of evaluation is set wrongly and accommodation of return on assets is done slowly and gradually. If one wanted to deliberately distort the result of valuation, no simpler way is than that to overstate investments and understate income (*et vice versa*) during the financial planning. Another possible event is if the valuation was based solely on terminal value and the return on assets were not adequate to the assumed return on investment (or cost of capital). Even though the parametric formula addresses this problem much better than the approach based on the constantly growing parts of the income, it does not support us with quick enough reaction built in the computational algorithm.

There are many possible solutions of this problem, of which one is to adjust the value of assets; the other is to adjust the level of earnings. Range of possible solutions lies in-between. A simple test for consistency is possible through equating of the income f computed using (2) and income f computed using (13). The implications were already shortly mentioned above. Equating (2) and (13) we simultaneously assume the constant growth of all the parts of income and investments adequate to the assumed growth rate and to the return on investments. These conditions can be met only if $\Delta A = g \cdot A_1$, so that return on assets in any of the subsequent years in the infinite time horizon, for which the terminal value is computed are equal $r_{A,n} = r_{A,n+1}$. This test can give one signal that he (or she) should adjust the amount of investments in his (her) financial plan or to rise the level of earnings at the end of financial plan. The first solution is more consistent and accurate; the second one is simpler. If $r_{A,0} = r$ then the equation (14) can be used, which emerges from the above text.

Proposition II: *Let companies' income grow in each period in the future (since certain point) with constant rate. Let the investors who purchase and sell companies' shares at the stock market to estimate the value of a company in the timeframe for which the companies are assumed to be stable in terms of variation of return on capital, using formula*

$$V_0 = f \cdot \int_0^{\infty} e^{-t\cdot(r-g)} dt = \frac{f}{r-g}. \qquad (1)$$

Let the capital and real assets markets efficient.

Then the price and value of set of capital is given by ratio of earnings (profit) attributable to that set of capital to required rate of return of that capital if and only if at the time point 0 return on that set of capital is equal to the required return rate.

$$V = \frac{f \cdot (1-\alpha)}{r}. \qquad (14)$$

Derivation: see above.

Conclusion

If we accepted the assumptions that since certain point in time a company is "stable" in terms of ratio of profit to a set of capital to which the profit is attributable, that investors employ the Gordon's formula for evaluation of such a company at that point of time and finally that markets are efficient in the weak form, the proper formula for terminal value estimation is ratio of profit (earnings) attributable to that set of capital to the required return on that set of capital only if the one complies with the condition that return on the evaluated set of capital at the time to which the terminal value is estimated, is equal to the required return rate. We derive this solution on the general conceptual framework, however the steady state validity conditions can be extended to ratio of any two items of balance sheet or income statement. In these terms we come to the conclusion in more general way than Levin and Olsson (2000) and we propose the steady state conditions, which were not published in the central-European literature on valuation before (e.g. Mařík, 1998).

In some cases, when the gradual accommodation of the company's return on capital (equity or the sole capital) or investments is assumed, there is not need for stability of the income with respect to its structure. In such cases the parametric formula allows such an accommodation to course gradually.

Our conclusions are important mainly for terminal values computed from infinite horizon cash flow models (Penmann, 1998 or Mařík, 1998).

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ABSTRACT

Some textbooks and a just a couple of articles address the problems of evaluation of so called terminal value. The term terminal value describes the value of certain set of company's capital at the point in the time, from which the company is assumed to be stable in the terms of profit growth and investments or to converge by a stable rate to the "stable state". Much of the attention is concentrated on the estimation of cost of capital.

We concentrated on the other parameters of terminal value. If we accepted the assumptions that since certain point in time a company is "stable" in terms of ratio of profit to a set of capital to which the profit is attributable, that investors employ the Gordon's formula for evaluation of such a company at that point of time and finally that markets are efficient in the weak form, the proper formula for terminal value estimation is ratio of profit (earnings) attributable to that set of capital to the required return on that set of capital, but only if one complies with the condition that return on the evaluated set of capital at the time to which the terminal value is estimated, is equal to the required return rate. We derive this solution on the general conceptual framework, however the steady state validity conditions can be extended to ratio of any two items of balance sheet or income statement.

Key words: Valuation; Investments; Terminal value.

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