Mean-Variance Hedging for Pricing European Options Under Assumption of Non-continuous Trading

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Abstract

We consider a portfolio with call option and the corresponding underlying asset under the standard assumption that stock-market price represents a random variable with lognormal distribution. Minimizing the variance (*hedging risk*) of the portfolio on the date of maturity of the call option we find a fraction of the asset per unit call option. As a direct consequence we derive the statistically fair lookback call option price in explicit form.

In contrast to the famous Black-Scholes theory, any portfolio can not be regarded as risk-free because no additional transactions are supposed to be conducted over the life of the contract, but the sequence of independent portfolios will reduce risk to zero asymptotically. This property is illustrated in the experimental section using a dataset of daily stock prices of 18 leading Australian companies for the period of 3 years.

1 Introduction

A typical asset S as a geometric Brownian motion process [15], [16], [17] has its price governed by the following equation:

$$\frac{dS}{S} = \mu dt + \sigma dz,$$

where $\mu \in R$ and $\sigma \in R_+$ are *appreciation* and *volatility* coefficients, z is a standard Wiener process with

$$\mathbf{E}dz = 0, \mathbf{E}(dz)^2 = dt.$$

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According to the Ito's lemma,

$$d\log\{S(t)\} = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dz. \quad (1)$$

Therefore,

$$\log\{S(t+T)\} \sim \mathcal{N}(\log\{S(t)\} + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}),$$

where $\mathcal{N}(a,b)$ is the distribution function of a normal random variable with mean a and standard deviation b. Let us denote the corresponding density by

$$f_S(x) = \frac{1}{\sqrt{2\pi}\cdot x\cdot b(T)}\exp\{-\frac{(\log\{x\} - a(T))^2}{2b^2(T)}\},$$

where

$$a(T) = \log\{S(t)\} + (\mu - \frac{\sigma^2}{2})T, b(T) = \sigma\sqrt{T}.$$

Definition 1 *A European call option contract allows its owner to purchase one unit of the underlying asset at a fixed price K after date $t+T$ in the future or the owner of the call option may decide not to exercise if the price of the underlying asset is less than strike price K. Respectively, the value of a European call option with maturity date $t+T$ is*

$$C(t+T) = \psi(S(t+T) - K) = \max\{0, S(t+T) - K\}.$$

The fundamental problem in mathematical finance [8] is to find the fair hedger or price of such an option at a time t prior to expiry.

1.1 Expectations hedging and Black-Scholes formula

According to [14] and [19] one suggestion would be that

$$C_{exp}(t) = e^{-rT}\cdot \mathbf{E}\psi(S(t+T) - K) \quad (2)$$

where r is the riskless rate.

Proposition 1 *Suppose that parameters t, T, r and K are arbitrary fixed. Then,*

$$e^{-rT}\cdot\mathbf{E}\psi(S(t+T) - K) = S(t)\cdot e^{(\mu-r)T}\cdot\Phi(\alpha) - K\cdot e^{-rT}\cdot\Phi(\beta) \quad (3)$$

where Φ is a distribution function of the standard normal law, and

$$\alpha = \frac{\log\{\frac{S(t)}{K}\} + (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \beta = \alpha - \sigma\sqrt{T}.$$

The formula (3) was proved by [16] who also noted that (3) will coincide with Black-Scholes formula [3] in the particular case $\mu = r$:

$$C_{BS}(t) = S(t) \cdot \Phi(\alpha_r) - K \cdot e^{-rT} \cdot \Phi(\beta_r), \tag{4}$$

where

$$\alpha_r = \frac{\log\{\frac{S(t)}{K}\} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \beta_r = \alpha_r - \sigma\sqrt{T}.$$

Remark 1 *Similar results for variance gamma processes may be found in [11]. Also, we note papers [6] and [5] where formulas for call options were obtained using methods based on Fourier transformations.*

However, suggestion (2) ignores the fact that the seller can himself continue to trade actively on the stock market. The BS-formula (4) provides a unique price of the European contingent claim [10] in an ideal, complete and unconstrained market. Under these conditions the contract is self-financing and risk-free, to seller as well to buyer. In the given financial market, the mean-variance hedging problem in continuous time [7], [18] is to find for a given payoff a best approximation by means of a self-financing trading strategies where the optimality criterion is the expected squared error [4]. In a series of recent papers, this problem has been formulated and treated as a linear-quadratic stochastic control problem, see for instance [9], [1], [2].

According to [19] the original Black-Scholes formula is criticized on the grounds that it holds out the quite unrealistic prospect of risk-free operation, that it can sacrifice asset maximization to exact meeting of the contract.

2 Mean-variance hedger

Let us consider portfolio F consisting of the option C and h units of the underlying asset S. The value of the portfolio (seller case) is therefore

$$F(t) = -C(t) + h \cdot S(t) \tag{5}$$

or

$$F(t) = \begin{cases} (h-1) \cdot S(t) + K & \text{if } S(t) \geq K; \\ h \cdot S(t), & \text{otherwise.} \end{cases}$$

According to the fundamental principles of mean-variance model [12] we consider the rule that the investor considers expected return as a desirable thing and variance of return as an undesirable thing. The above rule may be implemented using different methods. For example, we can define fractions of the portfolio by maximizing the ratio of expected return to the standard deviation of the portfolio, or we can minimize variance assuming that the expected return is fixed. In our case we will minimize the variance of the portfolio (5) assuming that the number of call options is arbitrary fixed. In order to simplify notations we consider a portfolio with one call option.

The following Theorem represents the main result of the paper. It will establish the value of the parameter h in order to minimize variance of the portfolio (2). Then, we find hedging call option price or simply *hedger*

$$C_{MV}(t) = h \cdot S(t) - e^{-rT}\mathbf{E}F(t+T). \tag{6}$$

Theorem 1 *Suppose that the portfolio F is defined in (2). Then, the hedging problem*

$$\min_h Q_{var}(F(t+T)) \text{ where } Q_{var}(F(t+T)) := \mathbf{E}\left[F(t+T) - \mathbf{E}F(t+T)\right]^2$$

has the unique solution

$$h = \frac{A_4 - K \cdot A_2 + (A_2 + A_3)(K \cdot A_1 - A_2)}{A_4 + A_5 - (A_2 + A_3)^2}, \tag{7}$$

where

$$A_1(K) := \int_K^\infty f_S(x)dx = \Phi(\frac{a(T) - \log K}{b(T)});$$

$$A_2(K) := \int_K^\infty x f_S(x)dx = \exp\{a(T) + \frac{b^2(T)}{2}\}\Phi(b(T) + \frac{a(T) - \log K}{b(T)});$$

$$A_3(K) := \int_0^K x f_S(x)dx$$

$$= \exp\{a(T) + \frac{b^2(T)}{2}\}\left(1 - \Phi(b(T) + \frac{a(T) - \log K}{b(T)})\right);$$

$$A_4(K) := \int_K^\infty x^2 f_S(x)dx = \exp\{2(a(T) + b^2(T))\}\Phi(2b(T) + \frac{a(T) - \log K}{b(T)});$$

$$A_5(K) := \int_0^K x^2 f_S(x)dx$$

$$= \exp\{2(a(T) + b^2(T))\}\left(1 - \Phi(2b(T) + \frac{a(T) + \log K}{b(T)})\right).$$

Proof: According to the definition of variance (*hedging risk*)

$$Q_{var}(F(t+T)) = \mathbf{E}F^2(t+T) - (\mathbf{E}F(t+T))^2\,, \tag{8}$$

where

$$\mathbf{E}F^2(t+T) = (h-1)^2 A_4 + 2K(h-1)A_2 + K^2 A_1 + h^2 A_5;$$
$$\mathbf{E}F(t+T) = h(A_2 + A_3) - A_2 + K \cdot A_1.$$

Minimizing (8) as a function of h we find the required solution (7). ■

Finally, we find a mean-variance hedger according to (6)

$$C_{MV}(t) = h \cdot S(t) - e^{-rT}\left[h(A_2 + A_3) - A_2 + K \cdot A_1\right] \tag{9}$$

where parameter h is defined in (7).

Next, we can re-write (3) using new notations which were introduced in this section:

$$C_{exp}(t) = e^{-rT}\left(A_2 - K \cdot A_1\right). \tag{10}$$

The above call option relates to the portfolio combined with riskless asset only:

$$F(t) = -C(t) + h, \tag{11}$$

where h is a constant parameter. The corresponding standard deviation is invariant under h and is given by the following formula (see Figure 2):

$$S_{dev}(F(t+T)) = \sqrt{A_1(1-A_1)K^2 + 2A_2K(A_1 - 1) + A_4 - A_2^2}.$$

Remark 2 *Note that the price (9) may be negative, in contrast to the price (10) which is always positive by definition.*

Using relations

$$A_2 + A_3 = \exp\{a + 0.5b^2\}, A_4 + A_5 = \exp\{2(a + b^2)\},$$

we can simplify (7):

$$h(K) = \frac{A_4 - K \cdot A_2 + \exp\{a + 0.5b^2\}(K \cdot A_1 - A_2)}{\exp\{2a + b^2\}(\exp\{b^2\} - 1)}. \tag{12}$$

Let us consider some marginal properties of the coefficients $A_i(K), i = 1..5$:

$$A_1(K) \underset{K\to 0}{\longrightarrow} 1, A_3(K) \underset{K\to 0}{\longrightarrow} 0, A_5(K) \underset{K\to 0}{\longrightarrow} 0.$$

It follows from above that

$$h(K) \underset{K\to 0}{\longrightarrow} 1 \text{ and } S_{dev}(K) \underset{K\to 0}{\longrightarrow} 0 \text{ (see Figure 2).}$$

Proposition 2 *Assuming that $\sigma > 0$, the following range $0 < h < 1$ is valid where asset fraction parameter h is defined in (7).*

The proof of the above Proposition 2 follows from the following two Lemmas.

Lemma 1 *Assuming that Φ is a distribution function of standard normal law the following relation is valid:*

$$\frac{\Phi(v+b)-\Phi(v)}{\Phi(v)-\Phi(v-b)} < \exp\{0.5b^2 - bv\}, \tag{13}$$

for any $v \in R$ and $b \in R_+$.

Proof: We have

$$\Phi(v) - \Phi(v-b) = \frac{1}{\sqrt{2\pi}} \int_v^{v+b} \exp\{-\frac{(t-b)^2}{2}\}dt = \frac{e^{-0.5b^2}}{\sqrt{2\pi}} \int_v^{v+b} e^{-0.5t^2} e^{bt} dt$$

$$< \frac{e^{-0.5b^2+bv}}{\sqrt{2\pi}} \int_v^{v+b} e^{-0.5t^2} dt = \frac{e^{-0.5b^2+bv}}{\sqrt{2\pi}} \left[\Phi(v+b) - \Phi(v)\right].$$

Therefore, the proof is completed. ■

Lemma 2 *Assuming that Φ is a distribution function of standard normal law the following relation is valid:*

$$\Phi(v) < \frac{e^{b^2}\Phi(v+b) + \exp\{0.5b^2 - bv\}\Phi(v-b)}{1 + \exp\{0.5b^2 - bv\}}, \tag{14}$$

for any $v \in R$ and $b \in R_+$.

Proof: We have

$$e^{b^2}\Phi(v+b) - \Phi(v) > e^{b^2}\left[\Phi(v+b) - \Phi(v)\right] = \frac{e^{b^2}}{\sqrt{2\pi}} \int_v^{v+b} \exp\{-\frac{t^2}{2}\}dt$$

$$= \frac{e^{b^2}}{\sqrt{2\pi}} \int_{v-b}^{v} e^{-0.5(t+b)^2} dt = \frac{e^{0.5b^2}}{\sqrt{2\pi}} \int_{v-b}^{v} e^{-0.5t^2 - bt} dt$$

$$> \exp\{0.5b^2 - bv\}\left[\Phi(v) - \Phi(v-b)\right].$$

Therefore, the proof is completed. ■

Using definition of the coefficients $A_i(K), i = 1..5$, we can re-write (12) in the following form:

$$h = \frac{e^{b^2}\Phi(v+b) - \Phi(v) - \exp\{0.5b^2 - bv\}\left[\Phi(v) - \Phi(v-b)\right]}{e^{b^2} - 1},$$

where $v = b + \frac{a - \log\{K\}}{b}$.

Then, a strict upper and a lower bounds for h (as it is stated in the Proposition 2) follows from (13) and (14) if $\sigma > 0$.

Remark 3 *The left column of the Figure 1 illustrates descending property of the hedging call option price as a function of the strike price. It is interesting to note that the sum of the call option price and strike price is an ascending function of the strike price. This fact is quite explainable because the second part of the transaction (purchase of the stock) is not compulsory. According to the Figure 1 the formulas (3) and (9) are more flexible comparing with Black-Scholes formula which is independent of the appreciation coefficient* μ.

3 Experiments

Based on the representation (1) we can formulate an estimator for the historical (*moving*) volatility [13]:

$$\hat{\sigma}_{i,t} = \sqrt{\frac{\sum_{j=1}^{n}\left(R_{i,t-j} - \overline{R}_{i,t}\right)^2}{n-1}} \tag{15}$$

where

$$R_{i,t-j} = \log\frac{S_{i,t-j+1}}{S_{i,t-j}};$$

$S_{i,t}$ is a closing price of i-asset on the day $t > n$ and

$$\overline{R}_{i,t} = \frac{1}{n}\sum_{j=1}^{n}\log\frac{S_{i,t-j+1}}{S_{i,t-j}}.$$

Then, we can estimate historical (*moving*) appreciation:

$$\hat{\mu}_{i,t} = \overline{R}_{i,t} + \frac{1}{2}\hat{\sigma}^2_{i,t}. \tag{16}$$

Table 1: All prices are given in cents. The first column gives the name of asset, the second column gives the average price during period of 100 days ending on 10th January 2006. Columns 3-6 represent final profits of buyer and seller in cases of Expectations and MV approaches.

Asset	Average	Expectations		Mean-Variance	
Name	Price	Buy	Sell	Buy	Sell
ANZ Bank	2306.53	922.63	-922.63	1801.59	3125.66
CBA, Commonwealth Bank	3909.16	-1410.77	1410.77	-485.93	5702.04
CML, Coles Myers	990.6	24.78	-24.78	461.42	1477.59
DJS, David Jones	236.11	1428.42	-1428.42	1844.33	-513.65
FXJ, Fairfax	417.18	-816.73	816.73	-657.82	-58.73
HVN, Harvey Norman	277.67	671.77	-671.77	347.12	642.38
NAB, National Bank	3193.27	-4433.27	4433.27	-4172.65	6730.61
PBL, Publish.Brodcast.	1625.36	3416	-3416	2419.06	2119.44
QAN, Qantas	350.14	2234.44	-2234.44	1344.69	64.72
QBE Insurance	1813.72	1216.89	-1216.89	5023.97	1466.49
RIO, Rio-Tinto	5788.09	33156.66	-33156.66	47506.18	-10089.97
STO, Santos	1141.14	-9401.15	9401.15	-2086.32	3910.58
TAH, Tabcorp	1606.01	-183.37	183.37	-1558.01	2335.71
TEN Network	342.78	-1192.97	1192.97	-1816.85	818.41
TLS, Telstra	412.26	-1372.16	1372.16	-1924.31	-1068.74
WBC, Westpac Bank	2104.37	3293.2	-3293.2	2949.07	2272.26
WOW, Woolworth	1633.53	-3596.63	3596.63	-2517.34	3440.02
WPL, Woodside Petroleum	3361.07	-21427.93	21427.93	3208.89	11465.86

3.1 Expectations hedging

The call option price $C_{i,t}$ (see the right column of the Figure 3.1) was computed according to (10) subject to the following condition: $C_{i,t} \geq 0.03 \cdot S_{i,t}$ (administrative fees of not less than 3 %). The strike price was computed using historical appreciation and volatility coefficients

$$K_{i,t} = S_{i,t} \cdot \exp\{\frac{\mu_{i,t} \cdot T}{\beta + \gamma \cdot \sigma_{i,t}}\}, \beta = 1.1, \gamma = 20.$$

The left and middle columns of Figure 3.1 correspond to the profit of buyer $PB_{i,t}$ and seller $PS_{i,t}$ which were computed for the 100 consecutive days ending on 10th January 2006 ($j = 0..100$). The computations were conducted using the following rules:

$$PB_{i,t+T+j+1} = PB_{i,t+T+j}$$
$$+ \begin{cases} S_{i,t+T+j+1} - K_{i,t+j+1} - C_{i,t+j+1} & \text{if } S_{i,t+T+j+1} \geq K_{i,t+j+1}; \\ -C_{i,t+j+1}, & \text{otherwise;} \end{cases}$$

and

$$PS_{i,t+T+j+1} = PS_{i,t+T+j}$$

$$+ \begin{cases} y & K_{i,t+j+1} + C_{i,t+j+1} - S_{i,t+T+j+1} \text{ if } S_{i,t+T+j+1} \geq K_{i,t+j+1}; \\ & C_{i,t+j+1}, \text{ otherwise,} \end{cases}$$

where initial values of $PB_{i,t+T}$ and $PS_{i,t+T}$ are set to zero.

In order to estimate the performance of the system against the whole set of m assets, we computed average stock-prices $u_i, i = 1..m$, for the period under consideration. Then, we computed weights $w_i \propto (u_i)^{-1}, \sum_{i=1}^{m} w_i = 1$.

The average profits of buyers AB_t and sellers AS_t were computed using the following formulas:

$$AB_t = \sum_{i=1}^{m} w_i \cdot PB_{i,t}, \tag{17a}$$

$$AS_t = \sum_{i=1}^{m} w_i \cdot PS_{i,t}, \tag{17b}$$

$$AT_t = \sum_{i=1}^{m} w_i \cdot C_{i,t} \tag{17c}$$

where dash-dotted line corresponds to the average turnover AT_t (see the first two lines of the Figure 3).

3.2 Mean-variance hedging

Here we make modifications of (3.1) and (17c) (all other formulas remain the same as in the previous Section):

$$PS_{i,t+T+j+1} = PS_{i,t+T+j} + C_{i,t+j+1}$$

$$+ \begin{cases} (1 - h_{i,j+1})(K_{i,t+j+1} - S_{i,t+T+j+1}) \text{ if } S_{i,t+T+j+1} \geq K_{i,t+j+1}; \\ h_{i,j+1}(S_{i,t+T+j+1} - S_{i,t+j+1}), \text{ otherwise,} \end{cases}$$

and

$$AQ_t = \sum_{i=1}^{m} w_i \cdot (C_{i,t} + h_{i,t} S_{i,t}). \tag{18}$$

As a result, turnover of seller will be larger comparing with turnover of buyer. Third and Fourth lines of the Figure 3 illustrate average profits of buyer and seller.

4 Concluding remarks

The classical equation (2) establishes the hedging so that a transaction will be statistically profitable for buyer if price is smaller, or profitable for seller if price is higher. Any particular transaction is not risk-free, but the sequence of independent transactions may reduce risk essentially (see for details Figure 3 and Table 1.

In contrast, risk-free formula (4) was obtained under ideal assumption of absolute liquidity of the market. It means, any transaction represents a continuous sequence of trading, which (as it was noticed in many papers) can-not be achieved in real terms.

Combination of the call option with the corresponding asset represents an additional degree of flexibility. On the one hand, it will help to reduce risk for seller. On the other hand, the call option price will be reduced in the case if performance of the stock is good historically. Anyway, in accordance with MV approach a seller will calculate hedging call option price somewhere between prices computed according to the BS and Expectations approaches. Therefore, MV hedger may be regarded as a compromise between 2 base solutions (see Figure 1).

Comparing the third and fourth lines with first two lines of the Figure 3, which were developed using the same regulation parameters, we can see advantages of the MV approach against Expectations approach.

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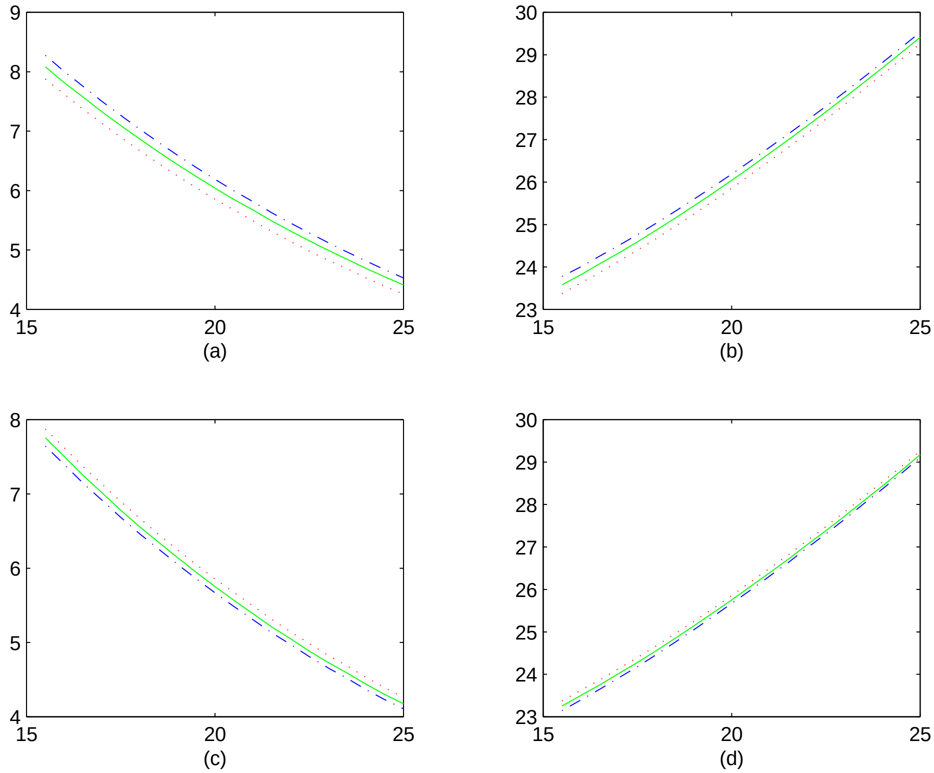

Figure 1: The left column illustrates behavior of call option C, the right column illustrates behavior of the sum of call option and strike price K as a function of K. The following parameters were used: (a-b) $S(t) = 20, \mu = 0.1, r = 0.05, \sigma = 1, T = 180/365$; (c-d) $S(t) = 20, \mu = 0.02, r = 0.05, \sigma = 1, T = 180/365$; green solid line, blue dash-dot line and red dotted line correspond to MV (9), Expectations (3) and BS (4) solutions, respectively.

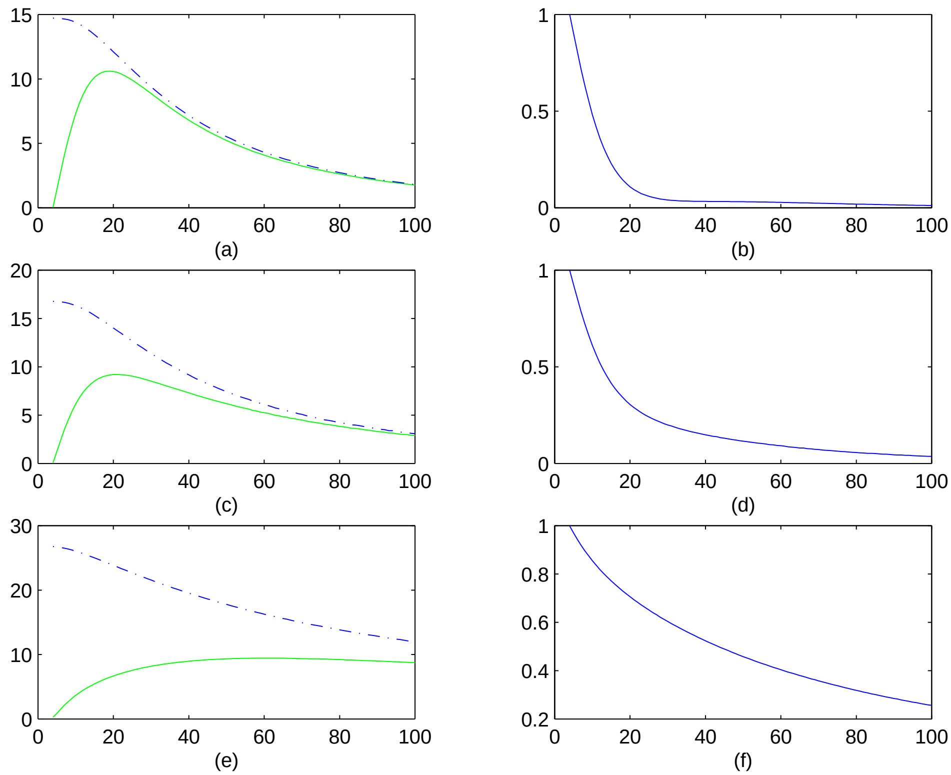

Figure 2: The left column: standard deviation of the portfolio as a function of K where the green solid line corresponds to (9), and the blue dash-dot line corresponds to the portfolio with riskless asset only (2) (Expectations approach); the right column: value of the parameter h as a function of strike price K. The following parameters were used: a-b) $\mu = 0.1, \sigma = 0.9$; c-d) $\mu = 0.1, \sigma = 1.0$; e-f) $\mu = 0.1, \sigma = 1.4$. All other parameters are the same as in the case of Figure 1.

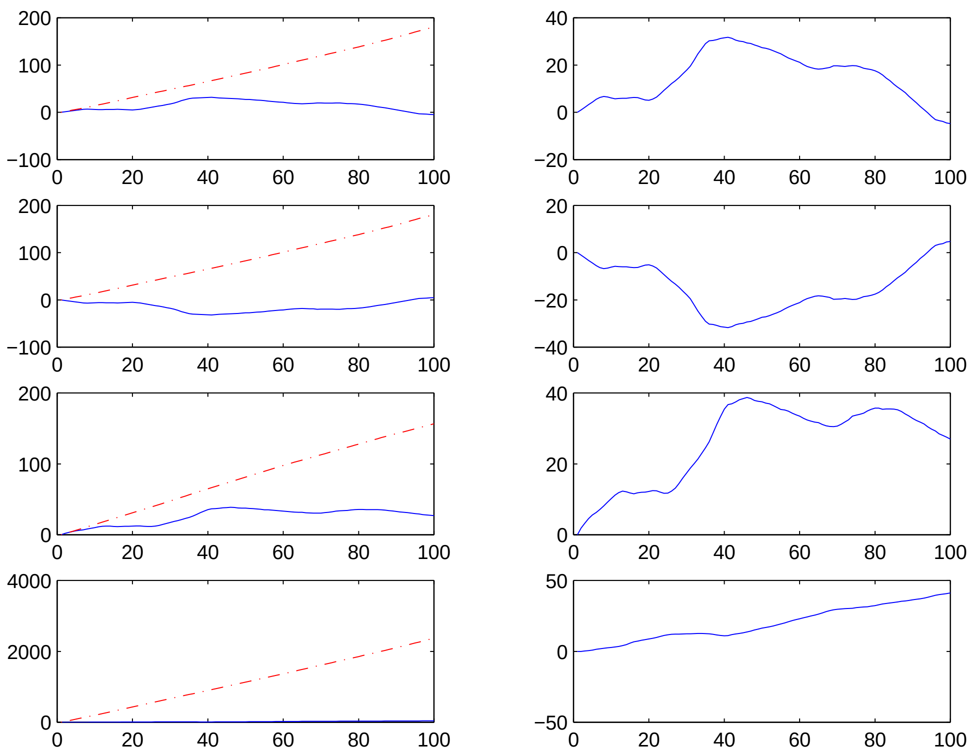

Figure 3: General case of $m = 18$ assets (see Table 3.1). Performance of portfolio based on Expectations (1st and 2nd rows) and MV (3rd and 4th rows) Algorithms where solid line represents profits of buyer (1st and 3rd rows) and seller (2nd and 4th rows), dash-dotted line represents an average turnover. All computations were done according to (17a - 17c) and (18).

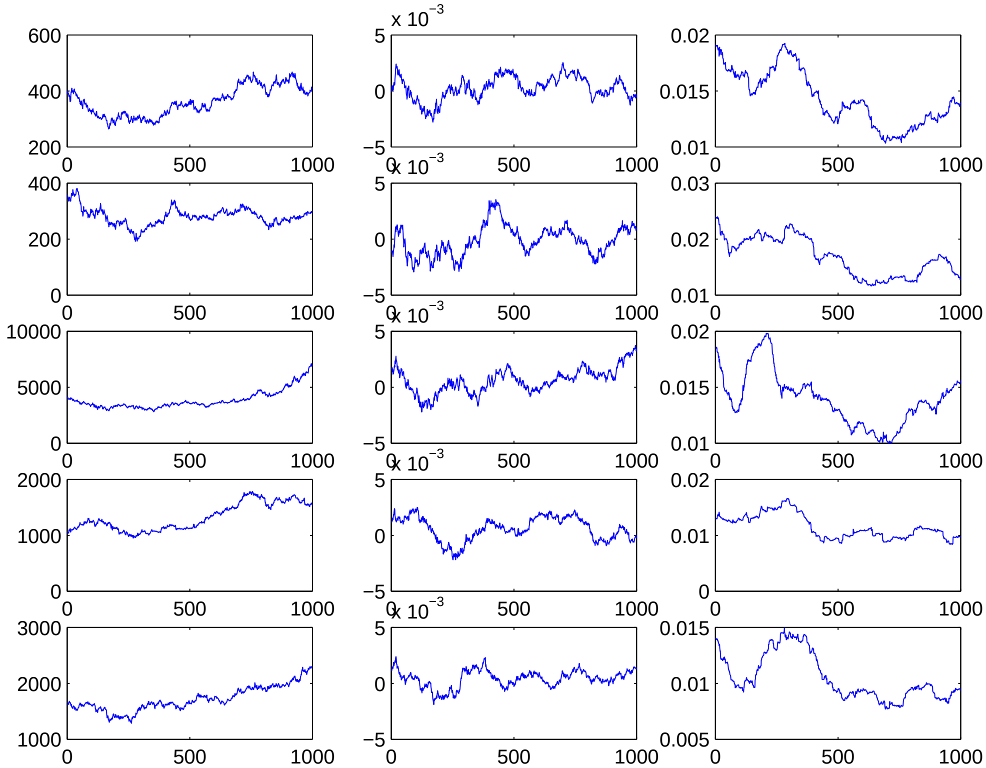

Figure 4: The left column illustrates stock market prices of Fairfax, Harvey Norman, Rio-Tinto, Tabcorp and Westpac for the period of 1000 days ending on 10th January 2006; the middle and right columns illustrate moving means and standard deviations which were computed according to (16) and (15) using smoothing parameter $n = 120$.

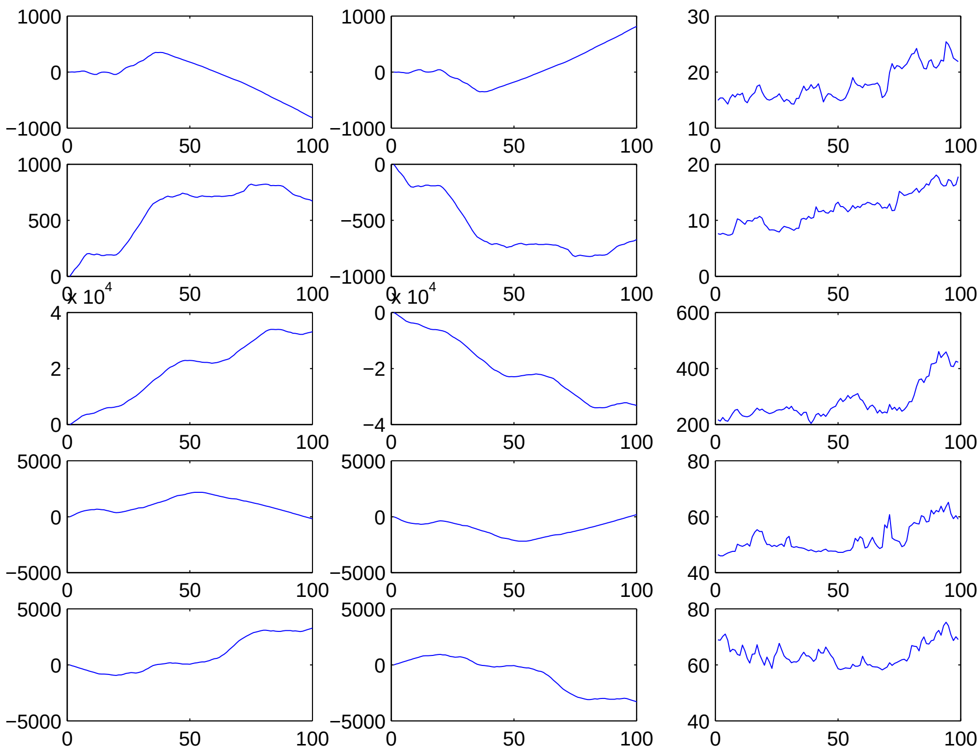

Figure 5: Expectations approach: the first 2 columns represent profits of buyer (3.1) and seller (3.1) during period of 100 days ending on 10th January 2006; the third column represents corresponding call options price (3).

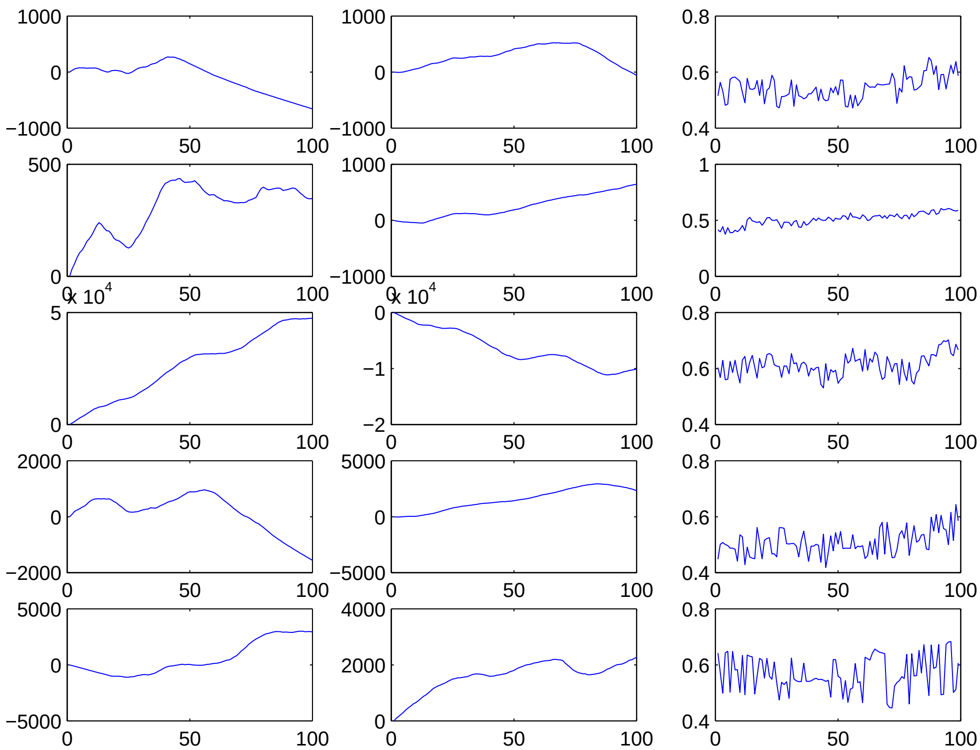

Figure 6: MV approach: the first 2 columns represent profits of buyer (3.1) and seller (3.2) during period of 100 days ending on 10th January 2006; the third column represents h parameter (7).