

# Solid state multi-ensemble quantum computer in waveguide circuit model

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The first realization of solid state quantum computer was demonstrated recently by using artificial atoms – transmons in superconducting resonator. Here, we propose a novel architecture of flexible and scalable quantum computer based on a waveguide circuit coupling many quantum nodes of controlled atomic ensembles. For the first time, we found the optimal practically attainable parameters of the atoms and circuit for 100% efficiency of quantum memory for multi qubit photon fields and confirmed experimentally the predicted perfect storage. Then we revealed self modes for reversible transfer of qubits between the quantum memory node and arbitrary other nodes. We found a realization of *i*SWAP gate via direct coupling of two arbitrary nodes with a processing rate accelerated proportionally to number of atoms in the node. A large number of the two-qubit gates can be simultaneously realized in the circuit for implementation of parallel quantum processing. Dynamic coherent elimination procedure of excess quantum state and collective blockade mechanism are proposed for realization of *i*SWAP and  $\sqrt{i$ SWAP quantum gates.

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## I. INTRODUCTION

Construction of large quantum computer (QC) is a synergetic physical and engineering problem which imposes a number of critical requirements on physical and spatial organization of interconnections between the qubits of QC and with its near environment necessary for a quantum transmission and readout of quantum calculations results [1, 2]. Quantum computing is based on delicate exploitation of number of various single- and two- qubit gates. Usually, single qubit gates are relatively easily fulfilled experimentally by using a well-known coherent control of single two-level atoms (atomic qubit) or molecular qubits in external resonant electromagnetic fields [3–5]. Also, the single photon qubit gates can be realized by the linear optics technique which provides simple procedures for rotation of the light phase and polarization with a control of light by mirrors and beam-splitters [6, 7].

Most complicated problem is an experimental realization of a high enough coupling constant between two arbitrary qubits in order to implement the deterministic sufficiently fast two-qubit gates. Few promising approaches have been proposed to increase a coupling constant of two qubit interactions. For example, using a single mode optical cavity provides the enhanced coupling constant of the interaction between single atom and photon in the cavity [8, 9]. Then cavity mediated photon-atom interaction that is also enhanced can be organized for implementation of basic quantum information processes [10, 11]. Very powerful method to increase the coupling constant with a photon is to use a Josephson

qubits characterized by superconducting current of mesoscopic magnitude [12] and by using the Josephson qubits (transmons) in superconducting resonators [13, 14]. The two qubit transmon processor has been demonstrated recently [15] for successful implementation of the Grover search and Deutsch–Jozsa quantum algorithms.

Another promising tool of the coupling constant enhancement is an encoding of qubits on multi-atomic coherent states. Here, the coupling constant of  $N$  atoms with a photon can be enhanced by factor  $\sqrt{N}$ . Initially the multi-atomic coherent ensembles have been used for optical quantum memory (QM) [16–24], implementation of robust quantum communication over long lossy channels [25] and for single photon generation [26]. Recently collective ensembles of multilevel systems have been discussed for QM cooperated with quantum computing [27, 28]. The idea has been developed to use the multi-qubit QM integrated in a hybrid superconducting QC for encoding of the qubits cooperated with transmon cooper pair box used for quantum processing [29]. Here, the single and two qubit gates are performed by coherent control of transmon qubit in the external electromagnetic field and via two-qubit *i*SWAP gate with the cavity photon state. Complete quantum computing for large number of qubits is realized by a transfer of any qubit pair on the transmon and photon qubits with their subsequent swapping and using single qubit gates. The hybrid approach had been proposed for quantum processing of more than 100 qubits stored in atomic ensemble embedded in superconducting transmission line resonator. Now, a spatial architecture of the multi-qubit transmon based on superconducting QC is under consideration [30].

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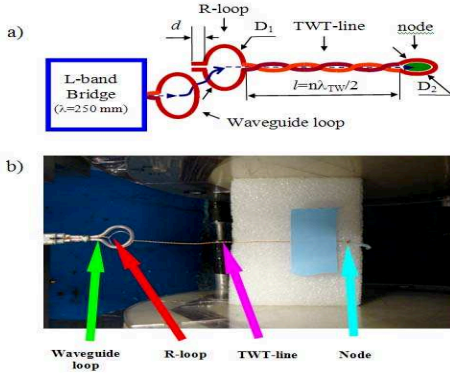


FIG. 1: Scheme of experiment with waveguide circuit. a) L-band Bridge is a source of microwave radiation coupled with primitive single node circuit. The circuit consists of capacity of the R-loop tuned experimentally by varying of length  $d$ , two waveguide transmission (TWT) line and small loop surrounding the resonant medium (green spot) - quantum memory (QM) node;  $D_1$  and  $D_2$  are the diameters ( $D_2/D_1 \ll 1$ ). The radiation is transferred from the L-band Bridge to the - (QM) node along arrow. b) Realization of efficient field transfer from Waveguide loop via R-loop to spatially distant Node for sufficiently large TWT-line length (rose); waveguide loop, R-loop and node are pointed by green, red and blue arrows.

## II. PRINCIPLE SCHEME OF MULTI-ENSEMBLE QUANTUM COMPUTER IN WAVEGUIDE CIRCUIT MODEL

Instead of the promising progress, the transmons are characterized by relatively fast decoherent processes and can work only in superconducting systems i.e. at very low temperatures that motivates a further search of new principle schemes for realization of solid state QC. Here, we propose a novel architecture of QC based on multi-atomic ensembles situated in different spatial nodes in single mode QED cavity. We demonstrate the QC architecture by using a waveguide circuit realization of the QED cavity which yields possibility to use available technologies for realization of multi-qubit QM and fast quantum processing satisfying the di-Vincenzo criteria [31]. In particular, scalability is easily achievable for the proposed circuit architecture.

The primitive waveguide circuit is shown in Fig. 1. The circuit consists of three basic elements: a usual copper receiver (R-) loop with diameter  $D_1$ , two-waveguide transmission (TWT-) line and second loop with highly reduced spatial diameter  $D_2$  containing the resonant atomic ensemble (node). Two waveguides of the TWT-line are twisted in order to suppress an irradiation to external space. In our experiments, we used diameter  $D_2 = 0.4$  mm which is more than 500 times smaller than the used wavelength  $\lambda = 250$  mm of the radiation transmitted through the external waveguide. The R-loop provides an efficient reception of the radiation from the closely situated waveguide output loop as shown in 1. At

small length of TWT-line, the waveguide circuit works as a single mode resonator. We have been urged experimentally in a robust work of the waveguide resonator in its numerous spatial architectures. R-loop with inductivity  $L$  and capacity  $C$  tuned by varying length of line  $d$  (see1) determines a resonant frequency  $\omega_o = 1/\sqrt{LC}$  of the circuit. TWT-line was adjusted to R-loop by choosing its spatial length  $\ell = n\lambda_{TW}/2 = n\pi c/(\omega_o\sqrt{\epsilon_o\epsilon})$ , where  $\lambda_{TW}$  is a wavelength of radiation in the TWT-line,  $c$  is the speed of light,  $\epsilon$  is a nondimensional permittivity of TWT-line volume,  $n$  is an integer. We note that a distance between two twisted waveguides in TWT-line was much smaller than the wavelength  $\lambda_{TW}$  so the coupling between TWT-line and R-loop didn't change the resonant frequency  $\omega_o$  while the line provided an effective transmission of the electromagnetic field between the R- and small loops. Since the small loop had a negligibly small spatial size the electromagnetic field evolved as a standing wave in the TWT-line with the same electrical current in the circuit waveguide. Thus the waveguide circuit works as a spatially distributed single mode resonator on the resonant frequency  $\omega_o$  with Q-factor  $Q = r^{-1}\sqrt{L/C}$ , where  $r$  is a total losses resistance in the circuit (the losses were negligibly weak in our experiments) and the circuit provides an effective quantum electrodynamics of the atomic ensemble with electromagnetic field of the single resonator mode.

## III. MULTI-MODE QUANTUM MEMORY

By following cavity mode formalism [32] we describe an efficient multi-qubit QM in our circuit. We use the generalized Tavis-Cumming Hamiltonian [33, 34]  $\hat{H} = \hat{H}_o + \hat{H}_1$  for  $N$  atoms, field modes and their interactions generalized by taking into account inhomogeneous broadening of atomic frequencies and continuous spectral distribution of the field modes where  $\hat{H}_o = \hbar\omega_o\{\sum_{j=1}^N S_z^j + \hat{a}_\sigma^+\hat{a}_\sigma + \sum_{n=1}^2 \int \hat{b}_n^+(\omega)\hat{b}_n(\omega)d\omega\}$  are main energies of atoms ( $S_z^j$  is a  $z$ -projection of the spin operator), energy of cavity  $\sigma$ -field mode ( $\hat{a}_\sigma^+$  and  $\hat{a}_\sigma$  are arising and decreasing operators), energy of waveguide field ( $n=1$ ) and energy of free space field ( $n=2$ ) ( $b_n^+$  and  $b_n$  are arising and decreasing operators of the waveguide modes),

$$\begin{aligned} \hat{H}_1 = & \hbar \sum_{j=1}^N \Delta_j S_z^j + \hbar \sum_{n=1}^2 \int (\omega - \omega_o) \hat{b}_n^+(\omega) \hat{b}_n(\omega) d\omega \\ & + i\hbar \sum_{j=1}^N [g_\sigma S_-^j \hat{a}_\sigma^+ - g_\sigma^* S_+^j \hat{a}_\sigma^-] \\ & + i\hbar \sum_{n=1}^2 \int \kappa_n(\omega) [\hat{b}_n(\omega) \hat{a}_\sigma^+ - \hat{b}_n^+(\omega) a_\sigma] d\omega. \end{aligned} \quad (3.1)$$

The first two terms in (3.1) comprise perturbation energies of atoms (where  $\Delta_j$  is a frequency detuning of  $j$ -th

atom) and the field modes; the third and fourth terms are the interaction energy of atoms with cavity mode ( $S_+^j$  and  $S_-^j$  are the transition spin operators,  $g_\sigma$  is a coupling constant) and interaction energy of the cavity mode with the waveguide and free propagating modes characterized by coupling constants  $\kappa_n(\omega)$ .

We note that  $[\hat{H}_o, \hat{H}_1] = 0$  and Hamiltonian  $\hat{H}_o$  characterizes a total number of excitations in the atomic system and in the fields which is preserved during the quantum evolution where  $\hat{H}_o$  gives a contribution only to the evolution of common phase of the wave function.  $\hat{H}_1$  determines a unitary operator  $\hat{U}_1(t) = \exp\{-i\hat{H}_1 t/\hbar\}$  causing a coherent evolution of the atomic and field systems with dynamical exchange and entanglement of the excitations between them. In spite of huge complexity of the compound light-atoms system here we show that their quantum dynamics governed by  $\hat{H}_1$  in (1) can be perfectly reversed in time on our demand in a simple robust way.

We assume that initially all atoms ( $j = 1, 2, \dots, N$ ) stay on the ground state  $|0\rangle = |0_1, 0_2, \dots, 0_N\rangle$  and free mode fields are in the vacuum state and we launch a signal multi-mode weak field to the circuit through the waveguide at  $t=0$  as shown in Fig.1. Total temporal duration of the signal field should be shorter than decoherence time of the atomic system ( $\delta t \ll T_2$ ). We assume that the spectral width  $\delta\omega_f$  of the signal field is narrow in comparison with inhomogeneously broadened width of the resonant atomic transition ( $\delta\omega_f \ll \Delta_{in}$ ). Then the consideration of the atoms and field evolution for  $\delta t < t \ll T_2$  gives an efficiency  $Q_{eff}$  of the signal field storage (a ratio of stored in atomic system energy to incoming energy of the signal field) [35]:

$$Q_{eff} = \frac{\gamma_1}{(\gamma_1 + \gamma_2)} \frac{4\Gamma/(\gamma_1 + \gamma_2)}{|1 + \Gamma/(\gamma_1 + \gamma_2)|^2}, \quad (3.2)$$

plotted in Fig. 2, where  $\gamma_i = \pi\kappa_i^2(\omega_o)$  determines a coupling between the resonator mode with waveguide modes ( $i=1$ ) and with free propagating modes ( $i=2$ ),  $\Gamma = N_{qm}|g_\sigma|^2/\Delta_{in}$  - coupling of cavity mode with atomic system,  $N_{qm}$  is a number of atoms in the QM node.

As it is seen in Fig. 2, the quantum efficiency  $Q_{eff}$  reaches unity at  $\Gamma/\gamma_1 = 1$  and  $\gamma_2/\gamma_1 \ll 1$  that shows a promising possibility of perfect storage for multi-mode signal field at moderate atomic density. Note that  $\gamma_1 = \Gamma$  is a condition of *matched impedance condition* between the waveguide modes and the atomic system in QM node. Detailed spectral analysis for the storage of the field characterized by finite spectral width has been performed in paper [36]. Also, spectral matching condition

$$\gamma_1 = 2\Delta_{in} \quad (3.3)$$

was found for the inhomogeneous broadening width  $\Delta_{in}$  with Lorentzian shape and the coupling constant  $\gamma_1$  that is a second optimal condition for the QM. This condition provides a high efficiency of the QM even in rather broad

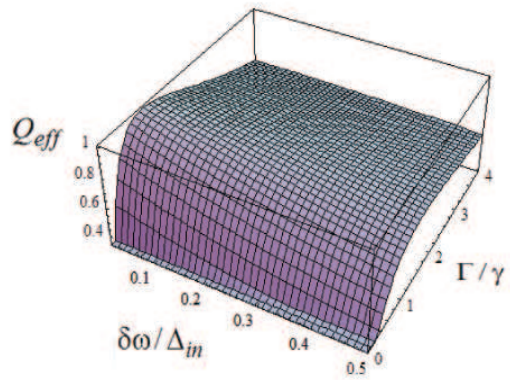


FIG. 2: Transfer efficiency of the input light field to the QM node from external waveguide as a function of spectral width  $\delta\omega/\Delta_{in}$  in units of inhomogeneous broadening and ratio of  $\Gamma/\gamma_1$  (where  $\gamma_1 + \gamma_2 \approx \gamma_1 \equiv \gamma$ ) for Lorentzian spectral shapes of the input field and inhomogeneous broadening for  $\Delta_{in} = \gamma$ . It is seen that efficiency  $Q_{eff} > 0.9$  for  $\delta\omega/\Delta_{in} < 0.2$  and the efficiency gets unity at  $\Gamma/\gamma_1 = 1$ .

spectral range. In this case all multi-mode signal fields incoming in the circuit transfer to the atomic system of the QM node. The efficient direct unconditional transfer of the multi-mode field is possible for inhomogeneously broadened atomic (electron spin) transition where the effective quantum storage of multi-mode fields occurs for arbitrary temporal profile of the modes.

We examined experimentally the signal storage for the radiation field with carrier frequency  $\nu = 1.2$  GHz. We varied parameter  $\gamma_1$  by changing a spatial distance between R-loop and external waveguide loop in 1 and found that all signal field energy was transferred to the electron spin systems of lithium phthalocyaninate (LiPc) molecule sample at the optimal matching value of  $\gamma_1 = 3.768 \cdot 10^7$  where the reflected field was absent in the external waveguide. At this condition we have observed a strong signal of the electron paramagnetic resonance with a spectral width  $\Delta\nu = 88.2$  KHz from  $4.51 \cdot 10^{13}$  LiPc molecules situated in one distant node in Fig. 1.

In order to construct an efficient QM for the multi-mode fields we follow the original protocol of the photon echo QM proposed in 2001 [21] and theoretically described [34] in most general way in the Schrödinger picture by exploiting symmetry properties of the light atoms Hamiltonian. Here, we exploit simplicity of this approach in description of the multi-mode QM in the proposed QC. The assumed atomic detunings  $\Delta_j$  are caused by a presence of the magnetic field gradient. By assuming a perfect storage in accordance with above coupling matching condition we change a sign of the detunings  $\Delta_j \rightarrow -\Delta_j$  at time moment  $t=t'$  by changing of the magnetic field polarity similar to recent experiments [37]. By using a substitution for the field operators  $\hat{a}_\sigma = -\hat{A}$  and  $\hat{b}_n(\omega_o - \Delta\omega) = \hat{B}_n(\omega_o + \Delta\omega)$  (with similar relations for the Hermit conjugated operators) we get a new Hamil-

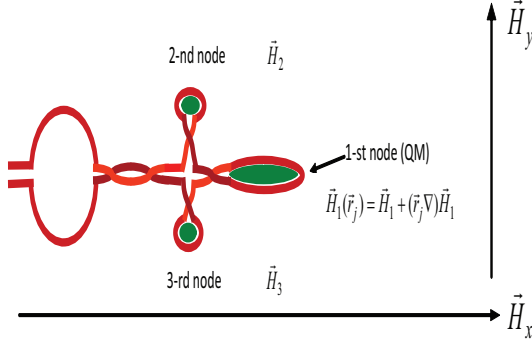


FIG. 3: Principle scheme of three node quantum computer. Left node is a QM node loaded in gradient magnetic field; Second and third nodes are the processing nodes situated in the constant different magnetic fields.

tonian  $\hat{H}'_1 = -\hat{H}_1$  which has an opposite sign with respect to initial Hamiltonian determining a reversed quantum evolution in accordance with a new unitary operator  $\hat{U}_2[(t-t')] = \exp\{-i\hat{H}'_1(t-t')/\hbar\} = \exp\{i\hat{H}_1(t-t')/\hbar\}$ . So the initial quantum state of the multi-mode signal field will be reproduced at  $t = 2t'$  in the echo pulse with the irradiated field spectrum inverted relatively to the frequency  $\omega_o$  in comparison with the original one. Thus we have shown for the first time that the multi-mode QM can be realized with nearly 100% efficiency by using the optimal matching condition  $\gamma_1 = \Gamma$  of the atoms in circuit with the waveguide modes. The possibility opens a door for practical realization of ideal multi-mode QM.

Let's consider a principle scheme of the quantum transport in QC architecture or three node circuit depicted in Fig. 3.

The circuit contains the QM node and two processing nodes. QM node is loaded in gradient magnetic field providing an inhomogeneous broadening of atomic frequencies  $\Delta_{in} \gg \delta\omega_f$  with central atomic frequency coinciding with the circuit frequency  $\omega_{qm} = \omega_o$ . The second and third nodes have  $N$  atoms in each node with equal frequencies  $\omega_{2,3}$  within each node tuned far away from the frequency  $\omega_o$ . We should provide a perfect reversible transfer of arbitrary qubits between QM node and each other two nodes. Initially, the multi-qubit states encoded in the  $M$  temporally separated photon modes  $E(t) = \sum_{m=1}^M E_m(t)$  with spectral width  $\delta\omega_f$  are recorded in the QM-node by excitation from the external waveguide. When the storage procedure is completed we tune away the atomic frequency of the QM node from resonance with the circuit  $\omega_{qm} \neq \omega_o$ . In order to transfer one arbitrary  $k$ -th qubit state from the QM node to the second node we switch off the waveguide circuit coupling with the external waveguide ( $\gamma_1 = 0$ ) and launch rephasing of the atomic coherence in QM node (by reversal of the atomic detunings  $\Delta \rightarrow -\Delta$ ). The  $k$ -th qubit state will be rephased at  $t=2t_k$ . At time moment

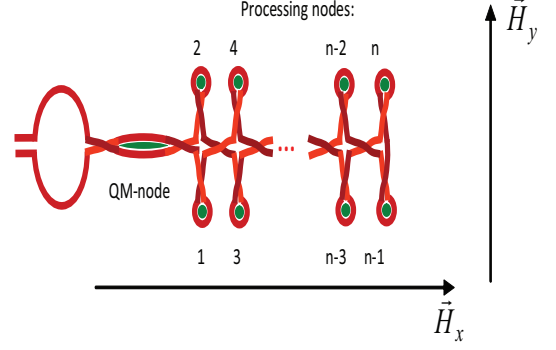


FIG. 4: Multi-qubit quantum computer. Magnetic fields  $\vec{H}_x$  and  $\vec{H}_y$  are used to control atomic frequencies in the nodes; atoms in the QM node exist in the magnetic field gradient that leads to inhomogeneous broadening of the atomic frequencies. The magnetic field magnitude in the nodes is varied during the quantum processing for rephasing the atomic coherence and to control the resonance conditions in the circuit-node and node-node interactions.

$t_k + t_{k-1}$  we equalize the frequencies of QM-node and 2-nd node with  $\omega_o$ . The quantum dynamics of atoms in QM and 2-nd nodes and circuit mode evolves to complete transfer of atomic excitation from the QM node to the second-node at  $t=2t_k$  when the temporal shape of rephased single photon wave packet is

$$E_k(t) = E_o \exp\{-\Gamma|t - 2t_k|/2\} \sin S(t - 2t_k)/S, \quad (3.4)$$

where  $S = \sqrt{N|g_\sigma|^2 - (\Gamma/2)^2}$ ,  $t < 2t_k$ ,  $E_o$  is an arbitrary small field amplitude.

These modes are the self quantum modes of the QC that provide the perfect reversible coupling of multi-mode QM with processing nodes. Similar temporal shape was proposed recently for single-mode QM [38]. After qubit transfer, we switch off the coupling of the 2-nd node with resonator by changing the frequency of atoms in the 2-nd node. The same procedure can be fulfilled for transfer of qubit from QM node to the 3<sup>rd</sup> node. The described picture of the quantum transport can be realized in more complicated 2D scheme of QC with larger number of nodes as depicted in Fig. 4.

#### IV. QUANTUM PROCESSING

For realization of two-qubit gates we transfer the two qubits from QM node to the 2-nd and 3-rd processing nodes and equalize the carrier frequencies of the nodes at time moment  $t=0$  with some detuning from the resonator mode frequency  $\omega_2 - \omega_o = \Delta_2 = \omega_3 - \omega_o = \Delta_3 = \Delta$ . It yields to the following initial state of 2-nd and 3-rd nodes in the interaction picture

$$\psi_{in}(0) = \{\alpha_2 |0\rangle_2 + \beta_2 |1\rangle_2\} \{\alpha_3 |0\rangle_3 + \beta_3 |1\rangle_3\}, \quad (4.1)$$

where  $|\alpha_{2,3}|^2 + |\beta_{2,3}|^2 = 1$ . Here, we have introduced the following states:  $|0\rangle_m = |0_1, 0_2, \dots, 0_{N_m}\rangle$  corresponding to the ground state of the m-th node,  $|1\rangle_m = \sqrt{1/N} \sum_{j=1}^{N_m} |0_1\rangle |0_2\rangle \dots |1\rangle_j \dots |0_{N_m}\rangle$  and  $|2\rangle_m = \sqrt{2/N(N-1)} \sum_{i \neq j}^{N_m} |0_1\rangle |0_2\rangle \dots |1\rangle_i \dots |1\rangle_j \dots |0_{N_m}\rangle$  are the collective states of m-th node with single and two atomic excitations. Equal frequencies of the two nodes results in interaction of the atoms via the virtual processes of resonant circuit quanta determined by effective Hamiltonian [39, 40]  $\hat{H}_{eff} = \sum_{m=1}^3 \hat{H}_{node}^{(m)} + \hat{H}_{int}$ , where  $\hat{H}_{node}^{(m)} = \hbar\Omega_\sigma \sum_{i_m, j_m} S_{i_m}^+ S_{j_m}^-$  is a long-range spin-spin interaction in m-th node,  $\hat{H}_{int} = \hbar\Omega_\sigma \sum_{j_1, j_2=1}^N (S_{j_1}^+ S_{j_2}^- + S_{j_1}^- S_{j_2}^+)$  (where  $\Omega_\sigma = |g_\sigma|^2/\Delta$ ) describes a spin-spin interaction between the two nodes ( $N_2 = N_3 = N$ ). The spin-spin interaction with atoms of QM-node is suppressed because of the absence of resonance.

Let's introduce the collective basis states of the two nodes:  $|\psi\rangle_1 = |0\rangle_2 |0\rangle_3$ ,  $|\psi\rangle_2 = |1\rangle_2 |0\rangle_3$ ,  $|\psi\rangle_3 = |0\rangle_2 |1\rangle_3$ ,  $|\psi\rangle_4 = |1\rangle_2 |1\rangle_3$  and  $|\psi\rangle_5 = (1/\sqrt{2})\{|2\rangle_2 |0\rangle_3 + |0\rangle_2 |2\rangle_3\}$ . It is important that the Hamiltonian  $\hat{H}_{eff}$  has a matrix representation in the basis of the five states which is separated from other states of the multi-atomic system

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & N\Omega_\sigma & N\Omega_\sigma & 0 & 0 \\ 0 & N\Omega_\sigma & N\Omega_\sigma & 0 & 0 \\ 0 & 0 & 0 & 2N\Omega_\sigma & 2\Omega_\sigma\sqrt{N(N-1)} \\ 0 & 0 & 0 & 2\Omega_\sigma\sqrt{N(N-1)} & 2\Omega_\sigma(N-1) \end{pmatrix}. \quad (4.2)$$

By using initial state (4.1) we find the unitary evolution of the atomic systems which couples independently two pairs of the quantum states  $|\psi\rangle_2 \leftrightarrow |\psi\rangle_3$  and  $|\psi\rangle_4 \leftrightarrow |\psi\rangle_5$

$$\begin{aligned} \Psi_1(t) = & \alpha_2\alpha_3 |\psi\rangle_1 + \exp(-i\Omega_\sigma Nt) \\ & \{\beta_2\alpha_3 [\cos(\Omega_\sigma Nt) |\psi\rangle_2 - i \sin(\Omega_\sigma Nt) \psi_3] \\ & + \alpha_2\beta_3 [\cos(\Omega_\sigma Nt) |\psi\rangle_3 - i \sin(\Omega_\sigma Nt) \psi_2]\} \\ & + \exp(-i2\Omega_\sigma Nt) \beta_2\beta_3 \\ & \{\cos(2\Omega_\sigma Nt) |\psi\rangle_4 - i \sin(2\Omega_\sigma Nt) |\psi\rangle_5\}, \quad (4.3) \end{aligned}$$

where we have assumed a large number of atoms  $N \gg 1$ . The solution demonstrates two coherent oscillations with the frequency  $\Omega_\sigma N$  for the first pair  $|\psi\rangle_2 \leftrightarrow |\psi\rangle_3$  and with the double frequency  $2\Omega_\sigma N$  for the second pair  $|\psi\rangle_4 \leftrightarrow |\psi\rangle_5$ . The oscillations are drastically accelerated N-times comparing to the case of two coupled two-level atoms so we can use even bad common resonator with relatively lower quality factor. By taking into account our experimental situation and by assuming  $\Delta \approx 10\Delta_{in}$  we get  $\Omega_\sigma N = \gamma_1/10 = 1.884 \cdot 10^6$ .

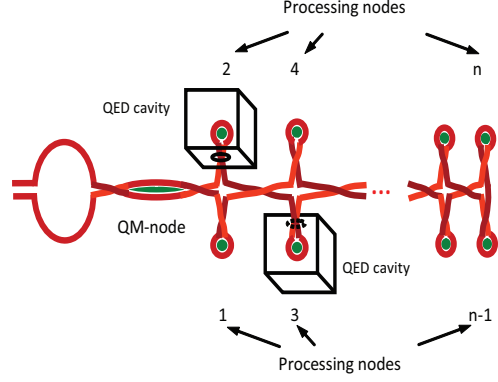


FIG. 5: Scheme of QC with processing gates for realization of  $\sqrt{iSWAP}$  gate. Scheme of QC with two (2-nd and 3-rd) processing nodes situated in local QED cavities characterized by the high quality factors. QM and all processing nodes are coupled via the common TW-line of the distributed electromagnetic resonator.

It is known [39, 40] that the evolution of the two coupled two level atoms can lead to  $iSWAP$  and  $\sqrt{iSWAP}$  gates. The  $iSWAP$  and  $\sqrt{iSWAP}$  gates work in the Gilbert space of four states  $|\psi\rangle_1, \dots, |\psi\rangle_4$  and these gates are important for realization of the complete set of the universal quantum gates [39, 40].  $iSWAP$  gate provides exchange of the two quantum states between the two nodes. In our case we get  $iSWAP$  gate occurs at shortened time  $t_{iSWAP} = \pi/2\Omega_\sigma N \cong 8.34 \cdot 10^{-7}$  sec.

$$\Psi_1(t_{iSWAP}) = \{\alpha_3 |0\rangle_2 - \beta_3 |1\rangle_2\} \{\alpha_2 |0\rangle_3 - \beta_2 |1\rangle_3\}. \quad (4.4)$$

We also note that by choosing different carrier frequencies we can realize the described  $iSWAP$  operation for many pairs of nodes simultaneously due to exploitation of the independent virtual quanta for each pair in the QED cavity. It is interesting that the  $iSWAP$  gate provides a perfect elimination of transfer of the initial state to the state  $|\psi\rangle_5$  that occurs only at  $t=t_{iSWAP}$ . The situation is more complicated for realization of  $\sqrt{iSWAP}$  gate because it is impossible to eliminate state  $|\psi\rangle_5$  with evolution based on matrix (3.4). Below, we propose a universal mechanism for *Collective Dynamical Elimination* (CDE -procedure) of the state  $|\psi\rangle_5$  for realization of  $\sqrt{iSWAP}$  gate by using the multi-atomic ensemble encoding single qubits and cavity mediated collective interaction.

Scheme of spatial arrangement of the processing nodes and cavities for realization of the  $\sqrt{iSWAP}$  is presented in Fig. 5.

Here, we insert the 2-nd and 3-rd nodes in two different single mode QED cavities characterized by high quality factors for  $\pi$ -modes. We assume that each  $\pi$ -mode interacts only with the atoms of one node and is decoupled from the basic circuit field mode that is

possible for large enough spectral detuning of the local QED cavity modes. Thus we take the additional field Hamiltonians  $H_\pi = \sum_m^{2,3} \hbar\omega_{\pi_m} \hat{a}_{\pi_m}^+ \hat{a}_{\pi_m}$  and the modes interaction with the atoms of 2-nd and 3<sup>rd</sup> nodes  $H_{r-a}^{(\pi)} = \sum_m^{2,3} \sum_{j_m=1}^{N_m} \left( g_\pi^{(m)} S_{j_m}^+ \hat{a}_{\pi_m} + g_\pi^{(m)*} S_{j_m}^- \hat{a}_{\pi_m}^+ \right)$  is a coupling constant of the interaction between atom and local m-th  $\pi$  mode). By assuming a large enough spectral detuning of atomic frequencies from the field mode and absence of real photons in the QED cavities we find the following effective Hamiltonian similar to previous section

$$\begin{aligned}
H_s = & \sum_m^{2,3} \sum_{j_m=1}^{N_m} \hbar\omega_m S_{j_m}^z \\
& + \sum_m^{2,3} \sum_{i_m, j_m}^{N_m} \frac{|g_\sigma^{(m)}|^2}{\hbar\Delta_m} S_{i_m}^+ S_{j_m}^- + \sum_m^{2,3} \sum_{i_m, j_m}^{N_m} \frac{|g_\pi^{(m)}|^2}{\hbar\Delta'_m} S_{i_m}^+ S_{j_m}^- \\
& + \frac{1}{2\hbar} \left( \frac{1}{\Delta_2} + \frac{1}{\Delta_3} \right) \sum_{j_1 j_2}^{N_m} (|g_\sigma^{(2)} g_\sigma^{(3)}| e^{i\varphi(j_2, j_3)} S_{j_2}^+ S_{j_3}^- \\
& + g_\sigma^{(2)*} g_\sigma^{(3)} e^{-i\varphi(j_2, j_3)} S_{j_2}^- S_{j_3}^+), \tag{4.5}
\end{aligned}$$

where  $\Delta_{2,3} = \omega_{2,3} - \omega_o$  are the atomic frequency detunings from the circuit mode and  $\Delta'_{2,3} = \omega_{2,3} - \omega_\pi$  are the atomic detunings from the frequency of the local QED cavities having the same frequency  $\omega_\pi$  ( $\omega_{\pi_2} = \omega_{\pi_3} = \omega_\pi$ );  $\varphi(j_2, j_3)$  is a phase which is assumed to be constant for the node size smaller than the mode wavelength. To be concrete below we take  $\Delta'_{2,3} = -\Delta_{2,3} = -\Delta$ ,  $\Delta > 0$ .

The second and third terms in Eq. (4.3) describes the atom-atom interactions insight each node via the exchange of  $\sigma$  and  $\pi$  virtual photons, while the last term describes the interaction due to the exchange of virtual  $\sigma$  photons between the atoms seated in different nodes. Again by assuming equal number of atoms in the two nodes  $N_2 = N_3 = N$ , we get the following matrix representation for the new effective Hamiltonian  $\hat{H}_{eff}$  in the basis of the five states

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & \Omega_s N & \Omega_\sigma N & 0 & 0 \\
0 & \Omega_\sigma N & \Omega_s N & 0 & 0 \\
0 & 0 & 0 & 2\Omega_s N & 2\Omega_\sigma \sqrt{N(N-1)} \\
0 & 0 & 0 & 2\Omega_\sigma \sqrt{N(N-1)} & 2\Omega_s(N-1)
\end{pmatrix}, \tag{4.6}$$

where  $\Omega_s = \Omega_\sigma + \Omega_\pi$ ,  $\Omega_\sigma = |g_\sigma|^2/\Delta$ ,  $\Omega_\pi = -|g_\pi|^2/\Delta$ .

For the initial state (4.1), the atomic wave function evolves as follows

$$\begin{aligned}
\Psi_2(t) = & \alpha_2 \alpha_3 |\psi\rangle_1 + \exp[-i\Omega_s N t] \\
& \{ \beta_2 \alpha_3 [\cos(\Omega_\sigma N t) |\psi\rangle_2 - i \sin(\Omega_\sigma N t) |\psi\rangle_3] \\
& + \alpha_2 \beta_3 [\cos(\Omega_\sigma N t) |\psi\rangle_3 - i \sin(\Omega_\sigma N t) |\psi\rangle_2] \} \\
& + \exp[-i\Omega_s (2N-1)t] \beta_2 \beta_3 \\
& \{ [\cos(St) - i \frac{\Omega_s}{S} \sin(St)] |\psi\rangle_4 \\
& - i \frac{2\Omega_\sigma \sqrt{N(N-1)}}{S} \sin(St) |\psi\rangle_5 \}, \tag{4.7}
\end{aligned}$$

where  $S = \sqrt{4\Omega_\sigma^2 N(N-1) + \Omega_s^2}$ .

We choose the following parameters for the evolution of (4.7) providing the dynamical elimination of the state  $|\psi\rangle_5$

$$1) \quad \Omega_\sigma N t = \pi \left( \frac{1}{4} + \frac{1}{2} \mu + n \right); \mu = 0, 1; n = 0, 1, \dots, \tag{4.8}$$

$$2) \quad St = \pi k, k = 1, 2, \dots, \tag{4.9}$$

that leads to the following entangled state of the nodes

$$\begin{aligned}
\Psi_2(t) = & \alpha_2 \alpha_3 |\psi\rangle_1 \\
& + (-1)^n \frac{1}{\sqrt{2}} \exp[-i\Omega_s N t] \{ [(-1)^\mu \beta_2 \alpha_3 - i \alpha_2 \beta_3] |\psi\rangle_2 \\
& + [(-1)^\mu \alpha_2 \beta_3 - i \beta_2 \alpha_3] |\psi\rangle_3 \} \\
& + (-1)^k \exp[-i\Omega_s (2N-1)t] \beta_2 \beta_3 |\psi\rangle_4, \tag{4.10}
\end{aligned}$$

where  $\Omega_s$  is determined by two conditions 4.8, 4.9. In particular we write three set of parameters for possible realizations of CDE procedure characterized by weaker coupling of atoms with  $\sigma$ -mode ( $n=0,1; \mu=0,1$ ):

$$\begin{aligned}
1) n = 0, \mu = 0, k = 1 : \Omega_\sigma N t = \pi/4, St = \pi \\
\rightarrow |\Omega_s| t = \sqrt{3}\pi, \frac{|\Omega_s|}{\Omega_\sigma N} = 4\sqrt{3} \approx 6.92; \\
2) n = 0, \mu = 1, k = 2 : \Omega_\sigma N t = 3\pi/4, St = 2\pi \\
\rightarrow |\Omega_s| t = \sqrt{7}\pi, \frac{|\Omega_s|}{\Omega_\sigma N} = \frac{4\sqrt{7}}{3} \approx 5.53; \\
3) n = 1, \mu = 0, k = 3 : \Omega_\sigma N t = 5\pi/4, St = 3\pi \\
\rightarrow |\Omega_s| t = \sqrt{11}\pi, \frac{|\Omega_s|}{\Omega_\sigma N} = \frac{4\sqrt{11}}{5} \approx 2,65.
\end{aligned} \tag{4.11}$$

Another interesting case occurs for stronger coupling of the atoms with local  $\pi$ -modes of the QED cavities when  $|\Omega_\pi| \gg N\Omega_\sigma$ . Here, we get a *Collective Blockade* of state  $|\psi\rangle_5$  that provides the following atomic evolution

$$\begin{aligned}
\Psi_2(t) = & \alpha_2 \alpha_3 |\psi\rangle_1 + \exp[-i\Omega_s N t] \\
& \{ \beta_2 \alpha_3 [\cos(\Omega_\sigma N t) |\psi\rangle_2 - i \sin(\Omega_\sigma N t) |\psi\rangle_3] \\
& + \alpha_2 \beta_3 [\cos(\Omega_\sigma N t) |\psi\rangle_3 - i \sin(\Omega_\sigma N t) |\psi\rangle_2] \} \\
& + \exp[-i2\Omega_s N t] \beta_2 \beta_3 |\psi\rangle_4, \tag{4.12}
\end{aligned}$$

yielding the entangled state of the two nodes if only the condition 4.8 is satisfied. So here, we can vary the coupling constant  $\Omega_\sigma$  and interaction time  $t$  in some possible

intervals providing a realization of general  $iSWAP$  gate with arbitrary tunable angle of rotation  $\Omega_\sigma Nt$ .

Thus we can perform  $\sqrt{iSWAP}$  gate which entangles the two qubits and provides a complete set of universal quantum gates together with single qubit operations. Here, we note that the single qubit gates can be performed by transfer the atomic qubit to photonic qubit in waveguide where it can be rotated on arbitrary angle by usual optical means [7]. We can also return the qubit back to QM node on demand as it has been shown above. Another possibility to implement the single qubit gates is to transfer it to the node with single resonant atom which state can be controlled by external classical field. Also we can mark the principle possibility to exploit the collective blockade mechanism for realization of the single qubit gate similar to approach developed for usual blockade mechanism [41].

## V. DISCUSSION

In this work we proposed a novel architecture of solid state QC based on multi-atomic ensembles (nodes) with integrated QM in the flexible circuit with local QED cavities. The evaluated scheme of QC can be realized in 2D and 3D architecture due to flexibility of the waveguide circuit providing a quantum transport between different nodes.

For the first time we found the optimal moderate parameters of the atoms and circuit for 100% efficiency of storage and retrieval of multi qubit photon fields that opens a door for practical realization of ideal multi-mode quantum memory. Then we revealed self quantum modes of the quantum computer for reversible transfer of the qubits between the quantum memory node and arbitrary other modes. Also, we show a realization of fast  $iSWAP$  gate for arbitrary pair of two nodes which drastically accelerates basic quantum processes. Direct coupling of large number of the node pairs can be simultaneously realized in the circuit that opens a practical road for fast parallel quantum processing of many accelerated  $iSWAP$  gates.

In this work, we also proposed new mechanisms for suppression of undesired (excess) state based on the collective coherent multi-atomic dynamics and demonstrate its application for realization of  $\sqrt{iSWAP}$  gate. In the first case we use a *Collective Dynamical Elimination* of the excess quantum state at some fixed moment of times. The second collective mechanism exploiting stronger coupling with additional local modes provides a permanent suppression of any transition to the excess state. We note that the two proposed collective blockade mechanisms are determined by the nonlinear collective interaction of the atoms with the electromagnetic field modes therefore the mechanisms are free from the decoherence in comparison with the well-known blockade based on the direct dipole-dipole interactions.

We have demonstrated that the proposed QC architecture is scalable for construction of many coupled distinct quantum circuits that opens a promising way for realization of multi-qubit QC with a number of coupled qubits limited only by the atomic coherence time. We anticipate that the proposed atomic QC can principally work at room temperatures for example on NV-centers in a pure diamond which looks now one of the promising candidates for the qubit carriers up to  $\sim 10^{-3} - 1$  sec. timescale [42]. We also believe that promising situation for realization of  $\sqrt{iSWAP}$  gate in our approach is in solid state media with atoms characterized by large dipole moments or the quantum dots in semiconductors.

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