Coherence of large-scale entanglement

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We report the creation of large Schrödinger-Cat states with up to 14 qubits. By investigating their coherence over time, we observe a decay proportional to the square of the number of qubits. The observed decay agrees with our theoretical model which assumes a system affected only by correlated, Gaussian phase noise. Our model holds for the majority of current experimental systems developed towards quantum computation and quantum metrology.

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Quantum states can show non-classical properties, for example, their superposition allows for (classically) counter-intuitive situations such as a particle being at two places at the same time. Entanglement can extend this paradox even further, e.g., the state of one subsystem can be affected by a measurement on another subsystem without any apparent interaction [1]. These concepts, although experimentally frequently verified, contrast with our classical perception and lead to several questions. Is there a transition from a quantum to a classical regime? Under which conditions does that transition take place? And why? The creation of large-scale multi-particle entangled quantum states and the investigation of their decay towards classicality may provide a better understanding of this transition [2, 3].

Usually, decoherence mechanisms are used to describe the evolution of a quantum system into the classical regime. One prominent example is the spontaneous decay of the excited state of an atom. In a collection of atoms, the decay of each would be expected to be independent of the others. Therefore, the number of decay processes in a fixed time window would intuitively be proportional to the number of excited atoms. This assumption, however, can be inaccurate. Decoherence effects can act collectively and produce "superradiance", a regime in which the rate of spontaneous decay is proportional to the square of the number of excited atoms [4]. Such collective decoherence can also occur in multi-qubit registers, an effect known as "superdecoherence" [5]. This particularly applies to most currently used qubits which are encoded in energetically non-degenerate states. In these systems, a phase reference (PR) is required to perform coherent operations on a quantum register. Noise in this PR thus collectively affects the quantum register.

In the following we introduce a model describing a quantum register in the presence of correlated phase noise. More specifically, we investigate N-qubit Schrödinger-Cat (SC) states of the form $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\dots0\rangle + |1\dots1\rangle)$. These states are the archetype of multi-particle entanglement and play an important role in the field of quantum metrology [6] for quantum-mechanically enhanced sensors. Employing up to 14 genuinely multi-particle-entangled ion-qubits in a SC state, we predict and verify the presence of superdecoherence which scales quadratically with the number of qubits N. This accelerated decay impacts any system of energetically non-degenerate qubits where a dominant correlated phase-noise affects the complete quantum register.

We model collective phase fluctuations acting on the quantum register with a Hamiltonian of the form

$$H_{\text{noise}} = \frac{\Delta E(t)}{2} \sum_{k=1}^{N} \sigma_z^{(k)},$$

where $\Delta E(t)$ denotes the strength of the fluctuations, and $\sigma_z^{(k)}$ a phase rotation on the k-th ion. Under this Hamiltonian, the initial state of the system $|\psi(0)\rangle$ evolves into $|\psi(t)\rangle = \exp(-i\int_0^t \mathrm{d}\tau\ H_{\text{noise}}(\tau))|\psi(0)\rangle$. As a measure of state preservation, we use the fidelity $F(t) = \overline{|\langle\psi(0)|\psi(t)\rangle|^2}$, where the bar refers to an average over all realizations of random phase fluctuations. The decay of this fidelity can be conveniently described by

$$F(t) = \frac{1}{2}(1 + \exp(-2\epsilon(N,t))),$$

where the effective error probability for stationary Gaussian random processes is given by

$$\epsilon(N,t) = N^2 \frac{1}{2\hbar^2} \int_0^t \mathrm{d}\tau (t-\tau) \overline{\Delta E(\tau)\Delta E(0)}. \quad (1)$$

The intuition of an error probability can be recovered in the limit of small $\epsilon(N,t)$ since the fidelity decays as $F \approx 1 - \epsilon(N,t)$. For correlated Gaussian phase noise the

TABLE I: Populations, coherence, and fidelity with a N-qubit SC-state of experimentally prepared states. Entanglement criterion supported by σ standard deviations. All errors in parenthesis, one standard deviation.

Number of ions	2	3	4	5	6	8	10	12	14
Populations, %	99.50(7)	97.6(2)	97.5(2)	95.0(7)	90(1)	85.50(8)	67.7(8)	37.80(5)	47.3(1)
Coherence, %	97.8(3)	96.5(6)	93.9(5)	92.8(8)	96.4(5)	78.7(7)	58.1(9)	41.5(10)	45.4(13)
Fidelity, %	98.6(2)	97.0(3)	95.7(2)	94.4(5)	93.1(5)	82.1(3)	62.9(5)	39.6(4)	46.3(6)
Entanglement criteria [7], σ	283	150	183	100	98	102	42	21	18

effective error probability is then always proportional to N^2. Therefore, initially negligible correlated phase noise can lead to unexpectedly high error probabilities as the size of the quantum register increases.

We can also obtain the Markovian and the static result for short times by considering a correlation function of the form $\overline{\Delta E(\tau)\Delta E(0)} = \overline{\Delta E^2}\exp(-\gamma t)$ in the error probability (Eq. 1). Our noise model then leads to a fidelity

$$F(t) = \frac{1}{2}(1 + \exp(-\frac{1}{T_2\gamma}\{\exp(-\gamma t) + \gamma t - 1\})),$$

where $T_2 = \hbar^2\gamma/(N^2\langle\Delta E^2\rangle) \propto 1/N^2$ corresponds to the decay time in the Markovian limit. For $\gamma t \gg 1$, we recover the Markovian limit $F_{\text{mark}}(t) \cong 1/2(1 + \exp(-t/T_2))$ and for $\gamma t \ll 1$, the static result for short times is $F_{\text{stat}}(t) \cong 1/2(1 + \exp(-\frac{1}{2}(t/\tau)^2))$ with a charactistic time $\tau = \hbar/(N\sqrt{\langle\Delta E^2\rangle}) \propto 1/N$.

Experimentally, we study correlated noise in an ion-trap quantum processor. Our system consists of a string of $^{40}Ca^+$ ions confined in a linear Paul trap where each ion represents a qubit [8]. The quantum information is encoded in the $S_{1/2}$(m=$-1/2$) $\equiv |1\rangle$ ground state and the metastable $D_{5/2}$(m=$-1/2$) $\equiv |0\rangle$ state. Each experimental cycle consists of three stages, (i) initializing the qubits and the center of mass mode in a well defined state, (ii) performing the entangling gate operation, and (iii) characterizing the quantum state. The qubits are initialized by optical pumping into the $S_{1/2}$(m=$-1/2$) state while the motion is brought to the ground state by Doppler cooling followed by sideband cooling. Qubit manipulation is realized by a series of laser pulses of equal intensity on all ions. The electronic and vibrational states of the ion string are manipulated by setting the frequency, duration, intensity, and phase of the pulses. Finally, the state of the ion qubits is measured by scattering light at 397 nm on the $S_{1/2} \leftrightarrow P_{1/2}$ transition and detecting the fluorescence simultaneously with a CCD camera and a photomultiplier tube (PMT). The camera detection effectively corresponds to a measurement of each individual qubit in the $\{|0\rangle, |1\rangle\}$ basis, while the PMT only detects the number of ions being in $|0\rangle$ or $|1\rangle$.

In our system, SC states of the form $(|0\dots0\rangle + |1\dots1\rangle)/\sqrt{2}$ are created from the state $|1\dots1\rangle$ through a high-fidelity Mølmer-Sørensen (MS) entangling interaction [9, 10]. Assessing the coherence, fidelity, and entanglement of SC states is straightforward as the density matrix ideally consists of only four elements: two diagonal elements corresponding to the populations of $|0\dots0\rangle$ and $|1\dots1\rangle$ as well as of two off-diagonal elements corresponding to the relative coherence. The diagonal elements of the density matrix ρ are directly measured by fluorescence detection and allow to infer the SC populations $P = \rho_{0\dots0,0\dots0} + \rho_{1\dots1,1\dots1}$. The off-diagonal elements of the density matrix are accessible via the observation of parity oscillations [11] as follows. After the SC-state is generated, all qubits are collectively rotated by an operation $\bigotimes_{j=1}^{N}\exp(i\frac{\pi}{4}\sigma_\phi^{(j)})$ where $\sigma_\phi^{(j)} = \sigma_x^{(j)}\cos\phi + \sigma_y^{(j)}\sin\phi$ is defined by the corresponding Pauli operators on the j-th qubit. By varying the phase ϕ, we observe oscillations of the parity $P = P_{\text{even}} - P_{\text{odd}}$ with $P_{\text{even/odd}}$ corresponding to the probability of finding the state with an even/odd number of excitations. The amplitude of these oscillations directly gives the coherence $C = \rho_{0\dots0,1\dots1} + \rho_{1\dots1,0\dots0}$ of the state. The fidelity of the SC state is then given by $F = (P + C)/2$, where a $F > 50\%$ implies genuine N-particle entanglement [11]. States with a $F < 50\%$ can still be genuinely N-particle entangled if they satisfy other criteria. We apply the criterion defined in Ref. [7] which can be used in conjunction with the procedure described above.

We have experimentally prepared SC states of {2,3,4,5,6,8,10,12,14} ions and achieved the populations, coherences, and fidelities shown in Table I. Figure 1 shows the observed parity oscillations. The obtained data support genuine N-particle entanglement according to the criteria in Ref. [7] for up to 14 qubits by at least 18 standard deviations and significantly more for fewer qubits. The Poissonian statistics of the PMT fluorescence data is accounted for by a data analysis based on Bayesian inference [12].

After creating these SC states, we investigated their coherence as a function of time, and thereby the error probability in a certain time window, by adding a waiting time between the creation of the quantum state and the parity analysis. Thus, the error probability of a SC state can be directly compared with that of a single qubit, yielding the relative error probability $\epsilon(N) = \epsilon(N,t)/\epsilon(1,t) = N^2$. A numerical fit to the data as shown in Fig. 2 leads to an

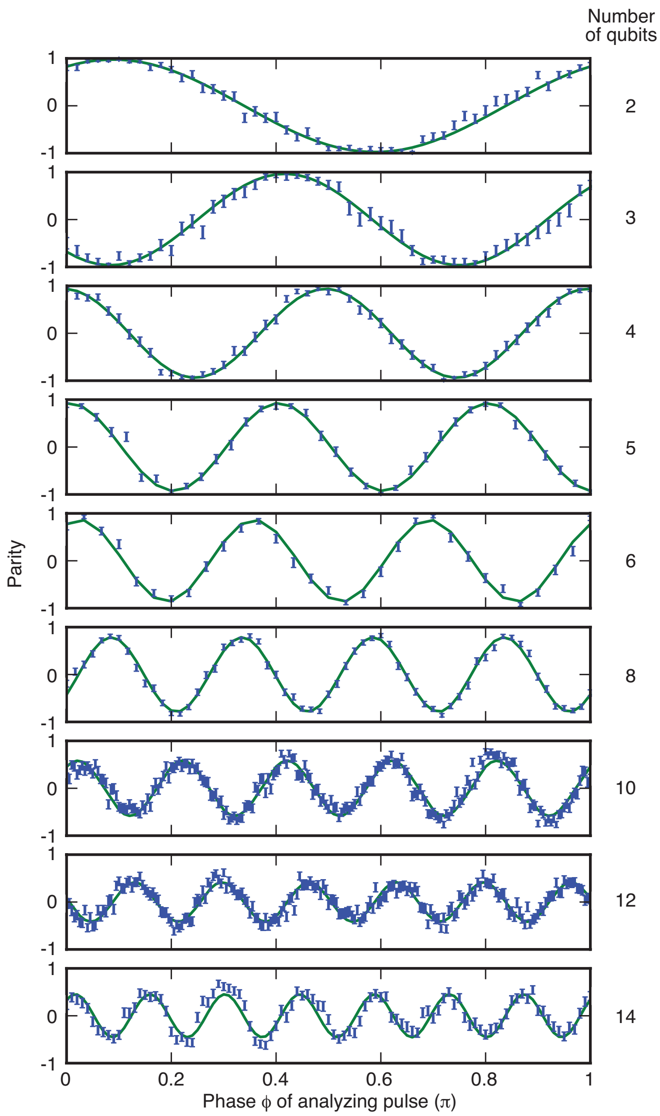

FIG. 1: Parity oscillations observed on {2,3,4,5,6,8,10,12,14}-qubit Schrödinger-cat states.

$N^{2.0(1)}$ scaling law, in agreement with our predictions for correlated Gaussian noise. An equivalent interpretation is that the coherence of an N-qubit SC state decays by a factor N^2 faster than for a single qubit.

As several systems experience correlated noise, this superdecoherence will eventually limit the overall performance of large-scale quantum registers (unless logical qubits are encoded in sufficiently noise-insensitive subspaces [13, 14]). In our experiment, the noise affecting the quantum register is mainly caused by fluctuations of the homogeneous magnetic field due to a varying current in the field generating coils. By decreasing this noise, the single-qubit coherence time had a ten-fold improvement from 8(1) ms to 95(7) ms. Such coherence time is approximately a factor of 1000 longer than the gate time of the MS interaction of approximately 100 μs. This long coherence time would, in principle, enable the implementation of algorithms with 10 and more qubits. In the presence of correlated noise, however, this N^2 scaling can potentially be the main limitation for several experiments. A correlated phase-noise environment with a single-qubit characteristic error probability of only 0.01 leads to a 10-qubit SC-state relative error probability $\epsilon(N = 10) = 0.01 \times 10^2 \approx 1$; most of the state's phase information is then lost.

We verify that correlated phase noise is dominant in our experiment by preparing a state which is insensitive to this noise. We create the state $|00001111\rangle + |11110000\rangle)/\sqrt{2}$, which is locally equivalent to an 8-qubit SC state. This state is realized by an MS interaction starting from the state $|00001111\rangle$. Its coherence properties are investigated as above using a local transformation into a SC state. The state shows a coherence time of 324(42) ms. This result is consistent with a lifetime-limited quantum state with an effective lifetime of one fourth of a single qubit of 1.17 s. In our apparatus, this extension of the coherence time relative to the SC state (or even the single-qubit case) can only be explained by correlated noise affecting the entire quantum register. Employing such insensitive states will therefore be crucial for large-scale quantum information processing affected by correlated phase noise.

Being able to efficiently generate entangled quantum states involving 10 and more qubits opens a new range of applications. Our system represents the basic building block for quantum simulation experiments [15] to investigate complex mechanism such as the magnetic sense of birds [16], to perform exponentially compressed spin-chain simulations [17], and to better understand cosmology and space-time [18]. It may serve as a very well-controlled testbed for fundamental questions in quantum physics such as the investigation of the cross-over from superpositions in quantum systems to defined states in macroscopic systems with SC states [2]. As an example of an even larger system we created a 64-ion string in a segmented ion trap [19]; as shown in Fig. 3. Since most applications require operations on individual ions, we shaped the trapping potential to achieve an even inter-ion spacing of 8(1) μm which is larger than the diffraction limit of the qubit-driving laser [20]. Storing the state of this 64-qubit quantum register already exceeds the memory capacity of existing supercomputers by several orders of magnitude [21].

In conclusion, we have analyzed the decay of SC-states in an ion-trap based quantum computer. We find a dependency that scales quadratically with the number of qubits and thus shows superdecoherence . This mechanism is present in every other experiment that relies on a phase reference for performing quantum information processing with energetically non-degenerate qubits. We achieve coherence times of about 100 ms on an optical

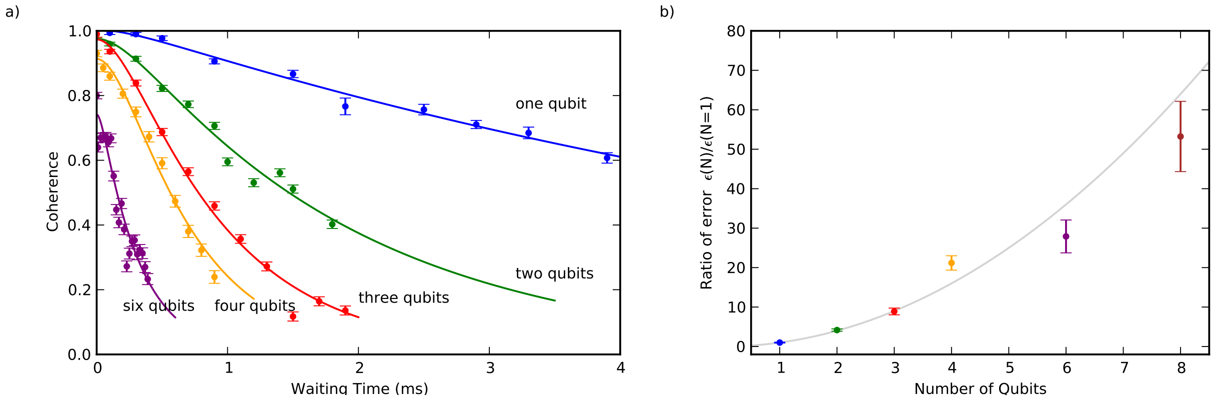

FIG. 2: Coherence decay and relative error probability $\epsilon(N)$ of Schrödinger-Cat states. a) Remaining coherence as a function of time for a single qubit (blue) and SC states of 2 (green), 3 (red), 4 (orange), and 6 (purple) qubits. b) The observed relative error probability is consistent with a scaling behavior proportional to N^2 as indicated by the gray line. The coherence of an N-qubit SC state then decays by a factor N^2 faster than the coherence of a single qubit.

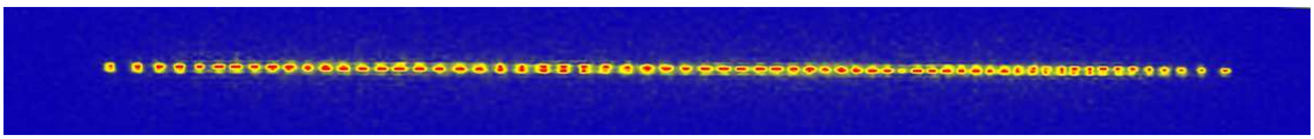

FIG. 3: Current investigations on large quantum registers performed in parallel. A quantum register consisting of 64 ions is shown. Creating a SC state in this quantum register could become the basis for advanced, fundamental investigations [2].

qubit which is a factor of 1000 longer than an entangling gate operation in the same system. Using a single-step entangling gate based on the ideas of Mølmer and Sørensen, we create and verify genuine multiparticle entangled states with up to 14 qubits. The employed techniques represent an encouraging building block for upcoming realizations of advanced quantum computation and quantum simulations with more than 10 qubits. We envision this to be of particular interest in segmented ion traps tailored for large quantum registers.

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