Simulation of nonuniform interconnects by harmonic differential quadrature method

Qin-Wei Xu, Zheng-Fan Li and Wen Chen

The harmonic differential quadrature (HDQ) method is employed to simulate the nonuniform interconnects in very large scale integration and multichip modules. The HDQ method reduces partial differential equations to ordinary differential equations. Being a direct numerical technique, it can easily be applied to the simulation of nonuniform interconnects.

Introduction: The simulation of interconnects is of great interest in modern VLSI (very large scale integration) and multichip module (MCM) design. Many methods have been proposed to deal with the uniform interconnects. When the nonuniform interconnects are analysed, they are generally segmented into many sections, each of which is regarded as being uniform. Such a process makes the computing quantity increase considerably [1]. The spectral method can be employed directly to analyse the nonuniform transmission lines. It approximates spatial or time derivatives by constructing a global interpolant through discrete data points. However, this method suffers from complexity of derivation and computation [2].

Classical numerical techniques, such as finite difference methods, can be employed easily in engineering and can provide very accurate results by using a large number of grid points. Despite their applicability, it is impractical to apply them in the transient analysis of transmission lines because the required computer capacity is always too large. In this Letter, a direct numerical technique called the harmonic differential quadrature (HDQ) method [3, 4] is employed to analyse the nonuniform transmission lines. It is based on the idea that a derivative, $\partial/\partial x$, can be expressed as a weighted linear sum of all the function values at all mesh points along the x-direction. Owing to the global approximation in the HDQ method, it usually requires fewer grid points than other numerical methods to achieve accurate results. Being a direct numerical technique, the method can be applied in almost any case.

Harmonic differential quadrature method: For a single lossy transmission line whose length is d, let $r(z)$, $l(z)$, $c(z)$ and $g(z)$ be the resistance, inductance, capacitance and conductance per unit length (PUL) at point z ($z \in [0, d]$), respectively. If the length is normalised to 1, then the other normalised parameters are: $R(x) = r(z)d$, $L(x) = l(z)d$, $C(x) = c(z)d$ and $G(x) = g(z)d$ ($x \in [0, 1]$). The distributed voltage $v(x, t)$ and current $i(x, t)$ at location x and time t can be described by Telegrapher's equations:

$$\frac{\partial}{\partial x}v(x,t) = -L(x)\frac{\partial}{\partial t}i(x,t) - R(x)i(x,t) \qquad (1a)$$

$$\frac{\partial}{\partial x}i(x,t) = -C(x)\frac{\partial}{\partial t}v(x,t) - G(x)v(x,t) \qquad (1b)$$

$$x \in [0,1], \;\; t \in [0, +\infty)$$

and boundary conditions

$$F(v(0,t), i(0,t)) = F_0(t) \qquad F(v(1,t), i(1,t)) = F_1(t) \qquad (2)$$

Assuming that the functions $v(x, t)$ and $i(x, t)$ satisfying the above equations are smooth [3], we can make the approximate relations of differential quadrature:

$$\frac{\partial}{\partial x}f(x_i,t) = \sum_{j=1}^{N} a_{ij} f(x_j,t) \qquad i = 1,2,...,N \qquad (3)$$

where $f(x, t) = v(x, t)$ or $f(x,t) = i(x, t)$, a_{IJ}s are global coefficients to be determined and N is the order of differential quadrature.

From eqns. 1 and 3 we can obtain:

$$\mathbf{Av} = -\mathbf{L}\frac{\partial}{\partial t}\mathbf{i} - \mathbf{Ri} \qquad (4a)$$

$$\mathbf{Ai} = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{v} - \mathbf{Gv} \qquad (4b)$$

where

$$\mathbf{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$$

$$\mathbf{v} = [v(x_1,t), v(x_2,t), ..., v(x_N,t)]^T$$

$$\mathbf{i} = [i(x_1,t), i(x_2,t), ..., i(x_N,t)]^T$$

$$\mathbf{L} = \mathrm{diag}\{L(x_1), L(x_2), ..., L(x_N)\}$$

$$\mathbf{C} = \mathrm{diag}\{C(x_1), C(x_2), ..., C(x_N)\}$$

$$\mathbf{R} = \mathrm{diag}\{R(x_1), R(x_2), ..., R(x_N)\}$$

$$\mathbf{G} = \mathrm{diag}\{G(x_1), G(x_2), ..., G(x_N)\}$$

However, eqn. 4*a* and *b* are not independent, and do not have nontrivial solutions. By analogy with the finite difference method using the backward Euler's method, we can substitute equations at $x = 0$ by the boundary conditions in eqn. 2.

In eqn. 3, $x_1 = 0$ and $x_N = 1$ hold. Hence, under the assumption that eqn. 3 is valid, we have succeeded in reducing the partial differential equations (PDEs) in eqn. 1 to ordinary differential equations (ODEs) in eqn. 4. It turns out that relatively low order differential quadrature is needed, so the total amount of storage and time required on the machine is thus quite low.

The most important step of the HDQ method is to determine the weighting coefficients a_{IJ}. In general, a set of orthogonal functions is selected to meet with eqn. 3. Because many solutions in circuits use Fourier series, sine and cosine functions (harmonic functions) are naturally selected to determine the weighting coefficients. We require that eqn. 3 is exact when $v(x, t)$ and $i(x, t)$ take the following forms:

$$g(x) = \Big\{1, \sin\pi x, \cos\pi x, \sin 2\pi x, \cos 2\pi x, \\ ..., \sin\frac{N-1}{2}\pi x, \cos\frac{N-1}{2}\pi x\Big\} \qquad (5)$$

where N is the number of grid points, which is normally an odd number, and $0 \le x \le 1$. As described in [3], N sets of N linear algebraic equations are obtained:

$$\sum_{j=1}^{N} a_{ij} g(x_j) = g'(x_i) \qquad i = 1,2,...,N \qquad (6)$$

Hence the weighting coefficients are completely determined. Such an approach is called the harmonic differential quadrature (HDQ) method [4]. In the process of the HDQ method, the points $x_1 = 0$ and $x_N = 1$ should be included for computing the responses of the two ends of the transmission lines, and the other can be either equally spaced or not.

It is significant that for the HDQ method, once the grid points are fixed, the global coefficients are determined. So the global coefficients for each approach can be computed precedingly and saved as fixed constants. In fact, equally spaced grid points are usually selected. In such cases, all the global coefficients can be totally determined.

For completeness, the error estimation is given here. In eqn. 3, there is an error $e(x)$ in the Nth order differential quadrature to approximate $\partial f/\partial x$. Provided that

(i) $\partial^N f/\partial x^N$ is continuous, and $\partial^N f/\partial x^N \le K$, K is a constant

(ii) among points x_J, $j = 1, 2, ..., N-1$, $\max(x_{J+1} - x_J) = h$

the error can be evaluated by [3]:

$$|e'(x)| \le K\frac{h^{N-1}}{(N-1)!}$$

As illustrated by the case of a single transmission line, the method can easily be applied to coupled lines.

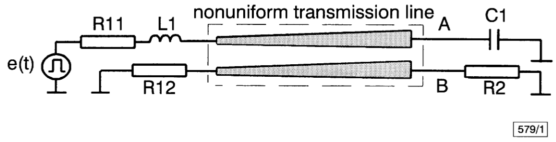

Fig. 1 *Circuit for example*

Numerical results: The circuit (see Fig. 1) includes a coupled nonuniform transmission line whose parameters are:
$d = 0.48$m, $R11 = R12 = R2 = 50\Omega$, $L1 = 10$nH, $C1 = 2$pF;
$L(x) = 387/(1 + K(x))$nH/m, $L_M(x) = K(x)L(x)$;
$C(x) = 104.3/(1 - K(x))$pF/m, $C_M(x) = -K(x)C(x)$;
$R(x) = 12\Omega$/m, $R_M(x) = 1\Omega$/m; $G(x) = G_M(x) = 0$;
$K(x) = 0.25\,(1 + 0.6\sin(\pi x + \pi/4))$;

where $L(x)$, $C(x)$, $R(x)$ and $G(x)$ are self-parameters of transmission lines; $Lm(x)$, $Cm(x)$, $Rm(x)$ and $Gm(x)$ mutual-parameters; $K(x)$ is a function of space. The signal is 1.5ns, rise/fall time, plus 3ns, 1V, top square voltage.

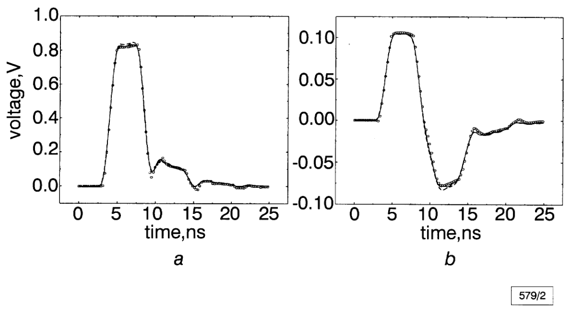

Fig. 2 *Results for example*

——— HDQ9
– – – – HDQ7
○ MC

We take seven equally spaced points in the HDQ method. That means that , besides the two end points, another five grid points are sampled along the transmission line. For comparison, the method of characteristics (MC), which is believed to be an efficient method, is also employed in this example. To apply the MC, we segmented the nonuniform transmission lines into eight sections, each of which is regarded as being uniform. The time-step method is employed in both the HDQ and the MC methods. For each time step, the seventh-order HDQ method needs to solve a set of 28×28 equations, while the MC method solves a set of 34×34 equations [5]. For more accurate results, we furthermore take the ninth-order HDQ method, adopting equally spaced points. The responses of the seventh-order HDQ metbod (labelled as HDQ7) and the ninth-order HDQ method (labelled as HDQ9) are shown in Fig. 2, altogether with those of the MC.

Conclusions: The harmonic differential quadrature method is first employed to simulate nonuniform interconnects in VLSI and MCMs. It is based on the idea that the derivative of a function with respect to a co-ordinate direction can be approximated by a weighted linear sum of all the function values at every mesh point. The process of the HDQ method is considerably simple. This method requires fewer grid points than other traditional numerical methods to achieve accurate results. Being a simple direct numerical technique, the HDQ method can easily be applied in the simulation of nonuniform interconnects. It can circumvent the difficulties of programming complex algorithms, as well as excessive use of storage and computer time. Numerical experiments show that considerable accurate solutions can be calculated rapidly at the cost of computing the quantities at a few points along the transmission lines.

Acknowledgment: This work has been supported by the National Natural Science Foundation, China.

 19 May 1998
Electronics Letters Online No: 19981530

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References

1 MAO, J.-F., and LI, Z.-F.: 'Analysis of the time response of nonuniform multiconductor transmission lines with a method of equivalent cascaded network chain', *IEEE Trans.*, 1992, **MTT-40**, pp. 948–954
2 PALUSINSKI, O.A., and LEE, A.: 'Analysis of transients in nonuniform and uniform multiconductor transmission lines', *IEEE Trans.* , 1989, **MTT-37**, pp. 127–138
3 BELLMAN, R., KASHEF, B.G., and CASTI, J.: 'Differential quadrature: a technique for the rapid solution of nonlinear partial differential equations', *J. Comput. Phys.*, 1972, **10**, pp. 40–52
4 STRIZ, A.G., WANG, X., and BERT, C.W.: 'Harmonic differential quadrature method and applications to analysis of structural components', *Acta Mechanica*, 1995, **111**, pp. 85–94
5 LI, Z.-F., XU, Q.-W., and ZHU, J.-Z.: 'Fast simulation of lossy transmission lines by the modified method of characteristics', *Electron. Lett.*, 1997, **33**, pp. 942–944