Exotic magnetic orders for high spin ultracold fermions

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Abstract. - We study Hubbard models for ultracold bosonic or fermionic atoms loaded into an optical lattice. The atoms carry a high-spin $F > 1/2$, and interact on site via strong repulsive Van der Waals forces. Making convenient rearrangements of the interaction terms, and exploiting their symmetry properties, we derive low energy effective models with nearest-neighbor interactions, and their properties. We apply our method to $F = 3/2$ fermions on two-dimensional square lattice at quarter filling, and investigate mean-field equations for repulsive singlet g_0 and quintet g_2 couplings. We find that the plaquette state appearing in the highly symmetric $SU(4)$ case ($g_0 = g_2$) does not require fine tuning, and is stable in an extended region of the phase diagram. This phase competes with an $SU(2)$ flux state, that is always suppressed for repulsive interactions in absence of external magnetic field. The $SU(2)$ flux state has, however, lower energy than the plaquette phase, and stabilizes in the presence of a weak applied magnetic field. Some preliminary results for $F = 5/2$ fermions are also presented.

Ultracold atoms in optical lattices provide controllable quantum many body systems that allow to mimic condensed matter [1, 2]. They may in particular serve as quantum simulators of various Hubbard models [3], including those that do not have condensed matter analogues. Prominent examples include Hubbard models for bosons or fermions with high spin F. Experimental progress in studies of high F Bose-Eistein condensates [4] and Fermi gases (cf. [5]) triggered a lot of interest in theoretical studies of such models. These studies go back to fundamental questions of large N limit of SU(N) Heisenberg-Hubbard model [6]; they have continued more recently in the context of ultracold atoms [7–9]. These papers discusses the interplay between the Néel, and valence bond solid (VBS), i.e. Peierls or plaquette ordering for antiferromagnetic systems. Several other exotic phase should be possible of earth alkali atoms (cf. [10–13]), where two orbital $SU(N)$ magnetism, and even chiral spin liquid states were predicted. Several authors predicted also a variety of novel, exotic phases from effective (generalized Heisenberg) spin Hamiltonians, obtained from spinor Hubbard models (cf. [14,15]). A lot of effort was devoted to the investigations of 1D and 1D ladder systems, where quantum effects are even stronger [16]. While for $F = 1/2$ Hubbard models quantum fluctuations suppress the conductor-Mott insulator transition [17], this is not the case for higher F, where dimer (Peierls) or valence bond crystal (Haldane) order, and in ladders even plaquette order are possible.

Fermi systems with $F = 3/2$ were also intensively studied [18]: first, because this is the simplest case beyond $F = 1/2$, second, because they can be realized with for instance with ultracold ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba, and ^{201}Hg (for an excellent review see Ref. [19]; Such systems exhibit a generic SO(5) or isomorphically, Sp(4) symmetry). In 1D there exist the quartetting phase, a four-fermion counterpart of the Cooper pairing phase. In some situations, counter intuitively quantum fluctuations in spin-3/2 magnetic systems are even stronger than those in spin-1/2 systems.

In this Letter we study the Hubbard model for $F = 3/2$ fermions with repulsive singlet g_0 and quintet g_2 interactions in 2D. First, by rearranging the interaction terms and exploiting their symmetry properties, we derive low energy effective Hamiltonians. In contrast to the standard approaches (see for instance [14,15]), we do not use the spin representation, but rather keep the description in terms of fermionic operators. This allows us to formulate mean field theory, somewhat analogous to slave-boson method [20], and show that the plaquette VBS state is stable in an extended region of the phase diagram, in agreement with the predictions of Ref. [18, 19]. In the presence of a weak applied magnetic field, however, the plaquette phase can be suppressed by an exotic $SU(2)$ flux state.

Let us consider a system described by a Hamiltonian with nearest-neighbor hopping $H_{kin} = -t\sum_{<i,j>} c^\dagger_{i,\alpha}c_{i,\alpha}$ and strong on-site repulsive interaction

$$H_{int} = \sum_i V^{\alpha,\beta}_{\gamma,\delta} c^\dagger_{i,\alpha} c^\dagger_{i,\beta} c_{i,\delta} c_{i,\gamma}. \tag{1}$$

$c^\dagger_{i,\alpha}$ ($c_{i,\alpha}$) are the usual creator (annihilator) operators of fermions with spin α at site i, and t is the hoping amplitude between the neighboring sites. Here, and in the following automatic summation over the repeated greek indices is assumed. The interactions depend on the spin of the scattering particles:

$$V^{\alpha,\beta}_{\gamma,\delta} = \sum_{S=0}^{\tilde{S}} g_S \left[P_S\right]^{\alpha,\beta}_{\gamma,\delta}. \tag{2}$$

This means that the scattering processes can happen at different spin channels which are determined by the total spin S of the scattering particles. P_S projects to the total spin-S subspace and g_S is the coupling constant in the corresponding scattering channel. Due to the on-site interaction the only contributing terms are either antisymmetric (as) or symmetric (s) for the exchange of the spin of the colliding particles depending on the fermionic or bosonic nature of the particles. In the following we exploit this property of the on-site interaction that is preserved for the effective strong repulsion model with nearest-neighbour interaction, too.

Starting from the fundamental relation between the P_S projection operator and the product of the $\mathbf{F}$ spin operators: $(2\mathbf{F}_1\mathbf{F}_2)^l = \sum_S \left[S(S+1) - 2F(F+1)\right]^l P_S$, the P_S projector can be expressed as a degree of S polynomial of the $\mathbf{F}_1\mathbf{F}_2$ product:

$$P_{S'} = \sum_{l=0}^{S} a_{S',l} \left(\mathbf{F}_1\mathbf{F}_2\right)^l \tag{3}$$

for all $S' = 0, 1, \ldots, S$ and with $(\mathbf{F}_1\mathbf{F}_2)^0 \equiv \mathbf{E}$. $a_{S',l}$ are the coefficients of the expansion. Note that for a given value of S' the $P_{S'}$ projector is either symmetric or antisymmetric in the spin indices of the scattering particles. This expansion is usually applied in order to express the high-spin two-particle interaction with effective multispin-exchange. In contrast, we will use the expansion of the projector operator in eqs. (1) and (2) in order to collect and treat adequately the two-particle interaction terms that describe different spin exchange and spin flip processes. In this case $\mathbf{F}$ denotes the three generators of the SU(2) Lie algebra in the appropriate representation: for high-spin fermions or bosons they are the SU(2) generators represented by the proper even/odd dimensional matrices. In case of pure boson or fermion system all processes take place only in the symmetric or antisymmetric part of the total spin space, respectively. Therefore, the following decomposition can be used:

$$\left(\mathbf{F}_1\mathbf{F}_2\right)^l = \left(\left(\mathbf{F}_1\mathbf{F}_2\right)^l\right)^{(as)} + \left(\left(\mathbf{F}_1\mathbf{F}_2\right)^l\right)^{(s)}, \tag{4}$$

and the symmetric and antisymmetric projectors can be expressed as follows:

$$P_S^{(as)} = \sum_{l=0}^{N_{(as)}-1} b_{S,l} \left(\left(\mathbf{F}_1\mathbf{F}_2\right)^l\right)^{(as)}, \tag{5a}$$

$$P_S^{(s)} = \sum_{l=0}^{N_{(s)}-1} c_{S,l} \left(\left(\mathbf{F}_1\mathbf{F}_2\right)^l\right)^{(s)}. \tag{5b}$$

Here $N_{(as)}$ and $N_{(s)}$ denotes the number of antisymmetric and symmetric subspaces of the total spin space. The antisymmetric and symmetric part of an operator A can be constructed by the exchange of two spin indeces:

$$\left[A^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} = A^{\alpha,\beta}_{\gamma,\delta} - A^{\alpha,\beta}_{\delta,\gamma}, \tag{6a}$$

$$\left[A^{(s)}\right]^{\alpha,\beta}_{\gamma,\delta} = A^{\alpha,\beta}_{\gamma,\delta} + A^{\alpha,\beta}_{\delta,\gamma}. \tag{6b}$$

It is obvious that the above decomposition leads to the polynomials eq. (5) having significantly smaller degree than eq. (3). $N_{(as)} - 1$ or $N_{(s)} - 1$, respectively, determines the minimum degree of the polynomial of the product $\mathbf{F}_1\mathbf{F}_2$ which is equivalent to the interaction eq. (2).

Now let us apply the above procedure to a 2 dimensional $F = 3/2$ fermion system. In this case the interaction has to be antisymmetric therefore the only contributing terms are the total spin-0 (singlet) and the spin-2 (quintet) scatterings:

$$V^{\alpha,\beta}_{\gamma,\delta} = \sum_{S=0}^{3} g_S \left[P_S^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} = g_0 \left[P_0\right]^{\alpha,\beta}_{\gamma,\delta} + g_2 \left[P_2\right]^{\alpha,\beta}_{\gamma,\delta}. \tag{7}$$

At quarter filling for strong repulsion the system can be described by an effective Hamiltonian with nearest-neighbor interaction. The effective model based on perturbation theory up to second (leading) order in the hopping t is the following:

$$H_{int} = \sum_{<i,j>} V^{\alpha,\beta}_{\gamma,\delta} c^\dagger_{i,\alpha} c^\dagger_{j,\beta} c_{j,\delta} c_{i,\gamma}, \tag{8a}$$

where $V^{\alpha,\beta}_{\gamma,\delta} = \sum_S G_S \left[P_S^{(as,s)}\right]^{\alpha,\beta}_{\gamma,\delta}$, and $G_S = -4t^2/g_S$ gives the energy shift due to the weak nearest-neighbor hopping. Since the effective model preserves the symmetry of the on-site model, it remains antisymmetric for the exchange of two spin indices. Now the components of $\mathbf{F}$ vector are the well known 4×4 matrices and eq. (3) has the following form: $\mathbf{E}^{(as)} = P_0 + P_2$, and $(\mathbf{F}_1\mathbf{F}_2)^{(as)} = -15P_0/4 - 3P_2/4$. The interaction part of the effective Hamiltonian has the form

$$H_{int} = a_n \sum_{<i,j>} \mathbf{E}^{(as)}_{i,j} + a_s \sum_{<i,j>} (\mathbf{F}_1\mathbf{F}_2)^{(as)}_{i,j} \qquad (9)$$

where $a_n = (5G_2 - G_0)/4$, and $a_s = (G_2 - G_0)/3$. The two-particle nearest-neighbor interaction terms are:

$$\mathbf{E}^{(as)}_{i,j} = c^\dagger_{i,\alpha} c^\dagger_{j,\beta} c_{j,\delta} c_{i,\gamma} \left[\mathbf{E}^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} \qquad (10a)$$

$$(\mathbf{F}_1\mathbf{F}_2)^{(as)} = c^\dagger_{i,\alpha} c^\dagger_{j,\beta} c_{j,\delta} c_{i,\gamma} \left[(\mathbf{F}_1\mathbf{F}_2)^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} \qquad (10b)$$

and their explicit spin dependence:

$$\left[\mathbf{E}^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} = \delta_{\alpha,\gamma}\delta_{\beta,\delta} - \delta_{\alpha,\delta}\delta_{\beta,\gamma}, \qquad (11a)$$

$$\left[(\mathbf{F}_1\mathbf{F}_2)^{(as)}\right]^{\alpha,\beta}_{\gamma,\delta} = [\mathbf{F}_1]_{\alpha,\gamma}[\mathbf{F}_2]_{\beta,\delta} - [\mathbf{F}_1]_{\alpha,\delta}[\mathbf{F}_2]_{\beta,\gamma}. \qquad (11b)$$

After straightforward calculations one arrives to the following form of the effective Hamiltonian:

$$H_{eff} = V^{(0)} + \sum_{<i,j>} \left[a_n \left(n_i n_j + \chi^\dagger_{i,j}\chi_{i,j} - n_i \right) + a_s \left(\mathbf{S}_i\mathbf{S}_j + \mathbf{J}^\dagger_{i,j}\mathbf{J}_{i,j} - \frac{15}{4} n_i \right) \right] \qquad (12)$$

where $n_i = c^\dagger_{i,\alpha} c_{i,\alpha}$, and $\mathbf{S}_i = c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{i,\beta}$ are the usual particle number and spin operators on site i, and

$$\chi_{i,j} = c^\dagger_{i,\alpha} c_{j,\alpha}, \qquad (13a)$$

$$\mathbf{J}_{i,j} = c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{j,\beta} \qquad (13b)$$

are introduced for the U(1), and SU(2) nearest-neighbor link operators, respectively. Note, that in general the SU(2) link operators do not satisfy the spin commutation relations, however, they clearly are related to the bond-centered spin. The competition between the spin and particle fluctuations can be controlled by tuning of a_n and a_s. The effective Hamiltonian (12) can be applied for less than quarter filled system too, provided the kinetic term is added to the Hamiltonian (12). $V^{(0)}$ contains the on-site energies and shifts the ground state energy only, so we do not consider it in the following.

We studied the possible phases of the quarter filled system with the constraint $\sum_\alpha c^\dagger_{i,\alpha}c_{i,\alpha} = 1$ — only single occupied sites are allowed due to the strong on-site repulsion. Due to this local constraint the Hamiltonian is invariant under the rotation of the phase of the fermions at each sites. This means that the Lagrangian of the system $\mathcal{L} = \sum_i c^\dagger_{i,\sigma}\partial_\tau c_{i,\sigma} + H$ is invariant under the U(1) gauge transformation $c_{i,\sigma} \to c_{i,\sigma}e^{i\phi_i}$ reflecting the local constraint for the particle number.

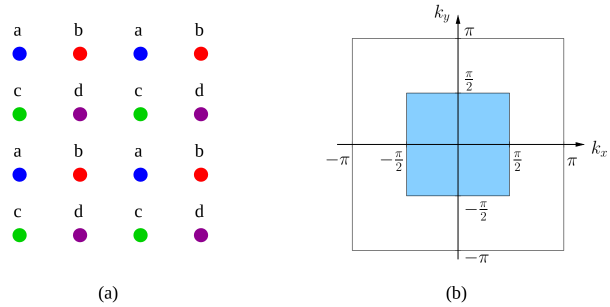

Fig. 1: (Color online) The lattice was splitted into 4 sublattices as shown in subfigure (a) and due to this splitting the Brillouin zone shrinked. (b) The shadow area depicts the reduced Brillouin zone.

Considering the Hamiltonian (12) the terms containing n_i do not give contribution at quarter filling and the remaining 4-fermion terms can be decoupled via a mean-field treatment by introducing the expectation values of the link operators $\langle\chi_{i,j}\rangle$ and $\langle\mathbf{J}_{i,j}\rangle$, and the spin operator $\langle\mathbf{S}_i\rangle$. Now the mean-field Hamiltonian is:

$$H^{MF} = \sum\nolimits_{<i,j>} H_{i,j} \qquad (14)$$

with

$$H_{i,j} = a_n\Big(\langle\chi_{j,i}\rangle c^\dagger_{i,\alpha}c_{j,\alpha} + \langle\chi_{i,j}\rangle c^\dagger_{j,\alpha}c_{i,\alpha} - |\langle\chi_{i,j}\rangle|^2 \Big) + a_s\Big(\langle\mathbf{J}_{j,i}\rangle c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{j,\beta} + \langle\mathbf{J}_{i,j}\rangle c^\dagger_{j,\alpha}\mathbf{F}_{\alpha,\beta}c_{i,\beta} - |\langle\mathbf{J}_{i,j}\rangle|^2 + \langle\mathbf{S}_i\rangle c^\dagger_{j,\alpha}\mathbf{F}_{\alpha,\beta}c_{j,\beta} + \langle\mathbf{S}_j\rangle c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{i,\beta} - \langle\mathbf{S}_i\rangle\langle\mathbf{S}_j\rangle \Big).$$

Note that the mean-field Lagrangian also has to remain invariant under the gauge transformation mentioned above. Thus, the link variables must transform as $\langle A_{i,j}\rangle \to \langle A_{i,j}\rangle e^{-i(\phi_j-\phi_i)}$.

The expectation values of the spin and link operators were determined self-consistently. Anticipating the appearance of a plaquette phase similar to that which is the ground state of the system for $G_0 = G_2$, it is reasonable to split the lattice into 4 sublattices (see Fig. 1) leading to the shrinking of the area of the Brillouin zone to the quarter of its original value. We assumed different values of the order parameters for the different sublattices and for the alternating links as the only space-dependence of them.

We found the following gauge non-equivalent states: Neel order, two plaquette orders, two SU(2) plaquette or flux states, and SU(2) dimer order. The phase diagram of the system is shown in fig. 2. If the effective interaction of the singlet channel is significiantly stronger than that of the quintet channel, the dominant order is purely antiferromagnetic without any bond order. For $a_n \geq 0$ stable U(1) link order neither can be expected because it would lead to the increase of the energy. Moreover, we found that the SU(2) bond-order parameter also remains zero in this regime. For $a_n < 0$ and $a_s > 0$ the spin and particle

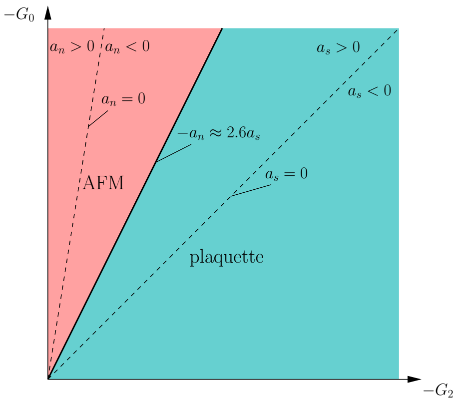

Fig. 2: (Color online) The phase diagram of $3/2$ fermion system with strong on-site repulsion on 2D square lattice at quarter filling. The Neel order (AFM) suppress any other order for dominant antiferromagnetic spin-coupling ($a_s > 0$ and $a_s > 1.9|a_n|$). In the plaquette phase the a_n coupling is dominant. Here the zero flux and the π-flux state have the same energy.

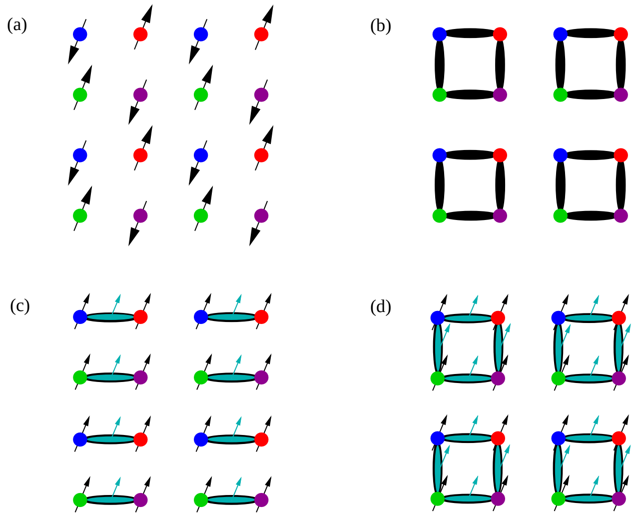

Fig. 3: (Color online) The different configuration that can be stable in the presence or without magnetic field. (a) In the Neel order the only nonzero expectation values are of the spin operators $\mathbf{S}_i$ and they alternate on the neighbouring sites. (b) In the plaquette order bond centered density waves form disconnected boxes. In the ground state the flux passing through the plaquettes can be 0 or π. In both the SU(2) dimer (c) and plaquette (d) phases the site centered ferromagnetic order coexists with bond centered density wave that also carries spin — both $\langle \chi_{i,j} \rangle$ and $\langle \mathbf{J}_{i,j} \rangle$ are nonzero.

order compete with each other. In ref. [18] a magnetically ordered dimer phase was suggested to appear in this regime, however, such a state was found to be instable within our calculation. When the antiferromagnetically ordered Neel phase becomes instable, plaquette order appears. The phase border is at around $G_0 \approx 1.9 G_2$ or equivalently $-a_n \approx 2.6 a_s$. In the plaquette phase the nonzero U(1) links form "boxes" as shown in fig. 3. In this phase one can define the U(1) plaquette as $\Pi = \chi_{i,j}\chi_{j,k}\chi_{k,l}\chi_{l,i}$, where i, j, k and l denote the sites of an elementary plaquette of the square lattice, and χ is defined for nearest neighbors only. The U(1) flux Φ is defined by the phase of the plaquette. The plaquette and therefore the flux are invariant under the U(1) gauge transformations. We found two different gauge-non-equivalent states in the plaquette phase labelled by $\Phi = 0$ and $\Phi = \pi$, respectively, and both states have the same energy. For $|G_0| < 1.9|G_2|$ we did not find any order (in the ground state) controlled by the a_s coupling. This is possibly due to the fact that for repulsive on-site interaction in this part of the parameter space, the coupling constant a_n is always the dominant one compared to a_s — independently on the sign of a_s.

While the plaquette order is the ground state of the system for $|G_0| < 1.9|G_2|$, we found for $a_s \neq 0$ two other phases having 10-15% higher energy than the ground state: the SU(2) dimer phase and the SU(2) flux phase, where the latter corresponds in fact to two gauge-non-equivalent states, similarly to the two states of the U(1) plaquette phase. Both the dimer and the flux phases have the same energy. In the SU(2) dimer state, in addition to weak ferromagnetic order, both types of the link operators χ and $\mathbf{J}$ have nonzero expectation values on every second link in one direction (see fig. 3). Here and in the following we use the term "weak ferromagnetic order" for the case of $\langle \mathbf{S}_i \rangle < F$. In the SU(2) flux phase the link operators with nonzero expectation values constitute plaquettes. Both states violate the spin-rotation invariance of the plaquette phase, and the SU(2) dimer state — contrary to the plaquette phases — preserves the translational invariance by one lattice site in one spatial dimension. At this point let us pay some attention of the denomination of these states. It is clear that $\mathbf{J}$ is not a member of the SU(2) therefore it is reasonable to ask why do we use the terms SU(2) plaquette, flux or dimer for the states where the expectation value of $\mathbf{J}$ is nonzero? To answer this question let us consider the mean-field Hamiltonian eq. (14). The non-local part of the one-particle excitations appears in the Hamiltonian as

$$\Big(a_n \langle \chi_{j,i} \rangle \delta_{\alpha,\beta} + a_s \langle \mathbf{J}_{i,j} \rangle \mathbf{F}_{\alpha,\beta}\Big) c^\dagger_{i,\alpha} c_{j,\beta} + H.c. \qquad (15)$$

From this form it can be read that the excitations consist two branches with two different symmetries: $\langle \chi_{j,i} \rangle$ relates to the U(1) excitations, while $\langle \mathbf{J}_{i,j} \rangle$ to the SU(2) excitations. In order to define the SU(2) flux let us introduce the new link parameter according to eq. (15) in the following way:

$$U_{i,j} = \langle \mathbf{J}_{i,j} \rangle \mathbf{F}, \qquad (16)$$

with the usual inner product of the vectors in the 3 dimensional space of the generators $\mathbf{F}$. $U_{i,j}$ is a member of SU(2) and a 4×4 matrix and the same holds for the plaquette $\Pi^{SU(2)} = U_{i,j}U_{j,k}U_{k,l}U_{l,i}$. The flux $\mathbf{\Phi}$ passing through the plaquette defined by the form: $\Pi^{SU(2)} = e^{i\mathbf{\Phi}\mathbf{F}}$. In order to determine the ground state it is worth to express the mean-field Hamiltonian with the $\mathbf{J}_{i,j}$ operators, while the

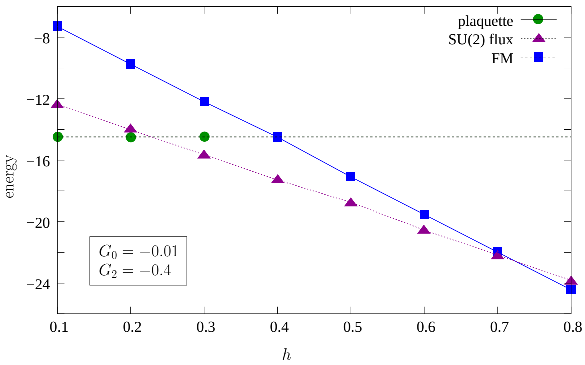

Fig. 4: (Color online) The magnetic field dependence of the energy of the different states in the unit $t = 1$. From a critical value of the magentic field the U(1) plaquette phase becomes instable and is completely suppressed by the SU(2) flux phase. For strong magnetic field the ferromagnetic order is the only one stable order. The lines are only guides of the eyes.

excitations and the SU(2) flux can be expressed with $U_{i,j}$. Note that the SU(2) plaquette $\Pi^{SU(2)}$ is also invariant under the U(1) gauge transformation defined above:

$$c_{i,\sigma} \to c_{i,\sigma} e^{i\phi_i},$$
$$\langle \chi_{i,j} \rangle \to \langle \chi_{i,j} \rangle \, e^{i(\phi_j - \phi_i)},$$
$$U_{i,j} \to U_{i,j} e^{i(\phi_j - \phi_i)}.$$

Considering the definition of $U_{i,j}$, the last relation is obviously equivalent to the transformation $\mathbf{J}_{i,j} \to \mathbf{J}_{i,j} e^{i(\phi_j - \phi_i)}$.

The SU(2) phases can patently claim to great interest, but they are suppressed by the U(1) plaquette state. Nevertheless, since the SU(2) flux, as well as the SU(2) dimer order coexist with ferromagnetic order, it can be expected that weak magnetic field does not destroy the SU(2) orders, but it can stabilize that. To check this let us investigate the changes of these states in the presence of external magnetic field h, and include it as a Zeeman term in the Hamiltonian:

$$H^h = H^{MF} + h \sum_i \mathbf{S}_i. \qquad (17)$$

The magnetic field dependence of the energy of the SU(2) flux state compared to the U(1) plaquette state and the ferromagqxnetic order is shown in fig. 4 for a typical value of the couplings in the unit of the nearest-neighbor hopping. The U(1) plaquette phase remains the ground state for nonzero, but only very small magnetic fields. The SU(2) plaquette state, as well as SU(2) dimer order, become stable as the applied magnetic field h is increased, and remain the ground states of the system in an extended region of the phase diagram. However, although both the dimer and the flux states have the same energy, starting from the U(1) plaquette state by increasing the applied magnetic field, the evolving state is always the SU(2) flux. The flux passing through the plaquette is determined by the flux of the initial U(1) flux. Further increasing the magnetic field, as it is expected — the strong ferromagnetic order destroys the SU(2) plaquettes, and suppresses any other order in the system.

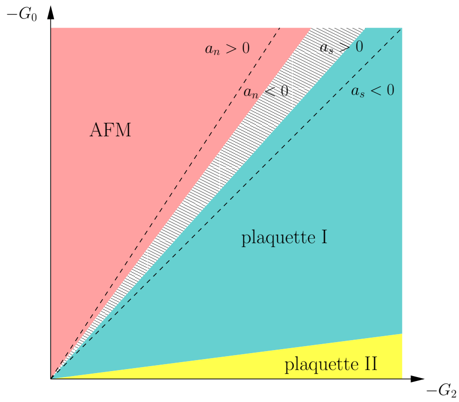

Fig. 5: (Color online) The expected phase diagram of the $F = 5/2$ fermion system with strong on-site repulsion on 2D square lattice at 1/6 filling. Similarly to the spin-3/2 system, the Neel order (AFM) is the dominant one over any other order for sufficiently strong antiferromagnetic spin-coupling. In the plaquette I. phase there is no any spin order while in the plaquette II. phase the SU(2) link oprators also form plaquettes and the bond orders coexist with a ferromagnetic order.

Similar analysis can be easily made for $F = 5/2$ fermions for the special values of the coupling constants $G_4 \approx (-7G_0 + 10G_2)/3$. G_4 is the coupling of the interaction with 9-fold spin multiplicity that appears in the Hamiltonian for spin-5/2 system in addition to the singlet (G_0) and quintet (G_2) scatterings. In the plane of the parameter space that defined by $G_4 = (-7G_0 + 10G_2)/3$, the structure of the Hamiltonian is exactly the same as eq. (14), there is no term containing higher order of the product $\mathbf{F}_1\mathbf{F}_2$, and the couplings take the values: $a_n = (-23G_0 + 35G_2)/12$ and $a_s = (-G_0 + G_2)/3$. The possible phases in the ground state of this system based on our preliminary calculations are shown in fig. 5. The phase diagram is clearly reacher than that of the $F = 3/2$ system, even though that our analysis for the spin-5/2 fermions is confined to the fixed value of $G_0/t = -0.2$. In case of dominant singlet scatterings the ground state is purely antiferromagnetic, at least while $a_n > 0$. A weak negative a_n seems to lead to an instability in the system, but we could not find any stable order in this narrow region. Further decreasing $|G_0|$, a quasi-plaquette phase appears. In this phase the expectation value of the U(1) link operator $\langle \chi_{i,j} \rangle$ is non-zero at every links, but stronger and weaker links alternate forming a weak plaquette structure. The flux passing through the plaquette is zero and there is no any spin order in this phase. For very weak singlet coupling (increasing the value of $|a_s|$) weak ferromagnetic order appears in addition to the plaquette order. Here the plaquettes are formed by not only the alternating zero and non-zero U(1) link operators $\langle \chi_{i,j} \rangle$, but by the SU(2) operators $\langle \mathbf{J}_{i,j} \rangle$, too. The flux passing through the plaquettes remains zero. Note, that in the same regime ($|G_0| \ll |G_2|$) we found another stable plaquette phase with π flux and with stronger ferromagnetic order, however $\langle \mathbf{S}_i \rangle$ remains smaller than 5/2. In this state along the links of the plaquettes the SU(2) order parameter is the dominant one, the value of $\langle \mathbf{J}_{i,j} \rangle$ is twice that of the corresponding $\langle \chi_{i,j} \rangle$. The energy of this spin ordered plaquette state is higher by about 5% than the zero flux spin ordered plaquette state.

To summarize, we used a decomposition of the total spin space into its symmetric and antisymmetric part with respect to the exchange of two spin indices of the high-spin scattering particles. This decomposition was used for strongly repulsive system to derive the effective low energy Hamiltonian. This task was achieved remaining within the two-particle representation. The main advantage of the treatment is that it does not require to introduce complicated effective multiparticle/multispin interactions, but relies only on rearrangements of the usual two-particle interactions. The effectiveness of the treatment does not depend on the statistics of the considered particles, and it allows to identify the different processes in the spin channel within the concept of site and bond spin. Applying this method to $F = 3/2$ fermions, we determined the ground state phase diagram of the system on mean-field level to complete the earlier results known for some regimes of the couplings. We found that the VBS state, which is stable in an extended region of the phase diagram, becomes instable in the presence of weak magnetic field. Instead, there appears an exotic SU(2) flux state. We made also some preliminary calculations for $F = 5/2$ fermions in the plane determined by the condition $G_4 = (-7G_0+10G_2)/3$ in the 3-dimensional parameter space of coupling constants. Also in this case we have predicted apearance of novel, exotic phases.

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