Localized Non-Linear Solutions in Multiply-Charged Dusty Plasmas

Paulo Hiroshi Sakanaka and Iglika Spassovska
Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, Unicamp
13083-970, Campinas, São Paulo, Brasil

Received on 14 August, 2002. Revised version received on 13 December, 2002.

Finite amplitude localized electrostatic solitons in a multi-component unmagnetized dusty plasma are presented. Assuming that the constituents of dusty plasmas are warm electrons, warm positive ions, and an admixture of cold dust grains with negative and positive charges, it is shown that stationary solutions of the fluid equations combined with Poisson's equation can be expressed in terms of the energy integral of a classical particle with a modified Sagdeev potential. The latter is analyzed both analytically and numerically to demonstrate the coexistence of rarefactive and compressive electric potential pulses which travel faster than the effective dust-acoustic velocity. Compressive dust-acoustic solitons exist only when there is a significant fraction of positively charged dust grains. Furthermore, the four-fluid dusty plasma system, with both negative and positively charged dust grains, also provides the possibility of double layers. Conditions under which solitons and double layers arise are given, and their profiles are displayed graphically. The results of investigation should be helpful in identifying the salient features of nonlinear structures in low-temperature space and laboratory dusty plasmas in which positive and negatively charged dust grains coexist. In particular, we have applied the theory in the laboratory plasma and we can predict that a double layer might be possible to be launched if a trace ions component is added.

I Introduction

Since the discovery of the dust acoustic waves (DAW) by Rao, Shukla, and Yu, [1], there has been a great interest in investigating numerous collective processes in dusty plasmas. In their paper, Rao et al. also introduced a theory for dust-acoustic solitons in a three-component dusty plasma with negatively charged dust grains. They pointed out the possibility of a finite-amplitude rarefactive dust-acoustic potential, in contrast to a compressional potential that is associated with the usual ion-acoustic soliton in a plasma without the dust component. Recently, it has been suggested that positively and negatively charged dust grains can co-exist in space [2]-[4] and laboratory [5] plasmas. Therefore, it is desirable to investigate the linear and nonlinear properties of dust-acoustic waves in a four component plasma that consists of electrons, ions and positively and negatively charged dust grains.

Here we sumarize the governing equations for DAW when both the negative and positive dust components are simultaneously present as given by Sakanaka and Shukla in [6]. We discuss the properties of DAW in the presence of positive and negative dust components, and define parameters that are relevant for the analysis of the nonlinear DAW. Using the reductive perturbation technique, the evolution equations for small, but finite, amplitude nonlinear DAW are derived. Explicit expressions for DAW solitons and dust-acoustic double-layers (DADLs) are presented. Stationary solutions of the governing nonlinear equations for arbitrary large amplitudes are discussed. Here, we derive the energy integral with a modified Sagdeev potential. The latter is analyzed both analytically and numerically to obtain the parameter regimes where dust acoustic (DA) solitons and DA double-layers are possible. It turns out that the presence of a positive dust component in a multi-component dusty plasma gives rise to such interesting features of the nonlinear structures as the compressional DA potential distribution and the monotonic double-layers, which otherwise are absent. Finally, possible applications of our investigation in space and laboratory plasmas are given.

Our paper is organized in the following way. In section II we present the governing equations for DAW with the presence of positive and negative dust components. In section III we define parameters that help the analysis of the nonlinear DAWs and we discuss the properties of the DAW with the simultaneous presence of positive and negative dust particles. Using the reductive perturbation technique, the evolution equations for small, but finite, amplitude nonlinear DAW are derived. Second order amplitude expansion for dust acoustic solitons and third order expansions for dust acoustic double-layers are presented. Stationary solutions of the governing nonlinear equations for arbitrary large amplitudes are discussed in Section IV. Also, we derive the energy integral with a modified Sagdeev potential. It is analyzed analytically and numarically to obtain the parameter regimes for existance of DA solitons and DA double-layers. The last section contains summary and possible applications in laboratory plasmas.

II Governing equations

We consider an unmagnetized dusty plasma consisting of the electrons, the ions, negatively and positively charged massive dust particles, with similar masses.

The quasi-neutrality at equilibrium is written

$$N_{e0} + Z_n N_{n0} = N_{i0} + Z_p N_{p0}, \tag{1}$$

where, N_{e0} and N_{i0} are the electrons and ions number density, Z_n and Z_p are the negative and positive dust particle charge, N_{n0} and N_{p0} are the dust particles number density, respectively.

The dust particules are assumed to be point charges and their sizes are much smaller than the effective Debye length. For low phase velocity (compared to the electron and ion thermal velocities) dust-acoustic waves, both the electrons and ions can be considered inertialess fluid and their number densities can be given by the Boltzmann distribution, respectively,

$$N_e = N_{e0} e^{e\Phi/T_e}, \tag{2}$$

and

$$N_i = N_{i0} e^{-e\Phi/T_i}, \tag{3}$$

where, Φ is the electrostatic potential and e is the magnitude of the electron charge.

The dynamics of charged dust grains are governed by the equations of the continuity and the momentum, which are, respectively,

$$\frac{\partial N_p}{\partial t} + \frac{\partial (N_p V_p)}{\partial x} = 0, \tag{4}$$

and

$$\frac{\partial V_p}{\partial t} + V_p \frac{\partial V_p}{\partial x} = -\frac{Z_p e}{M_p} \frac{\partial \Phi}{\partial x} \tag{5}$$

for positively charged dust grain, and

$$\frac{\partial N_n}{\partial t} + \frac{\partial (N_n V_n)}{\partial x} = 0, \tag{6}$$

and

$$\frac{\partial V_n}{\partial t} + V_n \frac{\partial V_n}{\partial x} = \frac{Z_n e}{M_n} \frac{\partial \Phi}{\partial x} \tag{7}$$

for negatively charged dust grain. Here V_p, V_n, M_p, M_n are the fluid velocities and mass of the positively and negatively charged dust grains, respectively. We are assuming a cold dust particules.

The system of equations is closed with the Poisson's equation

$$\frac{\partial^2 \Phi}{\partial x^2} = 4\pi e (N_e - N_i + Z_n N_n - Z_p N_p). \tag{8}$$

III Finite amplitude linear and nonlinear dust acoustic waves

We are looking for a plane wave solution for the set of equations (1) to (8) for small amplitude disturbances with angular frequency ω and wave number k. We linearize (2) to (8) and Fourier transform them by assuming that the first order quantities of N_e, N_i, N_n, N_p, V_n, V_p, and Φ are proportional to $exp[i(kx - \omega t)]$. Processing the resulting equations gives

$$\frac{\omega}{k} = \frac{C_{da}}{\sqrt{1 + \lambda_{Dd}^2 k^2}}, \tag{9}$$

where

$$C_{da}^2 = \frac{T_0}{M_0} \tag{10}$$

and

$$\lambda_{Dd} = \frac{C_{da}}{\omega_{pd}}. \tag{11}$$

We have introduced the symbols ω_{pd}, N_0, T_0 and M_0 as

$$\omega_{pd}^2 = \frac{4\pi e^2 N_0}{M_0}, \quad N_0 = N_{e0} + N_{i0},$$

$$\frac{N_0}{T_0} = \frac{N_{e0}}{T_e} + \frac{N_{i0}}{T_i}, \text{ and } \frac{N_0}{M_0} = \frac{Z_n^2 N_{n0}}{M_n} + \frac{Z_p^2 N_{p0}}{M_p}. \tag{12}$$

Here, C_{da} is the dust-acoustic velocity, λ_{Dd} the effective Debye length, ω_{pd} the dust plasma frequency, N_0, M_0, and T_0 are the effective number density, the mass and the temperature, respectively.

From the definition of M_0, we can see that the dust plasma frequency ω_{pd} is increased when both negative and positive charges are present and/or increased, because T_0 and M_0 are not dependent on the presence of dust, likewise the dust phase velocity, C_{da}, increases as dust increases.

III.1 Normalization

With the purpose of understanding the parametric space which limits the existence of DA solitons and double-layers we are normalizing all the parameters. The natural quantities for the normalization are T_0, N_0 and M_0, the effective temperature (in unit of energy), the plasma particle number density and the mass, respectively. From these we get the normalizing quantities for the time, $t \to t\omega_{pd}$, the space, $x \to x/\lambda_{Da}$, and the mass, $M_j \to M_j/M_0$. For velocities, the normalizing quantity is C_{da}, and the normalized velocities are expressed as the Mach number, M (the latter should not be confused with the plasma particle mass), which is the pulse velocity, V_0, normalized to C_{da}.

We define, then

$$\phi = \frac{e\Phi}{T_0}, \text{ and } u(\phi) = \frac{U(\Phi)}{4\pi N_0 T_0}, \tag{13}$$

where $U(\Phi)$ is the potential energy density, which will appear in the later context,

$$n_{e0} = \frac{N_{e0}}{N_0},\ n_{i0} = \frac{N_{i0}}{N_0},\ a_e = \frac{T_0}{T_e},\ a_i = \frac{T_0}{T_i} \tag{14}$$

for the electron and ion number densities and the temperatures,

$$n_n = \frac{N_{n0} Z_n}{N_0}, \text{ and } n_p = \frac{N_{p0} Z_p}{N_0} \quad (15)$$

for the dust particle number density,

$$a_n = \frac{Z_n T_0}{M_n V_0^2},\ a_p = \frac{Z_p T_0}{M_p V_0^2}, \text{ and } M = \frac{V_0}{C_{da}}, \quad (16)$$

for the nonlinear dust acoustic wave parameters and the Mach number.

From equation (1), (12) and (13)-(16) we have

$$\begin{aligned} n_{e0} + n_n &= n_{i0} + n_p && (17)\\ n_{e0} + n_{i0} &= 1 && (18)\\ n_{e0} a_e + n_{i0} a_i &= 1 && (19)\\ n_n a_n + n_p a_p &= \frac{1}{M^2}. && (20) \end{aligned}$$

III.2 Second order amplitude expansion and dust acoustic solitons

Now we use the reductive perturbation technique to derive the dynamical equations for nonlinear dust acoustic waves which have small, but finite amplitudes. Accordingly, we introduce in equations (2) to (8) the following expansion:

$$\begin{aligned} N_n &= N_{n0} + \epsilon N_{n1} + \epsilon^2 N_{n2} + \epsilon^3 N_{n3} + \ldots, && (21)\\ N_p &= N_{p0} + \epsilon N_{p1} + \epsilon^2 N_{p2} + \epsilon^3 N_{p3} + \ldots, && (22)\\ V_n &= \epsilon V_{n1} + \epsilon^2 V_{n2} + \epsilon^3 V_{n3} + \ldots, && (23)\\ V_p &= \epsilon V_{p1} + \epsilon^2 V_{p2} + \epsilon^3 V_{p3} + \ldots, && (24)\\ \Phi &= \epsilon \Phi_1 + \epsilon^2 \Phi_2 + \epsilon^3 \Phi_3 + \ldots, && (25) \end{aligned}$$

where ϵ indicates a small quantity. In the stationary wave frame, $x - V_0 t$, where V_0 is the velocity of propagation of the localized solution. Making change of the independent variables, t and x, to new ones, τ and ξ, respectively,

$$\begin{aligned} \xi &= \epsilon^{1/2}(x - V_0 t) \text{ and} && (26)\\ \tau &= \epsilon^{3/2} t, && (27) \end{aligned}$$

we can rewrite (2) to(8) in terms of the expanded variables (21) - (25), and analyze them order by order.

The $\epsilon^{1/2}$ -order equation is exactly the quasi neutrality condition (1) or (17).

The $\epsilon^{3/2}$ -order equation turns out to be

$$\delta \equiv \frac{N_{e0}}{T_e} + \frac{N_{i0}}{T_i} - \frac{Z_n^2 N_{n0}}{V_0^2 M_n} - \frac{Z_p^2 N_{p0}}{V_0^2 M_p} = 0 \quad (28)$$

yelding

$$V_0^2 = \frac{T_0}{M_0}, \quad (29)$$

which means that the moving stationary frame has exactly the velocity of the dust-acoustic phase velocity.

In $\epsilon^{5/2}$ -order, we obtain the Korteweg-de Vries (K-dV) equation

$$\frac{\partial \phi_1}{\partial \tau} + \frac{\alpha}{2}\frac{\partial \phi_1^2}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (30)$$

where

$$\alpha = \left(\frac{3}{2} a_{p1}^2 n_p + \frac{1}{2} a_i^2 n_{i0}\right) - \left(\frac{3}{2} a_{n1}^2 n_n + \frac{1}{2} a_e^2 n_{e0}\right), \quad (31)$$

and where a_{p1} and a_{n1} are values of a_p and a_n for $V_0 = C_{da}$ (that is, $M = 1$), respectively.

The solution of (30) is of a shifted hyperbolic secant square function

$$\phi_1 = \phi_{11} Sech^2[(\xi - \tilde{M}\tau)/w], \quad (32)$$

where $\tilde{M} = M - 1$, with $w = \sqrt{2/\tilde{M}}$ and $\phi_{11} = 3\tilde{M}/\alpha$. In (32), ξ and τ are in normalized units. The relative velocity $\tilde{M}$ to be a positive quantity is the necessary condition for the existence of a soliton. Its value is the relative velocity above the dust ion-acoustic phase velocity, and the Mach number of the soliton is $1 + \tilde{M}$, w represents the width of the soliton and ϕ_{11} is its amplitude. This potential can be either positive (compressive soliton) or negative (rarefactive soliton), depending on the sign of α. At this point, we cannot assure the sufficient condition for the existence of a soliton solution.

III.3 Third order expansion and dust acoustic double-layer

We can derive the so-called modified K-dV equation from (2) to (8), just as expansion used in the previous section using up to ε^3 terms. However here we make changes to the following new variables

$$\begin{aligned} \xi &= \epsilon(x - V_0 t) \quad \text{and} && (33)\\ \tau &= \epsilon^3 t. && (34) \end{aligned}$$

Substituting (21) to (25) into (2) to (8), we can analyze the resulting equations, order by order.

The ϵ^0-order equation is just the quasi neutrality equation (1) or (17).

The ϵ^1–order equation reproduces (28):

$$\delta \equiv \frac{N_{e0}}{T_e} + \frac{N_{i0}}{T_i} - \frac{Z_n^2 N_{n0}}{V_0^2 M_n} - \frac{Z_p^2 N_{p0}}{V_0^2 M_p} = O(\epsilon^1)$$

The next higher order equation, ϵ^2-order, is

$$\delta \Phi_2 - \alpha \Phi_1^2 = 0 \quad (35)$$

where α and δ are given by (31) and (28), respectively. Since δ is $O(\epsilon^1)$, we say that $\alpha \Phi_1^2$ is of $O(\epsilon^3)$. So, we can incorporate the term $\alpha \Phi_1^2$ to the ϵ^3 -order equations.

Thus, we obtain for the ϵ^3 -order equation, the modified Kortweg-de Vries (mK-dV) equation

$$\frac{\partial \phi_1}{\partial \tau} + \frac{\alpha}{2}\frac{\partial \phi_1^2}{\partial \xi} + \beta \frac{\partial \phi_1^3}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (36)$$

where

$$\beta = \frac{1}{2}\left[-\frac{1}{6}\left(n_{e0}a_e^3 + n_{i0}a_i^3\right) + \frac{5}{3}\left(n_n a_n^3 + n_p a_p^3\right)\right]. \tag{37}$$

In the stationary frame $\zeta = \xi - \tilde{M}\tau$, (36) takes the form

$$\frac{1}{2}\left(\frac{\partial \phi_1}{\partial \zeta}\right)^2 + u(\phi_1, \tilde{M}) = 0 \tag{38}$$

where $u(\phi_1, \tilde{M}) = -\tilde{M}\phi_1 + (\alpha/6)\phi_1^3 + (\beta/4)\phi_1^4$.

The conditions

$$u(\phi_{11}, \tilde{M}) = 0 \tag{39}$$

$$\left.\frac{\partial u}{\partial \phi_1}\right|_{\phi_1 = \phi_{11}} = 0 \tag{40}$$

produce the double-layer solution [8]

$$\phi_1(\zeta) = \frac{\phi_{11}}{2}[1 - \tanh(w\zeta)] \tag{41}$$

where $\phi_{11} = \alpha/(3\beta)$, $\tilde{M} = -(\alpha/4)\phi_{11}$ and $w = \sqrt{-(\beta/8)\phi_{11}^2}$. The double-layer solution exists only if $\beta < 0$.

IV Arbitrarily large amplitude nonlinear dust acoustic waves

IV.1 Compressive and rarefactive DA solitons

In this section we will be looking for arbitrary large amplitude solutions of the nonlinear equations (2) to (8) in the stationary frame $\xi = x - V_0 t$ (unnormalized). From (4) and (5), with the condition that at $\xi \to -\infty$, $N_p \to N_{p0}$ and $V_p \to 0$, we obtain

$$N_p = \frac{N_{p0}}{\sqrt{1 - 2Z_p e\Phi/(M_p V_0^2)}} \tag{42}$$

for the positive dust particle number density. Similarly, from (6) and (7) for the negative dust particle number density

$$N_n = \frac{N_{n0}}{\sqrt{1 + 2Z_n e\Phi/(M_n V_0^2)}} \tag{43}$$

Inserting expression N_p and N_n and Boltzmann distributions N_e and N_i into (8), we obtain, in the stationary frame

$$\begin{aligned}\frac{\partial^2 \Phi}{\partial \xi^2} = 4\pi e\Bigg[&N_{e0}\exp\left(\frac{e\Phi}{T_e}\right) - N_{i0}\exp\left(-\frac{e\Phi}{T_i}\right) + \frac{N_{n0}Z_n}{\sqrt{1 + 2Z_n e\Phi/(M_n V_0^2)}}\\ &-\frac{N_{p0}Z_p}{\sqrt{1 - 2Z_p e\Phi/(M_p V_0^2)}}\Bigg]\end{aligned} \tag{44}$$

Multiplying both sides of (44) by $\partial\Phi/\partial\xi$ and integrating once from -∞ to ξ, with the conditions that $\Phi \to 0$ and $\partial\Phi/\partial\xi \to 0$ at $\xi \to -\infty$, we obtain the Euler type energy integral

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial \xi}\right)^2 + u(\phi) = 0 \tag{45}$$

where

$$u(\phi) = -\left\{\frac{n_{e0}}{a_e}(e^{a_e\phi} - 1) + \frac{n_{i0}}{a_i}(e^{-a_i\phi} - 1) + \frac{n_n}{a_n}(\sqrt{1 + 2a_n\phi} - 1) + \frac{n_p}{a_p}(\sqrt{1 - 2a_p\phi} - 1)\right\}. \tag{46}$$

is the modified Sagdeev potential [9]. In (45) and (46) all the quantities have been normalized. The derivative of $u(\phi)$ with respect to ϕ is given by

$$\frac{du(\phi)}{d\phi} = -\left[n_{e0}e^{a_e\phi} - n_{i0}e^{-a_i\phi} + \frac{n_n}{\sqrt{1 + 2a_n\phi}} - \frac{n_p}{\sqrt{1 - 2a_p\phi}}\right], \tag{47}$$

whose right hand side is the normalized right hand side of (44). Equation (45) can have physically meaningfull solution only in the region of ϕ where $u(\phi)$ is negative, because $\phi'(\xi) = \sqrt{-2u(\phi)}$, from (45). Let us, then, assume that at $\phi = \phi_0$, $u(\phi_0) = u'(\phi_0) = 0$ and that at $\phi = \phi_1$, $u(\phi_1) = 0$, and analyze the solution, ϕ, near these values.

At ϕ_0 the leading ordet of Taylor expansion of $u(\phi)$ is

$u(\phi) \sim a_1(\phi - \phi_0)^2$. Plugging it to equation (45) we get

$$\frac{d(\phi - \phi_0)}{\phi - \phi_0} = \pm\sqrt{-2a_1}d\xi,$$

whose solution is

$$\phi = \phi_0 + A\exp(\pm\sqrt{-2a_1(\xi - \xi_0)}).$$

Since we are seeking a pulsed solution we choose "+ " for $\xi \to -\infty$, and "$-$" for $\xi \to +\infty$. From this we get the condition that a_1 is necessarily negative, that is $u''(\phi) < 0$, or that $u(\phi)$ is a maximum at ϕ_0.

At ϕ_1, the leading order for the Taylor expansion of $u(\phi)$ is

$$u(\phi) \sim b_1(\phi - \phi_1)$$

Plugging it to equation (45) we get

$$\frac{d(\phi - \phi_1)}{\sqrt{\phi - \phi_1}} = \pm\sqrt{-2b_1}d(\xi - \xi_1)$$

whose solution is

$$\phi = \phi_1 - \frac{b_1}{2}(\xi - \xi_1)^2$$

This shows that at ϕ_1 we have a maximum (soliton) of $\phi(\xi)$ if $\phi_1 > 0$, and a minimum (caviton) if $\phi_1 < 0$.

We can sumarize this analysis as in (48), for the existence of soliton (caviton).

$$\begin{array}{ll} (i) & u(\phi) = u'(\phi) = 0 \text{ at } \phi = 0 \\ (ii) & u(\phi) = 0 \text{ at } \phi = \phi_1 \neq 0 \text{ and} \\ & u'(\phi) < (>) 0 \text{ for } \phi_1 < (>) 0, \\ (iii) & u(\phi) < 0 \text{ for } 0 < |\phi| < |\phi_1| \end{array} \tag{48}$$

The condition (i) is automatically satisfied from the definition of $u(\phi)$. When $u''(0) < 0$ then, we may have soliton and/or caviton.

IV.2 Positive and negative double-layers

Double-layers can be understood as a sudden change of the electric potential due to the space charge. Finite amplitude double-layers exist provided that

$$\begin{array}{ll} (i) & u(\phi) = u'(\phi) = 0 \text{ at } \phi = 0 \\ (ii) & u(\phi) = u'(\phi) = 0 \text{ at } \phi = \phi_1 \neq 0 \\ (iii) & u(\phi) < 0 \text{ for } 0 < |\phi| < |\phi_1| \end{array} \tag{49}$$

This comes from the same analysis as in previous section. Now the conditions in the item (ii) provide two equations:

$$n_{e0}a_e + n_{i0}a_i - n_n a_n - n_p a_p = 0$$

and

$$n_{e0}a_e^2 - n_{i0}a_i^2 + 3n_n a_n^2 - 3n_p a_p^2 = 0. \tag{50}$$

which are conditions under which double-layers exist.

From conditions (48) for solitons and (49) for double-layers we have conducted a through parametric analysis to determine the regions where solitons and double-layers exist.

IV.3 Numerical Results

Here we will show the parametric boundaries for the existence of solitons and double layers.

A Sagdeev potential profile, $u(\phi)$, which results in coexistence of compressive and rarefactive (caviton) solitons is shown in Fig. 1, where the profile depicts the expression (46) with values of parameters: $M = 1.1$; $n_e = 0.571$; $n_i = 0.429$; $n_p = 0.761$; $n_n = 0.619$; $a_e = 0.4$; $a_i = 1.7986$; $a_n = 0.826$ and $a_p = 0.4141$, whose values are satisfy the relations (17) - (20). The Sagdeev potencial is a negative value in the interval of electric potential ϕ from 0 to ϕ_1 with one side ending on a maximum and the other side just crossing the zero value of $u(\phi)$. This will produce a soliton (compressive soliton) . On the other region, for ϕ from ϕ_2 to 0, a caviton is produced.

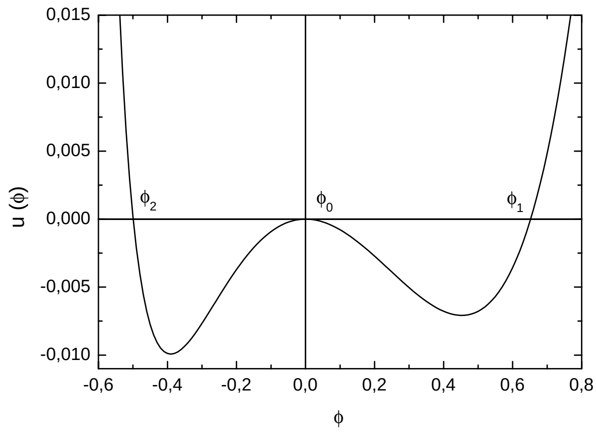


Figure 1. Sagdeev potential $u(\phi)$ versus electric potential ϕ, for soliton with parameters $M = 1.1$; $n_e = 0.571$; $n_i = 0.429$; $n_p = 0.761$; $n_n = 0.619$; $a_e = 0.4$; $a_i = 1.7986$; $a_n = 0.826$ and $a_p = 0.4141$.

Figure 2 shows a 3D plot of $u(\phi_m)$ as function of two parametrs n_{e0} and n_p for a fixed Mach number $M = 1.5$, where $\phi_m = \frac{1}{2a_p}$, ϕ_m is the maximum value of the ϕ such that $u(\phi)$ is a real number. Since $u(\phi)$ is a maximum at $\phi = 0$, so it is a negative value near this point, if $u(\phi)$ is a positive number at ϕ_m, it means that $u(\phi)$ has crossed a $u(\phi) = 0$ point somewhere before ϕ_m. This guarantees a soliton. In the figure the light gray color shows the region of positive $u(\phi_m)$ and the dark gray color shows the region of negative $u(\phi_m)$. Therefore the light gray color shows the region of soliton. The loci where $u(\phi) = 0$ is the boundary of the region of existence of solitons.

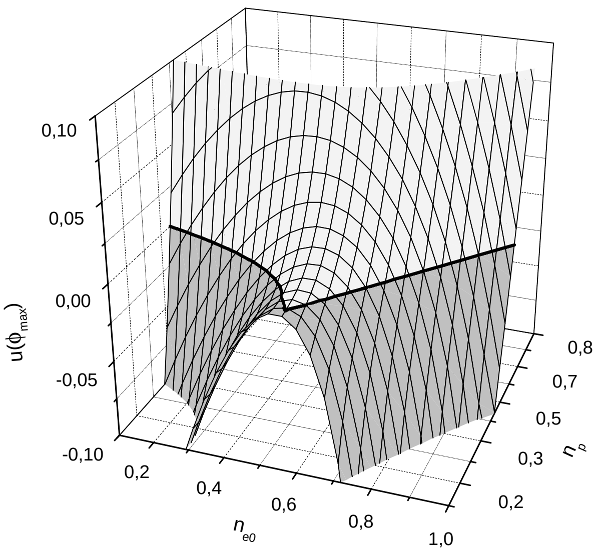


Figure 2. A 3D plot of $u(\phi)$ in function of two parametrs n_{e0} and n_p for a fixed Mach number $M = 1.5$ for the maximum value of ϕ, just before $u(\phi)$ turns into a complex value.

In Fig. 3 we show several of these boundary curves where $u(\phi_m) = 0$ for various values of M with the same paramerers as in the previous figure. The curve 3 - 0 is for $M = 1.5$, 1 - 0 is for $M = 1.01$, 2 - 0 for $M = 1.1$, and 4 - 0 for $M = 2.0$.

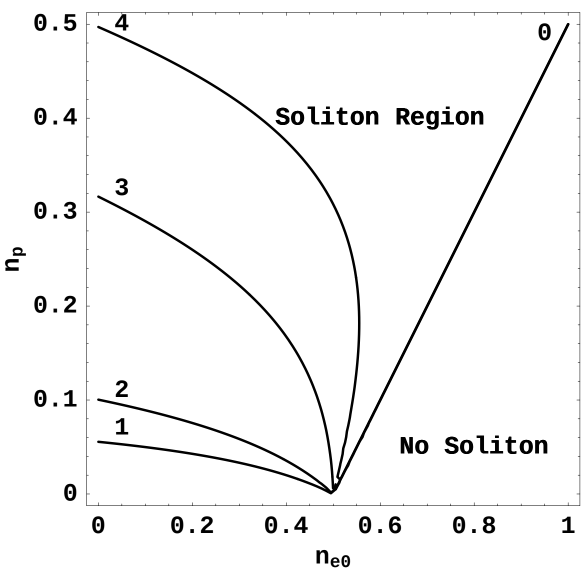


Figure 3. Contour plot of soliton solution for different M: 1 - $M = 1.01$; 2 - $M = 1.1$; 3 - $M = 1.5$; 4 - $M = 2.0$.

With the same way, we calculated the regions of existence of cavitons (Fig. 4) for different Mach number M: 1 - 0 for $M = 1.01$; 2 - 0 for $M = 1.1$; 3 - 0 for $M = 1.5$ and for the minimum value of ϕ.

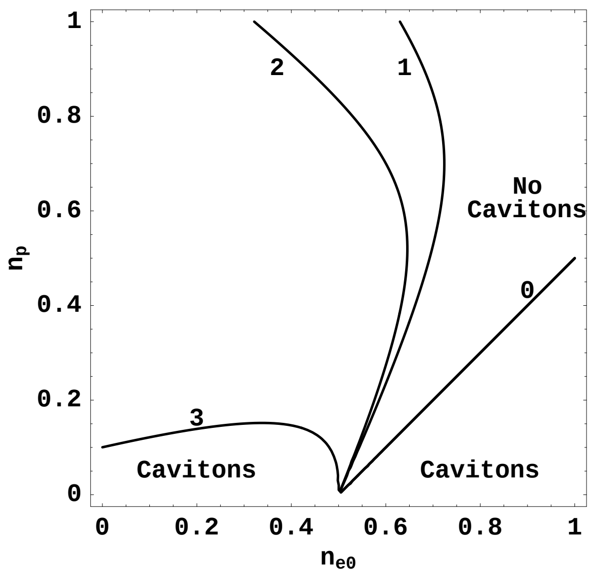


Figure 4. Contour plot of caviton solution for different M: 1 - $M = 1.01$; 2 - $M = 1.1$; 3 - $M = 1.5$.

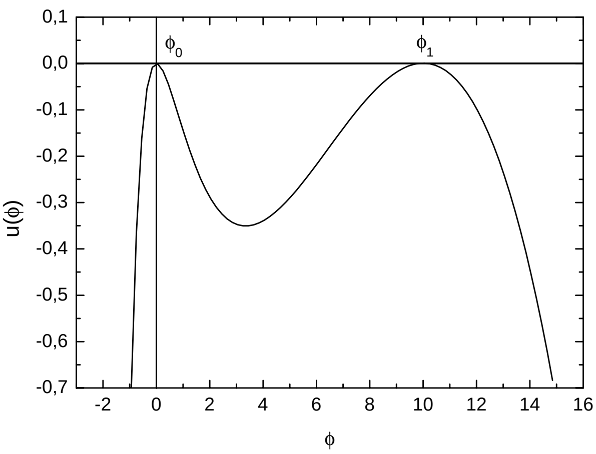


Figure 5. Potential energy versus electric potential, for positive double layer, with parameters $n_e = 0.525$; $n_i = 0.475$; $n_p = 1.665$; $n_n = 1.615$; $a_e = 0.1$; $a_i = 1.995$;$a_n = 0.2$; $a_p = 0.02$.

Figure 5 is a profile of $u(\phi)$ for the parameters $n_e = 0.525$; $n_i = 0.475$; $n_p = 1.665$; $n_n = 1.615$; $a_e = 0.1$; $a_i = 1.995$; $a_n = 0.2$; $a_p = 0.02$. The region between ϕ_0 and ϕ_1 , $u(\phi)$ is negative and are limited by two maxima. These conditions guarantee the existence of a double layer.

The case of negative double layer is presented on Fig. 6 with the value of the parameters $n_e = 0.9895$; $n_i = 0.01052$; $n_p = 1.691$; $n_n = 1.7116$; $a_e = 0.9$; $a_i = 10.4$; $a_n = 1.0$; $a_p = 0.1$.

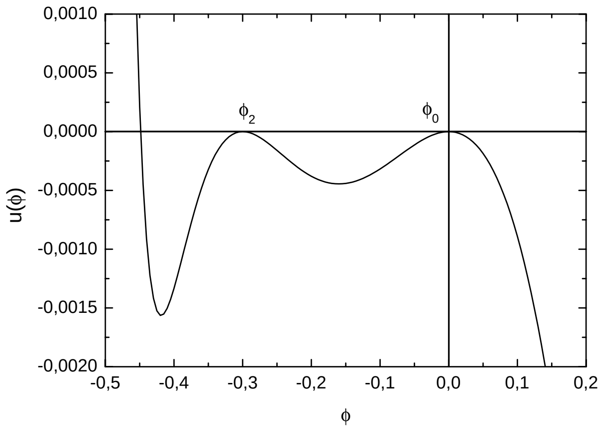


Figure 6. Potential energy versus electric potential, for negative double layer, with parameters $n_e = 0.9895$; $n_i = 0.01052$; $n_p = 1.691$; $n_n = 1.7116$; $a_e = 0.9$; $a_i = 10.4$; $a_n = 1.0$; $a_p = 0.1$.

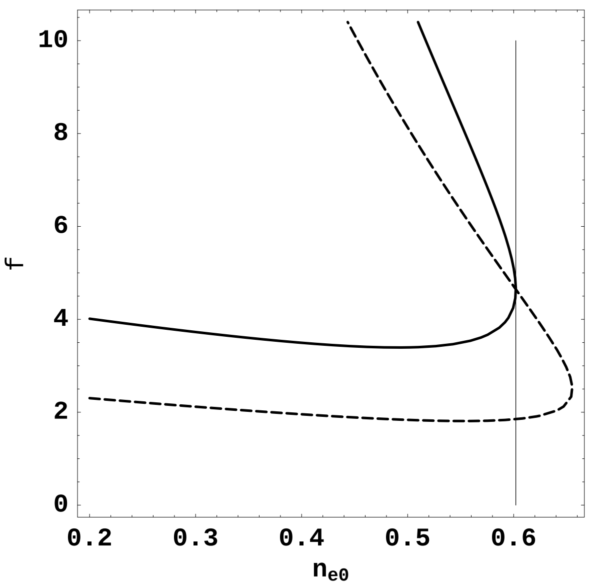


Figure 7. Loci of $u(\phi) = 0$ and $u'(\phi) = 0$ in the space (n_{e0}, ϕ) for $M = 1.5$ and $n_p = 4$.

Double layers exist if both ends of the interval for negative $u(\phi)$ are maxima as shown in figuras 5 and 6. Now, at $\phi = 0$, $u(\phi)$ is either a maximum ($M > 1$) or a minimum ($M < 1$) so if $M > 1$ we guarantee one maximum at $\phi = 0$. Now, the other end to be a maximum, we have to have both $u(\phi) = 0$ and also $u'(\phi) = 0$, at the same point. Figure 7 gives the loci of $u(\phi) = 0$ in solid line and $u'(\phi) = 0$ in dashed line in the space (n_{e0}, ϕ). It shows one point at $n_{e0} = 0.6$ the both conditions are met. This determinies the point where we have a double layer. After determining the point where both functions are zero, one has to check whether the function $u(\phi)$ is negative for all values of ϕ from zero to the crossing point. The calculation is for case Mach number $M = 1.5$ and parameter $n_p = 4$.

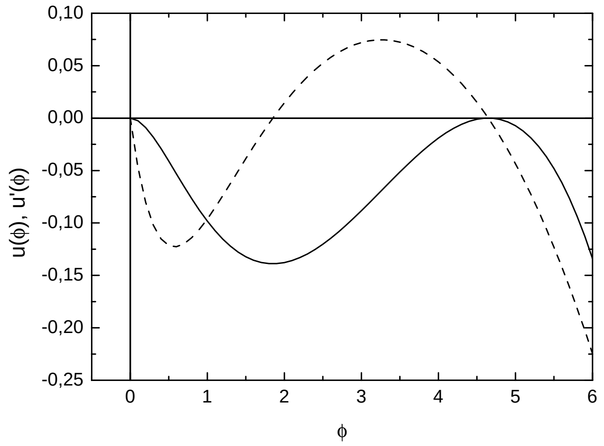


Figure 8. Ploting of $u(\phi)$ and $u'(\phi)$ as function of ϕ with $n_{e0} = 0.602$.

The necessary conditions for the double layer solution are met as you can see in the graph (Fig. 8) by ploting of $u(\phi)$ (solid line) and $u'(\phi)$ (dashed line) as function of ϕ with parameter $n_{e0} = 0.602$.

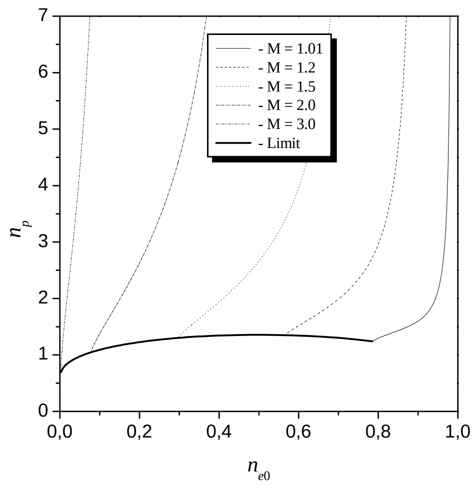
By changing parameters we can cover the regions where these conditions are satisfied. This is an extensive calculation and data collecting which will be done in another context.

We proceed to obtain the parametric regions where conditions (49) are satisfied. Starting with 9 parameters defined in (14)-(16) with the inclusion of 4 equations (17)-(20), we have a 5-parameter region. We introduce parameters α and β in substitution of a_i and a_n, $\alpha = a_e/a_i = T_i/T_e$ and $\beta = a_p/a_n = Z_p M_n / Z_n M_p$. So, we have to deal which a function $f(n_{e0}, n_p, M, \alpha, \beta)$ which satisfies relations given in (49).

Furthermore we reduce the 5 parameters to a even smaller number by taking a reasonable physical values for α, β and M, resulting in a two parametric space: $g(n_{e0}, n_p)$. In Fig. 9a, we show the curves where the double layer solutions are found. We have chosen $\alpha = 0.09$ and $\beta = 0.10$. For each given value of M, from 1.01 to 3.0, a curve is drawn on $n_{e0} \times n_p$ space where double layer exists.

The same treatment was applied for the particular case of laboratory plasma reported by Oohara et al [10], where a fullerene-ion plasma of the same mass (C_{60}) was produced in the process of a hollow electron-beam impact ionization. Authors observed two low-frequency electrostatic waves. For calculations we used main characteristics of the dusty plasma, i.e. $n_e/n_p \sim 10^{-6}$, $n_e = 1.0$, $n_p = n_n \sim 10^6$ and $M_p = M_n$. Moreover, we introduce, on their experimental conditions, a small quantity of ions to fulfill conditions of the four component dusty plasma. Thus we have parameter $\alpha = 0.09$ and $\beta = 1.0$ that is different from the case discussed above.

In the Figure 9b the result of the double layer conditions for different M values is shown. As we can see, the authors [10] have possibility to obtain a double layer in laboratory plasma. It is interesting to observe the different comportment of the curves for small values of the n_p. In contrast to



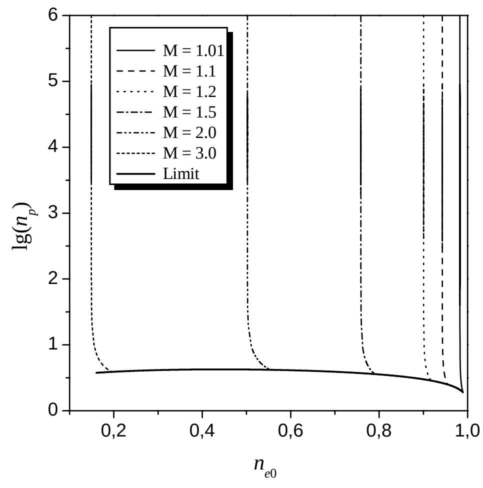


Figure 9. Relations between the parameters n_p and n_e for different values of Mach number M and parameter $\alpha = 0.09$: a) $\beta = 0.1$; b) $\beta = 1.0$.

the case of low β, for the double layer exist it is necessary increasing both n_e and n_p for some constant M. Furthermore, the limit of the double layer existence decrease with increasing β.

V Summary

The linear and nonlinear properties of dust-acoustic waves (DAW) were studied. We used the model of multi-component dusty plasma with inertialess electrons and ions as well as positively and negatively charged inertial dust grains. We found that in four component dusty plasma there are remarkable changes in the nonlinear properties of the DAW. We show that rarefactive and compressive DA potentials can simultaneously appear, which amplitudes are the function of the Mach number and the ratio between the positive and negative dust components. Moreover, the presence of positively charged dust grains produces double layers in those parameter regimes in which localized DA solitons are absent. The theory was applied to the laboratory plasma reported by Oohara et al. We predict that a double-layer might be possible to be launched in their experiment if a trace ions component is added. The results of the investigation can be useful for designing laboratory experiments dealing with the demonstration of DAW in multi-component dusty plasma with the positive and negative dust grains. Our parametric studies of solitons and double layers should be useful in identifying coherent nonlinear structures in the Earth's mesosphere. Furthermore, non-stationary double layers could be potential accelerators for dust particulates in space plasmas.

Acknowledgments: We acknowledge financial support of CAPES (Coordenação de Aperfeiçoamento de Pessoal do Ensino Superior) and FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo).

References

[1] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).

[2] Y. Nakamura, T. Odagiri, and 1. Tsukabayashi, Plasma Phys. Control. Fusion **39**, 105 (1997); Y. Nakamura and 1. Tsukabayashi, Phys. Rev. Lett. **52**, 2356 (1984).

[3] S. Watanabe, J. Phys. Soc. Japan **53**, 950 (1984); M. Tajiri and M. Tilda, ibid. **54**,19 (1985).

[4] T. E. Sheridan, J. Plasma Phys. **60**, 17 (1998).

[5] P. K. Shukla, Phys. Plamnas **1**, 1362 (1994).

[6] P.H. Sakanaka, and P.K. Shukla, Phys. Scripta, **84**, 181 (2000).

[7] V. V. Chow, D. A. Mendis, and M. Rosenberg, J. Geophys. Res. **98**, 19,065 (1993); D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994); D. A. Mendis, M. Rosenberg, and V. W. Chow, in Physics of Dusty Plasmas, edited by M. Horany, S. Robertson, and B. Walch (American Institute of Physics, 1998), pp. 1-11.

[8] M. A. Raadu and G. Chanteur, Physica Scripta **33**, 240 (1986).

[9] R. Z. Sagdeev, *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), pp. 23-91.

[10] W. Oohara, N. Tomioka, T. Hirata, R. Hatakeyama, and N. Sato, *Proceedings of the 2000 International Congress on Plasma Physics*, Quebec, October, 2000., Vol 1, pag. 116-119 (2000).