Hidden possibilities in controlling optical soliton in fiber guided doped resonant medium

Anjan Kundu
Theory Group & CAMCS, Saha Institute of Nuclear Physics
Calcutta, INDIA
anjan.kundu@saha.ac.in

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Abstract

Fiber guided optical signal, propagating in a Erbium doped resonant medium, is known to produce cleaner solitonic pulse described by the self induced transparency (SIT) coupled to the nonlinear Schrödinger equation. We discover hidden possibilities in such a set up, for amplifying solitonic pulse and controlling its shape and dynamics, regulated by the initial population inversion of the dopant atoms. The effects can be enhanced by a novel arrangement of going from single to a hierarchy of coupled SIT system. These theoretical predictions are workable exactly, due to the integrability of the system.

Optical communication through fiber has achieved phenomenal development over the last two decades OptÇom. To counter dissipation and dispersion in the media, which are the main hindrance of signal transmission through optical fibers, the emphasis is focused presently more on the dispersion management techniques and devices OptCom¸dismang. However apart from the involvement of costly repetitive devices and other adversities like the lack of stability in such arrangements, it suffers from the loss of analytic methods, bringing in theoretical disadvantages agaŗwal. On the other hand, there are proposals for fiber optic communication, mediated by the solitonic modes of the nonlinear Schrödinger equation (NLS), advocated in earlier days nlsşolit, where the group velocity dispersion in the light pulse wave guide can be balanced by the self phase modulation in the nonlinear fiber medium agaŗwal. Though the integrability of this system is an added advantage, experiments revealed the insufficiency of such models for efficient practical application nlsḑrab. Among other proposals of theoretical and practical importance with improved signal transmission was the soliton solution due to the self-induced transparency (SIT), created by a coherent response of the medium to an ultra short optical pulse SIT,ļamb. Finally a pioneering idea was put forward, combining the benefits of both the NLS and the SIT systems Maiminitsov, where stable signal propagation can be achieved by transmitting an optical pulse through Erbium (Er^{3+}) doped nonlinear resonant medium, described by the soliton solution in a coupled NLS-SIT system nakaẓawa1. However, in spite of the theoretical and experimental success, optical communication by the solitonic mode, governed by the NLS-SIT nakazawa1,naķazawa2,kakei or the Hirota-SIT porsęzian equations, unfortunately, could not receive the needed response to become a leading method in fiber communication.

One of the reasons for the waning interest in soliton mediated communication, is perhaps, the lack of exciting proposals and failure to explore new opportunities in the existing theory.

Our aim here is to explore new possibilities, hidden in the NLS-SIT system, through exact analytic treatment, to put forward a proposal for the amplification and control of the solitonic signal by regulating the population inversion in the laser-active dopant atoms. We have detected serious lapses in the well known result on the NLS-SIT soliton nakazawa1,nakazawa2 and rectified them, bringing out unexplored details. We also propose, within the framework of integrable systems, a novel arrangement of replacing the single SIT system coupled to the NLS, by its hierarchy, for further amplification and control of the soliton pulse.

Propagation of a stable optical pulse through a fiber medium, serving as a dispersive and nonlinear wave guide with Kerr nonlinearity, may be described by the solitonic form lamb,soliton

$$E = \eta \text{sech} \zeta e^{i\theta}, \ \zeta = \eta(t - vz), \ \theta = \omega z + kt. \tag{1}$$

With inverse soliton velocity $v = v_{nls}$, phase wave length $\omega = \omega_{nls}$ and soliton width $(\eta v_{nls})^{-1}$, having constant values, (1) is an exact solution of the NLS $iE_z = E_{tt} + 2|E|^2 E$, where the role of space and time variables has been interchanged, as customary in nonlinear optics agarwal. Soliton parameters:

$$v_{nls} = -2k \quad \text{and} \quad \omega_{nls} = k^2 - \eta^2, \tag{2}$$

can be linked to the complex discrete spectral parameter $\lambda_1 = k + i\eta$, arising in the inverse scattering method (ISM) for extracting exact solution of the integrable NLS soliton.

On the other hand, coupling to the SIT system, generates a deformed NLS equation given by

$$iE_z = E_{tt} + 2|E|^2 E + s < p >, \tag{3}$$

where s is a combination of different physical parameters of the system and averaged induced polarization $< p >=< p(z, t; w) >= \int p(z, t; w) g(w) dw$, is related to the frequency spread in the energy level: $g(w)$; $\int g(w) dw = 1$ of the resonant atoms nakazawa1,nakazawa2. Assuming sharp resonance with $g(w) = \delta(w - w_0)$, we replace $< p >= p(z, t; w_0) \equiv p$ in equation (3) and take $s = 1$, for simplicity. Equations closing this system are given by the rate of change of p and population inversion N, represented by the SIT system

$$p_t = i(2NE - w_0 p), \ N_t = -i(Ep^* - E^* p). \tag{4}$$

The induced polarization due to two level atoms given by the matrix element $p = \nu_1 \nu_2^*$, and the population inversion $N = |\nu_2|^2 - |\nu_1|^2, \ -1 \leq N \leq 1$, are described by normalized wave functions ν_1 and ν_2 of the ground and the excited states, respectively.

In this coupled NLS-SIT system (3-4), the optical pulse signal entering the fiber with sufficient intensity and propagating through the Erbium doped resonant medium, adjusts itself to the stable solitonic form givan again as (1), but with soliton parameters modified due to the interaction, with contribution from both NLS and SIT parts:

$$v = v_{nls} + v_{sit}, \ \omega = \omega_{nls} + \omega_{sit}, \ k \to \tilde{k} = k - w_0. \tag{5}$$

Note that, in the absence of the SIT system we must have $v_{sit} = \omega_{sit} = w_0 = 0$, recovering the well known NLS soliton, as presented in (2). Pure SIT soliton can be similarly obtained also as (1), by switching off the NLS influence by setting $v_{nls} = \omega_{nls} = 0$ in (5). The soliton solution in the coupled NLS-SIT system on the other hand, gets contribution from both the parts as in (5). The coupled NLS-SIT system exhibits also an intriguing property that, it allows only the moving soliton to exist, since v_{sit} can not vanish due to its specific form (6). Such a soliton can only be stopped dynamically, by fine tuning v_{sit} to compensate for the other velocity parameter v_{nls}.

This rich interaction pattern of the NLS-SIT system is however misinterpreted, as we have detected, in a well known earlier work nakazawa1,nakazawa2, due to the loss of some crucial terms in their soliton solution, giving incorrect assumption for the pulse delay as $v = v_{sit}$ and phase rotation as $\omega = \omega_{nls}$, which becomes evident when compared with our exact result (1,5). This error appeared because of the improper comparison of the NLS and the SIT solutions made by the authors, without considering additional contribution due to their interaction, which led in turn to fundamentally wrong conclusion that, *pulse delay (and hence the inverse velocity) of the soliton is determined due to the SIT effect alone* and similarly, *the z dependence of the phase rotation* ω *for both the dipole and the input field is determined solely due to the NLS soliton* nakazawa1,nakazawa2. Our exact soliton solutions for the input optical field (1) and the dipole (7) in the NLS-SIT system, which include the effect of interaction with correct expressions (5), rectifies it to get the valid conclusion that, only a part of the pulse delay, e.g. v_{sit} is determined by the SIT effect, while an additional delay v_{nls} comes from the NLS part. Similarly, the phase rotation is contributed from both NLS and SIT parts as $\omega = \omega_{nls} + \omega_{sit}$.

While solitonic parameters related to the NLS are expressed as (2), those linked to the SIT part are usually given by $v_{sit} = -\frac{1}{\rho}$, $\omega_{sit} = -\frac{\tilde{k}}{\rho}$, where $\rho = |\lambda_1 - w_0|^2 = \tilde{k}^2 + \eta^2$, nakazawa1,nakazawa2,kakei. However, we find that, these solitonic parameters related to the SIT system may be given in a more general form involving a z-dependent arbitrary function $c(z) \neq 0$ as

$$v_{sit} = \frac{1}{z\rho} \int^z c(z')dz', \quad \omega_{sit} = \tilde{k} v_{sit}, \tag{6}$$

defined through the initial profile of the population inversion. We show below that, our generalization (6), which reduces for $c(z) = -1$ to the known expressions, can play important role in soliton management. The solution for dipole p in the coupled NLS-SIT equations, derived from (3) using soliton solution (1) with (5) and (6), takes also solitonic form

$$p = \frac{\eta}{\rho}\, c(z) \mathrm{sech}\zeta (\tilde{k} - i\eta \tanh\zeta) e^{i\theta}, \tag{7}$$

with explicit appearance of $c(z) \neq 0$. Similarly, the population inversion can be derived exactly from the set of equations (3-4), using the soliton solutions (1) and (7), as

$$N(z,t) = c(z)(1 - \frac{\eta^2}{\rho}\, \mathrm{sech}^2\zeta), \tag{8}$$

where arbitrary nonzero function $c(z) = N|_{t \to -\infty}$ appears as an integration constant. Therefore, at the initial moment the occupancy for the excited state would be $|\nu_2|^2 = \frac{1}{2}(1 + c(z))$ and that for the ground state $|\nu_1|^2 = \frac{1}{2}(1 - c(z))$. It shows that, for nonzero $c(z) > -1$ the dopant atoms could be prepared initially in the excited state, e.g., by optical pumping, resulting a laser-active amplifying medium, with intensity determined by $c(z)$. Note that, only in such a case when more active dopant atoms are in the excited state, the optical soliton can gain net energy. This important fact, though known and used for amplifying optical pulses in fiber media OptÇom, was surprisingly ignored so far in the theoretical description of the soliton in the coupled NLS-SIT system, by restricting to $c(z) = -1$, by assuming atoms to be initially in their ground states: $|\nu_1|^2 = 1$, $|\nu_2|^2 = 0$.

However, at the initial moment with population inversion $N = c(z) = const. > -1$, all dopant atoms will be excited equally, while in the general case when $c(z)$ varies with z, the excitation population will have a distribution profile along the fiber. Optical solitonic pulse during the first half of its propagation through a resonant medium with initially populated excited state would make the excited state more populated, while during the second half, the coherent stimulated emission would return the energy back to the light pulse, in addition to the prepumped energy stored initially in the excited atoms. The induced polarization p, which mediates in this process, would also change continuously as (7), following the dynamics of SIT.

Our proposed general initial condition with nontrivial $c(z) > -1$ for the NLS-SIT, as we see below, can play also a crucial role in controlling the shape and dynamics of the optical soliton (1), in addition to the soliton pulse amplification, discussed above. Consequently, it opens up an important possibility to address the pulse broadening problem by regulating the initial population inversion of the dopant atoms, and achieve path-dependent accelerated motion of the soliton, aided by the energy available from the optical pumping, For example, a solitonic pulse governed by the NLS equation alone, subjected to a perturbative term $-i\frac{\Gamma}{2}\,E,\ \Gamma << 1$, would suffer attenuation by a factor $\eta(z)$ and broadening by an inverse factor $(2k\eta(z))^{-1}$, which can be worked out through the perturbation theory as $\eta(z) = \eta\ e^{-\Gamma z}$agaŗwal. Though an attenuation results intensity loss, the broadening leads to more serious problem of information loss and bandwidth limitation. Therefore transforming the field $E \rightarrow \eta(z)^{-1}E$, we concentrate here only on the broadening problem of the NLS soliton, due to the increasing solitonic width $\frac{1}{2k\eta}\ e^{\Gamma z}$ along z, as shown in Fig 1.

Transmitting this solitonic pulse with increasing width through a doped resonant medium, described by an interacting NLS-SIT system (3-4), it is possible to control the pulse broadening, by suitably preparing the initial population inversion as required by function $c(z)$. Fig 2a shows this controlling effect, where the broadening of solitonic pulse suffered in Fig 1, is countered by the narrowing of the pulse due to variable width $v(z) = (v_{nls} + \frac{\eta}{\rho}c(z))^{-1}$, by taking $c(z) \sim \eta(z)^{-1}$. The soliton dynamics is also changed to a variable velocity $v(z)$ with accelerated motion, possible due to the energy supplied by optical pumping from outside. These hidden possibilities remained unexplored in earlier work nakazawa1,nakazawa2,kakei,porsezian, due to the restriction to $c(z) = -1$.

Another promising opportunity in managing soliton in fiber optic communication, involving the NLS-SIT system, that has been missed completely in all earlier investigations, is the proposal of recursively enhancing the effect of amplification and control of the optical soliton, by replacing the single SIT, the only case considered so far in the literature, by a hierarchy of the SIT system. To show that, such a system retains its integrability, we construct the associated linear system $\Phi_t = U\Phi,\ \Phi_z = V\Phi$, with Lax pair $U(\lambda), V(\lambda)$ U̧V, the compatibility condition of which $U_z - V_t + [U, V] = 0$, yields the the same deformed NLS (3), coupled to a hierarchy of the SIT system:

$$p_t = i(2NE - w_0 p + e),\ N_t = -i(Ep^* - E^*p),$$
$$e_t = i(2ME - w_0 e),\ M_t = -i(Ee^* - E^*e). \tag{9}$$

Here we consider only two coupled SIT systems, with e as induced polarization and M as population inversion linked to the second SIT system. This process can be continued to a hierarchy of SIT equations without spoiling the integrability. The exact soliton solution of the optical pulse for this more general NLS-SIT system, interestingly, can be expressed again in the form (1), where the soliton parameters are modified with contributions from all interacting parts as $v = v_{nls} + v_{sit1} + v_{sit2},\quad \omega = \omega_{nls} + \omega_{sit1} + \omega_{sit2}$. where v_{nls}, ω_{nls} and v_{sit1}, ω_{sit1} are the same expressions as (2) and (6), while the additional SIT contribution is given by $v_{sit2} = 2\frac{\tilde{k}}{z\rho^2}\int^z c_2(z')dz'$, and $\omega_{sit2} = \frac{1}{2\tilde{k}}(\tilde{k}^2 - \eta^2)v_{sit2}$, . Therefore the soliton width and dynamics can be regulated now by an additional control provided by the second SIT system, as shown in Fig. 2b.

In this letter we have shown a novel possibility of amplification by prepumping energy and control of the solitonic pulses in the coupled NLS-SIT system, by adjusting initial population inversion of dopant atoms in the resonant medium, given by a more general function $c(z)$, in place of the traditional restriction $c(z) = -1$. We have also presented an innovative scheme to enhance these effects, particularly for addressing the pulse broadening problem, by introducing a novel multiple SIT system, coupled to the NLS equation in a hierarchal way. This arrangement should be realizable through multiple doping, where one set of dopant atoms in the resonant medium would be coupled to another set by coherent induced stimulated emission, with all atoms interacting in turn with the soliton pulse of the input field via induced polarization. In this multi-doped medium requiring more intense threshold pulse intensity for the formation of solitonic pulse, one could possibly use also multi-level dopant like neodymium (Nd^{3+}). In such atoms with more than two available levels, unlike two levels, the energy can be pumped throughout the process resulting to higher gain OptÇom. Both these theoretical proposals with applicable potentials can be worked out analytically in minute details through ISM, due to the integrability of the underlying system.

References

[1] G. P. Agarwal, *Fiber Optic Communication Systems*, (John Wiley, NY, 2002); V. Alwyn *Fiber Optic Technologies* (Cisco Press, 2004); Encyclopedia of Laser Physics & Technology, (Virtual Web-Library, RP Photonics Consulting)

[2] B. J. Eggleton et al, J. Lightwave Tech. **18**, 1418 (2000); X. F. Chen et al, Photonics. Tech. Lett. IEEE **12** , 1013 (2000); F. Poletti et al, Photonics. Tech. Lett. IEEE **20** , 1449 (2008)

[3] G. P. Agarwal, *Nonlinear Fiber Optics* (Acad. Press, N.Y., 2007)

[4] L. F. Mollenauer, R.H. Stolen and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980); A. Hasegawa and F. D. Tappert, Appl. Phys. Lett. **23**, 142 (1973)

[5] F. M. Mitshke and L. F. Mollenauer, Opt. Lett. **11**, 657 (1986)

[6] S. L. McCall and E. L. Hahn, Phys. Rev. Lett. **18**, 908 (1967); Phys. Rev. **183**, 457 (1969)

[7] G. L. Lamb Jr., Rev. Mod. Phys. **43**, 99 (1971)

[8] A. I. Maimistov and E. A. Manyakin, Sov. Phys. JETP **58**, 685 (1983)

[9] M. Nakazawa, E. Yamada and H. Kubota, Phys. Rev. Lett. **66**, 2625 (1991)

[10] M. Nakazawa, E. Yamada and H. Kubota, Phys. Rev. **A 44**, 5973 (1991)

[11] S. Kakei and J Satsuma, J. Phys. Soc. Jpn. **63**, 885 (1994)

[12] K. Porsezian and K. Nakkeeran, Phys. Rev. Lett. **74**, 2941 (1995)

[13] M. Ablowitz and H. Segur, Solitons and Inverse Scattering Transforms (SIAM, Philadelphia, 1981) ; S. Novikov et al , Theory of Solitons (Consultants Bureau, NY, 1984)

[14] $U(\lambda) = U_{nls}(\lambda)$ is as in NLS soliţon. In $V(\lambda) = V_{nls}(\lambda) + V_{def}(\lambda)$, V_{nls} represents NLS soliţon, while $V_{def}(\lambda) = i\sum_j(\lambda - w_{0j})^{-j}G_j$, with $G_j = N_j\sigma^3 + p_j\sigma^+ + p_j^*\sigma^-$, linked to the j-th equation in the SIT hierarchy kunḑu09.

[15] A. Kundu, J. Math Phys. **50**, 102702 (2009)

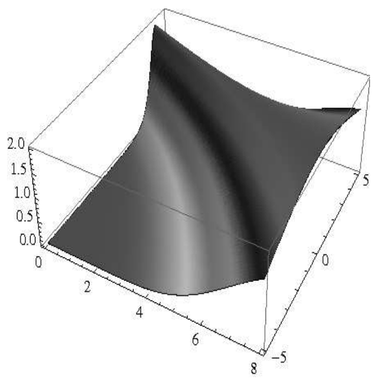

Figure 1: Broadening of the perturbed NLS soliton $|E(z,t)|$ along the fiber, moving with velocity $-\frac{1}{2k}$, for the choice of parameters $k = 0.25,\ \eta = 1.0,\ \Gamma = 0.3$

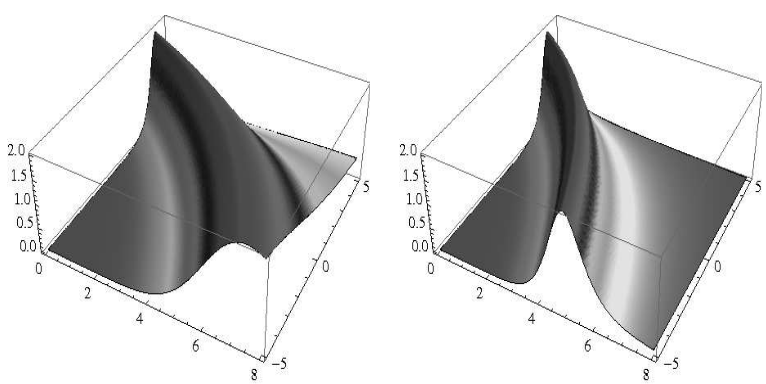

Figure 2: a) Broadening soliton pulse of NLS is controlled by coupling to a SIT system with $c(z) = 0.2e^{0.3z}$ and $w_0 = 1.5$. The soliton motion changes also to a variable velocity, evident from the bending of the pulse in the (z,t)- plane. b) Additional control by a coupled second SIT system with $c_2(z) = -e^{0.3z}$, showing distinctly the efficient restoration of the soliton width and a more prominent change in the soliton velocity.