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A NEW DELAY PARAMETER DEPENDENT ON VARIABLE ANALYSIS PERIODS AT SIGNALIZED INTERSECTIONS. PART 1: MODEL DEVELOPMENT

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Abstract. Delay is an important factor in the optimization of traffic signals and the determination of the level of service of a signalized intersection. This paper proposes a methodology and a new formulation to identify the delay parameter in signalized intersection delay models. In this study, the delay parameter is modeled as a function of analysis period instead of a fixed value used by the existing delay models. Therefore, the proposed delay model including new delay parameter can produce more reasonable delay estimations at signalized intersections for variable time periods. A comparative study of the proposed time-dependent model against the existing four different models was performed to present the improvements in this model.

Keywords: delay parameter, delay models, analysis period, simulation, signalized intersections, TRAF-NETSIM.

1. Introduction and background

Delay is one of the principal parameters used as the measure of effectiveness (MOE) to determine the level of service (LOS) at signalized intersections. The accurate prediction of delay is, therefore, important. Delay can be measured in the field or estimated using analytical models. Stochastic steady state, which was investigated by Webster (1958), Tanner (1962) and Miller (1968), and deterministic models, which were investigated by May and Keller (1967), Neuberger (1971) and Pignataro *et al.* (1978), have been commonly used in the estimation of delay at signalized intersections for undersaturated and oversaturated traffic conditions, respectively. However, as pointed out by Hurdle (1984), both types of delay models are entirely incompatible when degree of saturation is equal to 1.0. While the first model predicts infinite delay the latter estimates zero delay at this degree of saturation. To provide more realistic delay estimates and overcome the deficiencies in both models, time-dependent delay models have been developed, this problem is investigated by Burrow (1989), Catling (1977), Brilon, Wu (1990), Akcelik (1980, 1988), in Special Report (1994), Highway Capacity Manual (2000) and by Teply (1995). They are actually a mix of steady state and deterministic models utilizing the coordinate transformation techniques described by Kimber and Hollis (1978, 1979). The coordinate transformation is applied to the steady state curve, and smoothes it into deterministic line by making the steady state curve asymptotic to the deterministic line as shown in Fig. 1.

Thus, time-dependent delay models predict delay for both undersaturated and oversaturated conditions without having a discontinuity at the degree of saturation 1.0. Generally, time-dependent delay models consist of uniform and overflow delay terms given by Equation 1:

$$d = d_u + d_0 \tag{1}$$

in which d is average total delay (sec), d_u is uniform delay (sec) and d_0 is overflow delay (sec).

1.1. Uniform delay

Uniform delay resulting from the interruption of traffic flow by the traffic signals at intersections is estimated by assuming that vehicles arrive at a uniform and constant arrival rate during each signal cycle, and can be derived from a deterministic queuing theory. For undersaturated conditions, the uniform delay is calculated using the well-known delay model developed by Webster and Cobbe (1966) as follows:

$$d_u = \frac{C(1-\frac{g}{C})^2}{2\left(1-x\frac{g}{C}\right)}, \tag{2}$$

where C is cycle time (sec), g is green time (sec), and x is degree of saturation indicating the ratio of arrival flow (or demand) to capacity (i.e., v/c).

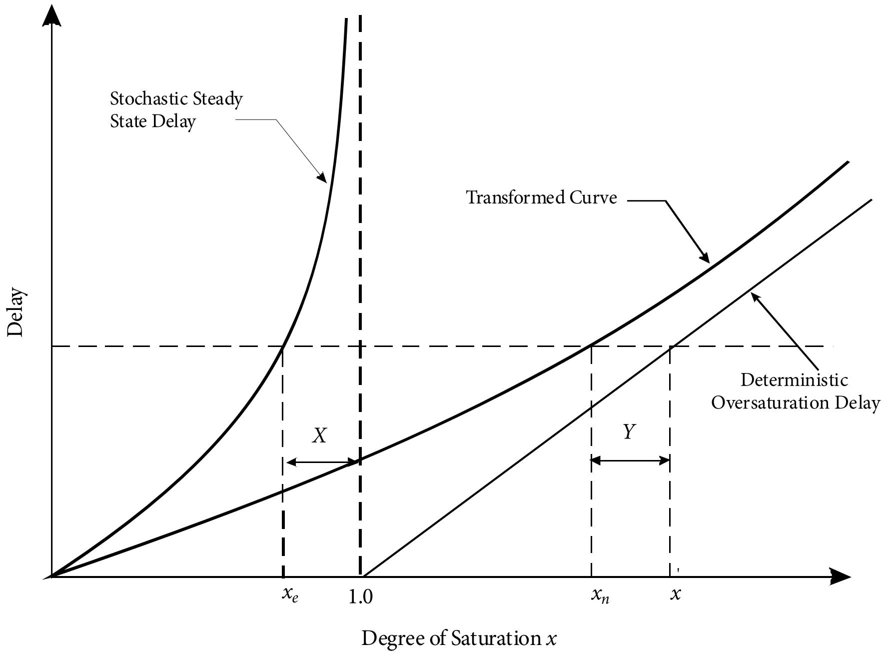

Fig. 1. Coordinate transformation technique (Kimber, Hollis 1979)

Values of x beyond 1.0 are not used in evaluating the delay in Equation 2. Therefore, uniform delay is computed by Equation for oversaturated conditions:

$$d_u = 0.5(C - g). \quad (3)$$

1.2. Overflow delay

Overflow delay is additional delay experienced by vehicles arriving during a specified flow period, which resulted from temporary oversaturation and continuous oversaturation. Overflow delay, which is defined in Equation 4 and estimated by Equation 5, consists of random and continuous overflow delays. Random overflow delay is caused by the fluctuation in vehicle arrivals. It occurs during some signal cycles in both undersaturated and oversaturated conditions. For low degrees of saturation, the effect of the randomness in arrivals is not important because total arrivals are much less than the capacity. Conversely, for high degrees of saturation and especially when arrival flow approaches the capacity, the effect of randomness increases sharply. Continuous overflow delay is the delay experienced by vehicles which are unable to discharge within the signal cycle since the arrival flow is greater than the capacity. Continuous overflow delay, as seen in Equation 6, is directly proportional to analysis period T.

$$d_o = d_{ro} + d_{co}, \quad (4)$$

$$d_o = 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{8kx}{cT}}\right], \quad (5)$$

$$d_{co} = \frac{T}{2}(x-1). \quad (6)$$

1.3. Delay parameter

Arrival and service characteristics at a signalized intersection determine level of delay, stop rate, emission consumption, and a queue on the approach. These characteristics are described by a delay parameter k. The analytical equations developed for estimating the delay parameter k are obtained by using either queuing analysis approach or empirical approach. The queuing analysis approach, which is mostly based on statistical distributions and probability theory, can be employed for a deterministic or a stochastic process, this problem is investigated by May (1990), Gerlough and Huber (1975). To estimate the delay parameter k with the queuing analysis approach, it is necessary to know the arrival, service and queue discipline characteristics on the road. On the other hand, if there is not any information about the queuing characteristics, an empirical approach is employed to estimate by calibrating the delay parameter k to fit the random simulated delay.

The coefficient C_p, known as Pollaczek-Khintchine expression, given in Equation 7, is used to represent the queuing characteristics at an intersection, and it also refers to the delay parameter k in overflow delay term of time-dependent delay models.

$$C_p = \frac{1}{2}\left(1 + v_s^2\right), \quad (7)$$

where v_s^2 – coefficient of variation of service time distributions.

Kimber *et al.* (1986) modified the Pollaczek-Khintchine expression utilizing computer simulation techniques to account for the effects of non-randomness in arrival pattern. For their study, they simulated a single service first come-first serve queuing process. Arrival and service times were drawn from a lognormal distribution. In addition, equivalent analytical calculations were performed for Erlang 2 and hyper-exponential arrivals. They found that the average delay parameter could be approximated closely by the Pollaczek-Khintchine expression. Their modified expression is given by:

$$k = \frac{1}{2}\left(v_a^2 + v_s^2\right), \tag{8}$$

where v_a^2 – coefficient of variation of arrival time distributions.

Kimber and Daly (1986) reported that v_s varies between 0.75 and 1.25 for road traffic. For random arrivals and departures, such as at unsignalized intersections, $v_a = v_s = 1$, so $k = 1$. On the other hand, for random arrivals and uniform departures, such as at fixed time signalized intersections, $v_a = 1$, $v_s = 0$ resulting in $k = 0.5$.

1.4. Previous studies relating delay parameter *k* to other factors

Akcelik and Rouphail (1993, 1994) used a cycle by cycle simulation model to develop an expression for the delay parameter k as a function of capacity per cycle. The parameters k and x_0 were derived from the simulation for random and platooned arrivals using the steady state delay model. Then, these two parameters were applied to the time-dependent delay model using a coordinate transformation technique. The expression k developed by Akcelik and Rouphail (1993) is given by:

$$k = 1.22(sg)^{-0.22}, \tag{9}$$

where s is saturation flow (veh/sec) and g is green time (sec).

The above expression is only applied when the degree of saturation x is greater than 0.5, and the k values in Equation 9 range from 1 to 0.5 while sg values vary from 3 to 60 vehicles per cycle.

Akcelik and Rouphail (1994) also developed a delay parameter k given in Equations 10 and 11 for platooned arrivals. In their model, k is not only a function of capacity per cycle but also a function of the magnitude of the platooning (PIP) and the cycle to cycle variation in the arriving stream. The magnitude of platooning is equivalent to the proportion of vehicles stopped at the upstream intersection.

$$k = (1.22 - 0.527PIP)(sg)^{-0.22} \quad \text{for } x_0 = 0.5, \tag{10}$$

$$k = \frac{0.302}{1 - PIP}(sg)^{-0.22} \quad \text{for } x_0 > 0.5. \tag{11}$$

Tarko *et al.* (1994) developed a model using the difference between the upstream and downstream intersection capacities to describe the delay parameter k. While they were developing a model related to the delay parameter k for platooned arrivals, accepting the basic theory that is random, overflow delay approaches to zero when the capacity at upstream intersection is less than or equal to the capacity at the downstream intersection. The model is then expressed as follows:

$$k = k_0 f, \tag{12}$$

where k_0 is model parameter for an isolated intersection, and f is the adjustment factor for upstream conditions as a function of the difference between the upstream and downstream capacities. The f term in the model is expressed as follows:

$$f = 1 \quad \text{when } (sg)_u >> (sg)_d, \tag{13}$$

$$0 < f < 1 \quad \text{when } (sg)_u > (sg)_d, \tag{14}$$

$$f = 0 \quad \text{when } (sg)_u < (sg)_d, \tag{15}$$

where $(sg)_u$ is the upstream capacity in vehicles per cycle and $(sg)_d$ is the downstream capacity in vehicles per cycle.

After calibration, the final expression of the delay parameter k in the stochastic steady state form is given in Equation 16.

$$k = 0.408\left\{1 - e^{-0.5\left[(sg)_u - (sg)_d\right]}\right\}. \tag{16}$$

The calibrated delay parameter k is true for $(sg)_u > (sg)_d$ and $x > (sg)_d / 100$. When these conditions are not met, it becomes zero.

Daniel *et al.* (1996) presented a model given in Equation 17 using an empirical approach for the three signal controller types (pre-timed, semi- actuated and fully- actuated) to express the delay parameter k at signalized intersections. Daniel *et al.* (1996) calibrated the delay parameter k by substituting the measured overflow delay in the time-dependent model and solving for k. As seen in Equation 17, the model was expressed as a function of degree of saturation and it is only valid for the values of degree of saturation from 0.5 to 1.0.

$$k = e^{\beta_0} x^{\beta_1} \tag{17}$$

in which β_0 and β_1 are regression coefficients, and x is degree of saturation.

Akgungor and Bullen (2007) performed another study utilizing empirical approach and simulation technique for pre-timed signalized intersections. The following model in Equation 18 provides an alternative form for the delay parameter k which is applicable to all degrees of saturation and for variable demand conditions.

$$k = 0.8x^2 - 1.4x + 1.1. \tag{18}$$

Kimber and Daly (1986) proved that the ratio between standard deviation and the mean of the arrival headway is not always 1.0, and it changes between 0.75 and 1.25 according to different traffic and time conditions. Therefore, it is not suitable to use a fixed value for k in the delay estimation models while traffic and time conditions vary during course of the day. Whereas, for pre-timed signalized intersections, Australian, Canadian and HCM 2000 delay models use a fixed value for the delay parameter k. The last two models take a fixed value of 0.5 for k based on a queuing model with random arrivals and uniform departures. The Australian model also uses a constant value of 1.5 instead of 0.5 for k. However, large k value in this model is compensated by an additional parameter x_0 called the degree of saturation below which the overflow delay is zero.

2. Research approach and methodology

The previous studies have analyzed the relationship between the delay parameter k and some factors affecting delay such as, capacity and degree of saturation. Besides these factors, arrival and service characteristics and amount of delay at a signalized intersection vary depending on analysis time period T. Therefore, this study is performed to investigate the relation between the delay parameter k and analysis time period T.

If there is no information about queuing characteristics of a signalized intersection, empirical approach is used to estimate the delay parameter k. In this approach, random delay is drawn from total delay which is obtained from simulation or observed in the field. Later, k values are calculated by using random delays and the delay model is developed by means of obtained results. The main principle of the method used in this study is to analyze different compounds of total delay (uniform, random overflow and continuous overflow delays). For unsaturated traffic flow conditions, Equation 2 is used to determine uniform delay and Equation 5 is utilized to determine overflow delay. While continuous overflow delay only takes place where flow is greater than capacity, random overflow delay can occur in all types of saturation conditions. Therefore, random overflow delay is a key term for the modeling of k parameter. Equations 1 and 4, along with Fig. 2, show mathematical and graphical meanings of delay terms.

In undersaturated traffic conditions, where flow is less than capacity, continuous overflow delay is zero. In this case, delay obtained from simulations is the sum of uniform and random overflow delays. In order to obtain

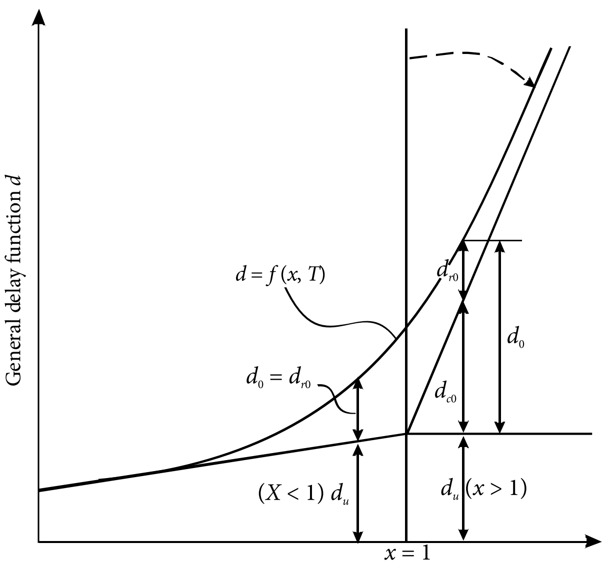

Fig. 2. A general delay function for delay components (Akcelik 1980)

the random overflow delay, the simulated delay should always be greater than the uniform delay. If uniform delay is less than the simulated delay, the random overflow delay is considered to be zero.

In oversaturated traffic conditions, the random overflow delay is estimated by taking the difference between the simulated delay and the sum of uniform and continuous overflow delays. For both traffic flow conditions, k values can be calculated through substitution of simulated random overflow values into time-dependent delay model.

3. Experimental design and model development

As mentioned earlier, a number of studies on the estimation of delay parameter k were presented in the literature. The relationship between k values and other parameters affecting delay, such as capacity and degree of saturation, was investigated. The analysis period T is an essential parameter to determine the level of delay at signalized intersections, especially during oversaturated traffic conditions. Therefore, in this study, k was modeled as a function of T to better describe variable traffic conditions in delay estimations.

The simulated intersection in this study consisted of one lane for every approach. The link length of the intersection was set to 3 000 ft (914 m) for each approach. The intersection operated under two phases with a cycle length of 90 seconds. Green time on the major and minor approaches was 45 and 35 seconds, respectively. Yellow and all red intervals for all approaches at the intersection were 3 and 2 seconds, respectively. Saturation flow rate was 1800 vphpl with mean discharge headway of 2 seconds per vehicle. A start-up lost time of 2 seconds and a free flow speed of 30 mph (~48 km/h) were used in the simulation runs. The entry link volumes were 630 vph for major approaches and 490 vph for minor approaches.

In the experiments, the analysis time period varied from 15 minutes to 1 hour and 15 runs were performed for each analysis time period. In each simulation run, a different random seed number was used to account for the variability in driver and vehicle characteristics. However, they were kept constant between runs to have identical traffic movements while different analysis time periods were being compared. By utilizing the methodology explained above, k values were computed for each analysis period and following that the delay parameter k is modeled using the best-fitted curve. The developed model is expressed in Equation 19. As shown in Table 1, the delay parameter k varied from 0.6159 to 0.6923 while analysis period ranged between 0.25 and 1 hour.

$$k = 0.6923T^{0.0844}. \quad (19)$$

4. Comparison of existing models with the proposed model

The data obtained from simulations for k values were analyzed statistically to evaluate the reliability of the developed model. The summary of the statistical analysis belonged to k values of proposed model is given in Table 2. The performance of the delay model proposed in

Table 1. Analysis periods *T* (in hours) and developed delay parameter *k* values

Analysis period *T* (in hours)	*k* values
0.25	0.6159
0.30	0.6254
0.35	0.6336
0.40	0.6408
0.45	0.6472
0.50	0.6530
0.55	0.6582
0.60	0.6631
0.65	0.6676
0.70	0.6718
0.75	0.6757
0.80	0.6794
0.85	0.6829
0.90	0.6862
0.95	0.6893
1.00	0.6923

Table 2. Summary statistics for *k* values

Analysis period *T* (in hour)	Mean	Range	Standard error	Standard deviation	Confidence interval for % 95	Tolerated error (%)
0.25	0.609	1.524	0.037	0.292	0.535–0.683	12.1
0.50	0.667	0.962	0.029	0.225	0.609–0.723	8.5
0.75	0.676	0.852	0.026	0.204	0.624–0.728	7.7
1.00	0.685	0.773	0.024	0.189	0.636–0.732	7.0

this research was also evaluated by comparing with the Australian, Canadian, HCM 2000 delay models as well as the deterministic delay model defined by Equation 6. Here, only the overflow delays were considered since the above delay models have a similar expression for the uniform delay. The performance study was accomplished for traffic conditions with a cycle length of 90 seconds, an effective green time of 30 seconds and a saturation rate of 1500 vph giving a capacity of 500 vph.

In the performance and validation studies of the proposed model to existing delay models, the progress adjustment factor (*PF*), the incremental delay calibration factor (*k*) and the upstream filtering adjustment factor (*I*) in the HCM 2000 delay model were taken as 1.0, 0.5 and 1.0, respectively. In addition, d_3 term, initial queue delay, in HCM 2000 delay model was assumed as zero. Thus, it was presumed that there was no queue at the beginning of analysis period. The degrees of saturation were selected from 0.1 to 2.0, so that arrival flows changed from 50 to 1000 vph. Four periods starting from 15 minutes to 1 hour by 15-minute increments were considered for analyses. A comparison of overflow delay estimates for a 15-minute analysis period is shown in Table 3.

From Table 3, as compared with other models, the proposed model slightly overestimates overflow delays for undersaturated conditions. The Australian delay model gives lower values and predicts zero overflow delay at x values below 0.7 because of x_0. When the degree of saturation x is at around 1.0, all the model estimations of overflow delay are around 40 seconds. On the other hand, for oversaturated traffic conditions, the overflow delay estimated by the proposed model is less than that of the Australian Model but over than that of the Cana-

Table 3. Comparison of estimated overflow delays produced by proposed and existing delay models (analysis time period T = 0.25h)

Degree of saturation	Proposed delay model estimates	Australian delay model estimates	Relative errors (%)	Canadian and HCM 2000 delay model estimates	Relative errors (%)	Deterministic delay model estimates	Relative errors (%)
0.1	0.49	–	–	0.40	18.37	–	–
0.2	1.11	–	–	0.90	18.92	–	–
0.3	1.89	–	–	1.54	18.52	–	–
0.4	2.92	–	–	2.38	18.49	–	–
0.5	4.35	–	–	3.54	18.62	–	–
0.6	6.42	–	–	5.25	18.22	–	–
0.7	9.66	0.32	96.69	7.93	17.91	–	–
0.8	15.18	5.54	63.50	12.63	16.80	–	–
0.9	25.48	16.51	35.20	21.82	14.36	–	–
1.0	44.67	38.75	13.25	40.25	9.89	–	–
1.1	74.47	72.44	2.73	70.34	5.55	45.00	39.57
1.2	111.48	112.07	–0.53	108.00	3.12	90.00	19.27
1.3	152.06	154.19	–1.40	149.12	1.93	135.00	11.22
1.4	194.37	197.45	–1.58	191.82	1.31	180.00	7.39
1.5	237.60	241.29	–1.55	235.33	0.96	225.00	5.30
1.6	281.35	285.48	–1.47	279.28	0.74	270.00	4.03
1.7	325.42	329.87	–1.37	323.51	0.59	315.00	3.20
1.8	369.91	374.40	–1.21	367.93	0.54	360.00	2.68
1.9	414.15	419.02	–1.18	412.46	0.41	405.00	2.21
2.0	458.70	463.72	–1.09	457.09	0.35	450.00	1.90

dian and HCM 2000 delay models. Similar comparison analyses were carried out for 30, 45 and 60 minutes of analysis periods to better describe the performance of the proposed delay model. For these analysis periods all of the delay models have almost the same trend similar to the one for the 15-minute analysis period.

As the previous studies related to the delay parameter *k* have shown that *k* has varied depending on traffic conditions, this study has presented that the delay parameter *k* changes depending on time conditions. Therefore, instead of using a constant value for delay parameter *k* in delay studies, the use of varying *k* values in these studies seems to be more reasonable.

5. Conclusion

Although delay estimation at signalized intersections is dependent on many parameters, analysis period (*T*) is one of the most effective ones. In this paper, the effect of the delay parameter *k*, on delay when it is time-dependent rather than a constant value has been investigated. The studies in the literature have shown that the delay per vehicle changes with respect to time periods in addition to the traffic conditions. Especially, in overflow conditions when the flow is greater than the capacity, the delay time increases in proportion to time periods and the delay parameter *k* dominates. In the developed model, at signalized intersections, especially, for oversaturated traffic conditions, the delay estimations appear to be more meaningful and realistic when the time variable *k* values are used.

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