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A New Method for Ranking of Fuzzy Decision Making Units by FPR/DEA Method

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Abstract

The goal of multi-attribute decision-making problems is the ranking of units. In real world, we usually deal with in inexact data. So, there are various ways for ranking of decision making units in fuzzy situation. In this article, a new way of ranking of fuzzy decision making units has been proposed. This method involves three stages. First, pairwise efficiency scores are computed using two data envelopment analysis (DEA) models. In the second stage, the pairwise efficiency scores are then utilized to construct the fuzzy preference relation. In the last stage, by use of row wise summation technique, fuzzy units are ranked. Finally, this proposed method has been used for decision making units in eight manufacturing enterprizes in China. Then, it was compared with Young's ranking.

Keywords: data envelopment analysis, multi-attribute decision-making, fuzzy preference relation

1 Introduction

In a multi-attribute decision-making problem, the aim is to find the best unit (attribute) among all units. One way is using decision makers views and constructing preference relations by which units are ranked. There exists four types of preference relation: Fuzzy preference relation[1], Multiplicative preference relation[3,4], Interval-valued preference relation[5], Linguistic preference relation[8]. Prevalent techniques used to construct preference relation are founded on subjective assessment, requiring much involvement of expert knowledge and time. An objective technique can highly decrease the cost in incurred by the involvement of expert knowledge and time in the evaluation process. DEA provides a technique for objective evaluation. By using CCR

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and revised cross-rated [2] models pairwise efficiency for units is computed. This proposed method for ranking inexact units is the fuzzy combination of FPR/DEA. In the Section 2, the FPR/DEA method has been summarized. This method is expanded for inexact situation in Section 3. In Section 4, an example has been solved using this method and conclusion is presented in Section 4.

2 The FPR/DEA method

One of the methods for ranking decision making units is using FPR/DEA method proposed by Wu. This method consists of three levels. In the first level, using DEA models, both CCR and revised cross-rated models the pairwise efficiencies are computed. In the second level, the pairwise comparison fuzzy preference relation(matrix) is constructed. Finally, by using row wise summation technique, units ranking is done. In this paper, we are going to apply this technique to fuzzy data[7].

3 The fuzzy FPR/DEA method

3.1 Fuzzy data envelopment analysis models

Suppose the inputs and outputs of the decision making units are L-R fuzzy numbers. CCR and revised cross-rated models are as follows:
CCR model:

$$\begin{array}{lll} Max & \sum_{r=1}^{s} \mu_{rd}\tilde{y}_{rd} & \\ s.t. & & \\ & \sum_{i=1}^{m} \omega_{id}\tilde{x}_{il} - \sum_{r=1}^{s} \mu_{rd}\tilde{y}_{rl} \geq 0 & l = 1, ..., n \\ & \sum_{i=1}^{m} \omega_{id}\tilde{x}_{id} = \tilde{1} & \\ & \omega_{id} \geq 0 & i = 1, ..., m \\ & \mu_{rd} \geq 0 & r = 1, ..., s \end{array} \qquad (1)$$

Revised cross-rated model:

$$
\begin{array}{lll}
Max & \sum\limits_{l\neq d}\sum\limits_{r=1}^{s} u_{rd}\tilde{y}_{rd} & \\
s.t. & & \\
 & \sum\limits_{i=1}^{m}\nu_{id}\tilde{x}_{il}-\sum\limits_{r=1}^{s}\mu_{rd}\tilde{y}_{rl}\geq 0 & l=1,...,n \\
 & \sum\limits_{l\neq d}\sum\limits_{i=1}^{m}\omega_{id}\tilde{x}_{id}=\tilde{1} & \\
 & \alpha_d\times\sum\limits_{i=1}^{m}\nu_{id}\tilde{x}_{il}-\sum\limits_{r=1}^{s}\mu_{rd}\tilde{y}_{rl}\geq 0 & l=1,...,n \\
 & \omega_{id}\geq 0 & i=1,...,m \\
 & \mu_{rd}\geq 0 & r=1,...,s
\end{array}
\tag{2}
$$

where $\tilde{x}_{id}=(x^l_{id},x^m_{id},x^u_{id})$, $\tilde{y}_{rd}=(y^l_{rd},y^m_{rd},y^u_{rd})$ are an m-dimensional L-R fuzzy input vector, an s-dimensional L-R fuzzy output vector of the dth DMU and $\tilde{1}=(\tilde{1}-\varepsilon,\tilde{1},\tilde{1}+\varepsilon)$ is L-R fuzzy number and $0\leq\varepsilon<1$. The upper models are fuzzy. By using α-cut these models are converted to the following linear models:
CCR model:

$$
\begin{array}{lll}
Max & \sum\limits_{r=1}^{s}\mu_{rd}\bar{y}_{rd} & \\
s.t. & & \\
 & \sum\limits_{i=1}^{m}\omega_{id}\bar{x}_{il}-\sum\limits_{r=1}^{s}\mu_{rd}\bar{y}_{rl}\geq 0 & l=1,...,n \\
 & \sum\limits_{i=1}^{m}\omega_{id}\bar{x}_{id}=\bar{1} & \\
 & \omega_{id}\geq 0 & i=1,...,m \\
 & \mu_{rd}\geq 0 & r=1,...,s
\end{array}
\tag{3}
$$

In the way that:

$$
\begin{array}{l}
\bar{x}_{id}=(x^m_{id}-x^l_{id}L^{-1}(\alpha),x^m_{id}+x^u_{id}R^{-1}(\alpha)) \\
\bar{x}_{il}=(x^m_{il}-x^l_{il}L^{-1}(\alpha),x^m_{il}+x^u_{il}R^{-1}(\alpha)) \\
\bar{y}_{rd}=(y^m_{rd}-y^l_{rd}L^{-1}(\alpha),y^m_{rd}+y^u_{rd}R^{-1}(\alpha)) \\
\bar{y}_{rl}=(y^m_{rl}-y^l_{rl}L^{-1}(\alpha),y^m_{rl}+y^u_{rl}R^{-1}(\alpha))
\end{array}
\tag{4}
$$

Revised cross-rated model:

$$
\begin{array}{lll}
Max & \sum\limits_{l\neq d}\sum\limits_{r=1}^{s} u_{rd}\bar{y}_{rd} & \\
s.t. & & \\
 & \sum\limits_{i=1}^{m}\nu_{id}\bar{x}_{il}-\sum\limits_{r=1}^{s}u_{rd}\bar{y}_{rl}\geq 0 & l=1,...,n \\
 & \sum\limits_{l\neq d}\sum\limits_{i=1}^{m}\nu_{id}\bar{x}_{id}=\bar{1} & \\
 & \alpha_d\times\sum\limits_{i=1}^{m}\nu_{id}\bar{x}_{il}-\sum\limits_{r=1}^{s}u_{rd}\bar{y}_{rl}\geq 0 & l=1,...,n \\
 & \nu_{id}\geq 0 & i=1,...,m \\
 & \mu_{rd}\geq 0 & r=1,...,s
\end{array}
\tag{5}
$$

In the way that:

$$\begin{array}{l}\bar{x}_{id} = (x_{id}^m - x_{id}^l L^{-1}(\alpha), x_{id}^m + x_{id}^u R^{-1}(\alpha)) \\ \bar{x}_{il} = (x_{il}^m - x_{il}^l L^{-1}(\alpha), x_{il}^m + x_{il}^u R^{-1}(\alpha)) \\ \bar{y}_{rd} = (y_{rd}^m - y_{rd}^l L^{-1}(\alpha), y_{rd}^m + y_{rd}^u R^{-1}(\alpha)) \\ \bar{y}_{rl} = (y_{rl}^m - y_{rl}^l L^{-1}(\alpha), y_{rl}^m + y_{rl}^u R^{-1}(\alpha)) \\ \alpha_d = [\alpha_d^l, \alpha_d^u]\end{array} \tag{6}$$

The resulted CCR model is an interval model and concerning the pessimistic and optimistic status, it is converted to the following two models.
Optimistic CCR model:

$$\begin{array}{lll} Max & \sum_{r=1}^{s} \mu_{rd} y_{rd}^U & \\ s.t. & & \\ & \sum_{i=1}^{m} \omega_{id} x_{il}^U - \sum_{r=1}^{s} \mu_{rd} y_{rl}^L \geq 0 & l = 1, ..., n \\ & \sum_{i=1}^{m} \omega_{id} x_{id}^L = 1^L & \\ & \omega_{id} \geq 0 & i = 1, ..., m \\ & \mu_{rd} \geq 0 & r = 1, ..., s \end{array} \tag{7}$$

Pessimistic CCR model:

$$\begin{array}{lll} Max & \sum_{r=1}^{s} \mu_{rd} y_{rd}^L & \\ s.t. & & \\ & \sum_{i=1}^{m} \omega_{id} x_{il}^L - \sum_{r=1}^{s} \mu_{rd} y_{rl}^U \geq 0 & l = 1, ..., n \\ & \sum_{i=1}^{m} \omega_{id} x_{id}^U = 1 & \\ & \omega_{id} \geq 0 & i = 1, ..., m \\ & \mu_{rd} \geq 0 & r = 1, ..., s \end{array} \tag{8}$$

For revised cross-rated model ,we take a similar way of CCR model. The optimistic revised cross rated model:

$$\begin{array}{lll} Max & \sum_{l \neq d} \sum_{r=1}^{s} u_{rd} \mathbf{y}_{rd}^U & \\ s.t. & & \\ & \sum_{i=1}^{m} \nu_{id} \mathbf{x}_{il}^U - \sum_{r=1}^{s} u_{rd} \mathbf{y}_{rl}^L \geq 0 & l = 1, ..., n \\ & \sum_{l \neq d} \sum_{i=1}^{m} \nu_{id} \mathbf{x}_{id}^L = 1^L & \\ & \alpha_d^U \times \sum_{i=1}^{m} \nu_{id} \mathbf{x}_{il}^U - \sum_{r=1}^{s} u_{rd} \mathbf{y}_{rl}^L \geq 0 & l = 1, ..., n \\ & \nu_{id} \geq 0 & i = 1, ..., m \\ & \mu_{rd} \geq 0 & r = 1, ..., s \end{array} \tag{9}$$

The pessimistic revised cross-rated model:

$$\begin{array}{lll}
Max & \sum_{l\neq d}\sum_{r=1}^{s} u_{rd}\mathbf{y}_{rd}^{L} & \\
s.t. & & \\
& \sum_{i=1}^{m} \nu_{id}\mathbf{x}_{il}^{L} - \sum_{r=1}^{s} u_{rd}\mathbf{y}_{rl}^{U} \geq 0 & l=1,...,n \\
& \sum_{l\neq d}\sum_{i=1}^{m} \nu_{id}\mathbf{x}_{id}^{U} = 1 & \\
& \alpha_d^L \times \sum_{i=1}^{m} \nu_{id}\mathbf{x}_{il}^{L} - \sum_{r=1}^{s} u_{rd}\mathbf{y}_{rl}^{U} \geq 0 & l=1,...,n \\
& \nu_{id} \geq 0 & i=1,...,m \\
& \mu_{rd} \geq 0 & r=1,...,s
\end{array} \tag{10}$$

By solving both of the models $(u_d^*, \nu_d^*)^U$, $(u_d^*, \nu_d^*)^L$ are obtained respectively.

3.2 Instruction of preference relations

For inexact inputs and outputs, with solving (7)(8)(9)(10) models, interval solutions are resulted. by using these solutions, we have:

$$\begin{array}{lll}
E_{dj}^{L} = \frac{u_d^{*L^T} y_j^L}{\nu_d^{*U^T} x_j^U} & d\neq j & d,j=1,...,n \\
E_{dj}^{U} = \frac{u_d^{*U^T} y_j^U}{\nu_d^{*L^T} x_j^L} & d\neq j & d,j=1,...,n
\end{array} \tag{11}$$

Therefore, the elements of pairwise comparison preference relation are computed like this:

$$\begin{array}{l}
r_{dj}^{L} = \frac{E_{dd}^{L}+E_{dj}^{L}}{E_{dd}^{U}+E_{jd}^{U}+E_{jj}^{U}+E_{dj}^{U}} \\
r_{dj}^{U} = \frac{E_{dd}^{U}+E_{dj}^{U}}{E_{dd}^{L}+E_{jd}^{L}+E_{jj}^{L}+E_{dj}^{L}}
\end{array} \tag{12}$$

so, the preference matrix will be an interval preference matrix. In that we have:

$$R^L = (r_{dj}^L)_{n\times n} \qquad R^U = (r_{dj}^U)_{n\times n}$$

In fact, the elements of this matrix are interval. This matrix is presented in these two matrices. In the first matrix,down bound elements of the first matrix and in the second matrix, upper bound elements of the first matrix.

$$\begin{array}{lll}
c_d^L = \sum_{j=1}^{n} r_{dj}^L = \sum_{j=1}^{n} \frac{E_{dd}^{L}+E_{dj}^{L}}{E_{dd}^{U}+E_{jd}^{U}+E_{jj}^{U}+E_{dj}^{U}} & & d=1,...,n \\
c_d^U = \sum_{j=1}^{n} r_{dj}^U = \sum_{j=1}^{n} \frac{E_{dd}^{U}+E_{dj}^{U}}{E_{dd}^{L}+E_{jd}^{L}+E_{jj}^{L}+E_{dj}^{L}} & & d=1,...,n \\
b_{dj}^L = \frac{c_d^L+c_j^L}{2(n-1)} + 0.5 & b_{dj}^U = \frac{c_d^U+c_j^U}{2(n-1)} + 0.5 & d,j=1,...,n
\end{array} \tag{13}$$

So, two pairwise comparison fuzzy preference matrixes are obtained. Now, using the following relations, fuzzy preference relations are converted to consistency fuzzy preference relations:

$$\begin{aligned} w_d^L &= \frac{\sum\limits_j b_{dj}^L}{\sum\limits_d \sum\limits_j b_{dj}^U} = \frac{\sum\limits_j b_{dj}^L + \frac{n}{2} - 1}{n(n-1)} \\ w_d^U &= \frac{\sum\limits_j b_{dj}^U}{\sum\limits_d \sum\limits_j b_{dj}^L} = \frac{\sum\limits_j b_{dj}^U + \frac{n}{2} - 1}{n(n-1)} \end{aligned} \tag{14}$$

Finally, two consistency summation matrixes are resulted. With the contribution of row wise summation techniques, we can calculate the weights vector.

$$w_d = [w_d^L, w_d^U]$$

So, weights vector is obtained in interval form.

Definition 3.1 *When $w_i = [w_i^L, w_i^U]$, $w_j = [w_j^L, w_j^U]$ are two interval numbers, the comparison function $F(\Re) \to R$ that $F(\Re)$ is the total of all interval numbers will be considered as follows:*

$$\begin{array}{lcl} F_1(\Re) \to R & \quad or \quad & F_2(\Re) \to R \\ F_1([w^L, w^U]) \to w^L \times w^U & & F_2([w^L, w^U]) \to w^L + w^U \end{array} \tag{15}$$

If $F_k(w_j) < F_k(w_i)$ then $w_j \prec w_i$, k=1,2.
If $F_k(w_j) = F_k(w_i)$ then $w_j \simeq w_i$, k=1,2.
In correspondence with the definition, if $F_k(w_j) < F_k(w_i)$ then w_i number is better than w_j number.

We consider these models for different α cuts and for each α cut we use the same method.

4 Example

Consider a performance assessment problem in China where eight manufacturing enterprises (DMUs) are to be evaluated in terms of two inputs and two outputs. The eight manufacturing enterprises all manufacturing the same type of product but with different qualities[8].

Enterprises (DMUs)	Input MC	NOE	Output GOV	PQ
1	(2120,2170,2210)	1870	(14500,14790,14860)	(3.1,4.1,4.9)
2	(1420,1460,1500)	1340	(12470,12720,12790)	(1.2,2.1,3)
3	(2510,2570,2610)	2360	(17900,18260,18400)	(3.3,4.3,5)
4	(2300,2350,2400)	2020	(14970,15270,15400)	(2.7,3.7,4.6)
5	(1480,1520,1560)	1550	(13980,14260,14330)	(1,1.8,2.7)
6	(1990,2030,2100)	1760	(14030,14310,14400)	(1.6,2.6,3.6)
7	(2200,2260,2300)	1980	(16540,16870,17000)	(2.4,3.4,4.4)
8	(2400,2460,2520)	2250	(17600,17960,18100)	(2.6,3.6,4.6)

Table1 : input and output data for eight manufacturing enterprises

By solving this method we will have follow results:

(DMUs)	Wang's ranking 1	Wang's ranking 2	Fuzzy DEA/FPR ranking F_1	Fuzzy DEA/FPR ranking F_2
DMU_1	3	3	4	4
DMU_2	1	1	1	1
DMU_3	5	7	6	6
DMU_4	7	8	8	8
DMU_5	2	2	2	2
DMU_6	8	5	7	7
DMU_7	4	4	3	3
DMU_8	6	6	5	5

Table1 : Results of proposed model

At first, we convert the suggested method from fuzzy to interval and then with $\alpha = 0.5$ cut exactly solve it.
Ranking results with suggested method and two proposed Wang methods are presented in the following table.
It is clear that the obtained results are not the same in these three methods

but in all of them, second, fourth and fifth units have the ranking of 1,8,2. In Wang's two methods, the third rank is given to the first unit and the fourth rank is given to the seventh while the suggested method for the first unit was the fourth rank and third rank for the seventh unit. Both of the F_1, F_2 functions have the same result.

5 Conclusion

In this article, a method for ranking inexact decision making units (interval and fuzzy) is proposed. One of the advantages of this method is the absolute ranking of units. In fact, using this method enables us to specify the most efficient unit, uniquely.

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