A Stochastic Model for the Expected Time to Recruitment in a Two Grade Manpower System Having Correlated Inter-Decision Times and Constant Combined Thresholds

A. Srinivasan and S. Vidhya

Department of Mathematics
Bishop Heber College, Tiruchirappalli-17, India
mathsrinivas@yahoo.com,   s.s.vidhya@gmail.com

Abstract

In this paper, an organization having two grades in which depletion of manpower occurs at every decision epoch is considered. Using univariate max policy of recruitment, based on shock model approach mathematical model is constructed. It is assumed that the system has constant combined threshold and the inter-decision times are exchangeable and constantly correlated exponential random variable. The mean and variance of the time to recruitment are derived for the above model. The analytical results are substantiated with numerical illustrations.

1. Introduction

Consider an organization having two grades in which the depletion of manpower occurs at every decision epoch. Instead of recruiting people after each occasion of manpower loss, the organization is sustained with the maximum loss of manpower and as and when the maximum loss crosses a constant combined threshold level, it signals the organization to go for recruitment. In [4], [9] and [10] the authors have obtained the mean time to recruitment in a two graded organization using univariate cum policy of recruitment. In [7] and

[8] the authors have obtained the mean time to recruitment using univariate max policy of recruitment. In all the above cited research works, the threshold levels are not combined and hence only one of the two thresholds can be crossed at a time but not both since transfer of persons are permitted. In [1], [6] and [11] the authors have obtained the mean and variance of the time to recruitment for a two graded manpower system when the thresholds for the loss of manpower in the two grades are combined together. For the combined threshold model, the authors in [1] have obtained the mean and variance of the time to recruitment using univariate cum policy of recruitment when i) the loss of manpower and the thresholds for the two grades are discrete random variables and ii) the inter-decision times for the two grades are i.i.d random variables forming the same renewal process.In [2] the auther have obtained the distribution of the maximum of arithmetic mean for correlated random variables and in [5], the mean time to recruitment for correlated renewal sequence is obtained. The objective of the present paper is to obtain the mean and variance of the time to recruitment using univariate max policy of recruitment when inter-decision times are correlated.

2. Model description

Consider an organization with two grades A and B taking decisions at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3,..., let X_i^A and X_i^B be the number of exits due to the ith decision in grades A and B respectively. Let $\{X_i^A\}$ and $\{X_i^B\}$ be a sequences of independent and identically distributed random variables with respective distributions $\phi_A(.)$ and $\phi_B(.)$. Write $Z_k^A = \sum_{i=1}^{k} X_i^A$ and $Z_k^B = \sum_{i=1}^{k} X_i^B$ for the cumulative number of exits in grades A and B in the first k decisions respectively. It is assumed that the inter-decision times for the two grades are exchangeable and constantly correlated exponential random variable with density function $f(x) = \lambda e^{-\lambda x}$. Let $f^*(.)$ be the Laplace transform of $f(.)$. Let $c > 0$ be a constant combined threshold level for the two grades.Let R be the correlation between any X_i and $X_j, i \neq j$ and $\psi(n,x) = \int_0^x e^{-\tau}\tau^{n-1}d\tau$. Let $b = a(1 - R)$ where a is a mean of X_i,i=1,2,... and let $m = 1/(1 + bs)$. Let T be a continuous random variable denoting the time to recruitment in the organization with probability density function $l(.)$ and distribution function $L(.)$. The recruitment policy employed in this paper is as follows. *Recruitment is done whenever the maximum of the maximum total number of exits due to the policy decisions in grades A and B crosses the combined constant threshold level c.* Let $V_k(t)$ be the probability that there are exactly k-decision epochs in $(0,t]$ .Since the number of decisions made in $(0,t]$ form a renewal process, from [2] we note that $V_k(t) = F_k(t) - F_{k+1}(t)$ where $F_0(t) = 1$. Let $E(T)$ and

$V(T)$ be the mean and variance of the time for recruitment respectively.

3. Main Results

The survior function of T is given by

$$P(T > t) = \sum_{k=0}^{\infty} \{\text{(Probability that exactly k decision epochs in (0,t] and the combined threshold level is not crossed)}\}$$

$$= \sum_{k=0}^{\infty} V_k(t) P(Z_k^A < c) P(Z_k^B < c))$$

$$= 1 - (1 - \phi_A(c)\phi_B(c)) \sum_{k=1}^{\infty} F_k(t)(\phi_A(c)\phi_B(c))^{k-1}$$

Therefore on simplification it can be shown that

$$L(t) = (1 - \phi_A(c)\phi_B(c)) \sum_{k=1}^{\infty} F_k(t)(\phi_A(c)\phi_B(c))^{k-1} \tag{1}$$

Taking Laplace-Stieltjes transform on both sides

$$L^*(s) = (1 - \phi_A(c)\phi_B(c)) \sum_{k=1}^{\infty} F_k^*(s)(\phi_A(c)\phi_B(c))^{k-1} \tag{2}$$

As in Gurland [4],

$$F_k(x) = (1 - R) \sum_{i=0}^{\infty} \left[\frac{(kR)^i}{(1 - R + kR)^{i+1}} \right] \left[\frac{\psi(k+1, x/b)}{(k + i - 1)!} \right] \tag{3}$$

From (3) it can be shown that

$$F_k^*(s) = \frac{m^k}{1 + \frac{kR(1-m)}{1-R}}$$

and

$$\left[\frac{d}{ds} F_k^*(s) \right]_{s=0} = -ak, \quad \left[\frac{d^2}{ds^2} F_k^*(s) \right]_{s=0} = a^2((1 + R^2)k^2 + (1 - R^2)k) \tag{4}$$

We know that

$$E(T^n) = \left[\frac{-d^n}{ds^n} L^*(s) \right]_{s=0}, r = 1, 2 \tag{5}$$

$$V(T) = E(T^2) - (E(T))^2 \tag{6}$$

Using (4) in (5)and (6) and on simplification it can be shown that

$$E(T) = \frac{a}{1 - \phi_A(c)\phi_B(c)} \tag{7}$$

$$V(T) = \frac{a^2}{(1 - \phi_A(c)\phi_B(c))^2} \tag{8}$$

(7) and (8) give the mean and variance of the time to recruitment for the present model.

4.Special Case
Case(i): Assume that X_i^A and X_i^B follow Poisson distribution with parameter α_1 and α_2 respectively.

$$Then \quad \phi_A(c) = \sum_{k=0}^{c} \frac{e^{-\alpha_1}(\alpha_1)^k}{k!} \quad and \phi_B(c) = \sum_{k=0}^{c} \frac{e^{-\alpha_2}(\alpha_2)^k}{k!} \tag{9}$$

Using (9) in (7) and (8)

$$E(T) = \frac{a}{1 - \sum_{k=0}^{c} \frac{e^{-\alpha_1}(\alpha_1)^k}{k!} \sum_{k=0}^{c} \frac{e^{-\alpha_2}(\alpha_2)^k}{k!}} = \frac{b}{1 - R} \left[\frac{1}{1 - \sum_{k=0}^{c} \frac{e^{-\alpha_1}(\alpha_1)^k}{k!} \sum_{k=0}^{c} \frac{e^{-\alpha_2}(\alpha_2)^k}{k!}} \right] \tag{10}$$

and

$$V(T) = (\frac{b}{1 - R})^2 \left[\frac{1}{1 - \sum_{k=0}^{c} \frac{e^{-\alpha_1}(\alpha_1)^k}{k!} \sum_{k=0}^{c} \frac{e^{-\alpha_2}(\alpha_2)^k}{k!}} \right]^2 \tag{11}$$

(10)and (11) give the mean and variance of the time to recruitment for case(i).
case(ii): Assume that X_i^A and X_i^B follow geometric distribution with parameter β_1 and β_2 respectively.

$$Then \quad \phi_A(c) = \sum_{k=1}^{c} \beta_1(1 - \beta_1)^{k-1} \quad and \phi_B(c) = \sum_{k=1}^{c} \beta_2(1 - \beta_2)^{k-1} \tag{12}$$

Using (12) in (7) and (8)

$$E(T) = \frac{b}{1 - R} \left[\frac{1}{(1 - \beta_1)^c + (1 - \beta_2)^c - (1 - \beta_1)^c(1 - \beta_2)^c} \right] \tag{13}$$

and

$$V(T) = (\frac{b}{1-R})^2 \left[\frac{1}{(1-\beta_1)^c + (1-\beta_2)^c - (1-\beta_1)^c(1-\beta_2)^c}\right]^2 \quad (14)$$

(13)and (14) give the mean and variance of the time to recruitment for case(ii).
case(iii): Assume that X_i^A and X_i^B follow exponential distribution with parameter λ and μ respectively.

$$Then \quad \phi_A(c) = 1 - e^{-\lambda c} \quad and \phi_B(c) = 1 - e^{-\mu c} \quad (15)$$

Using (15) in (7) and (8)

$$E(T) = \frac{b}{1-R} \left[\frac{1}{e^{-\lambda c} + e^{-\mu c} - e^{(-(\lambda+\mu)c)}}\right] \quad (16)$$

and

$$V(T) = (\frac{b}{1-R})^2 \left[\frac{1}{e^{-\lambda c} + e^{-\mu c} - e^{(-(\lambda+\mu)c)}}\right]^2 \quad (17)$$

(16)and (17) give the mean and variance of the time to recruitment for case(iii).

5.Numerical Illustrations and Conclutions

The influence of parameters on the performance measures namely the mean and variance of the time to recruitment are studied numerically.

In the following table these performance measures are calculated by varying the parameters R,α_1 and α_2 one at a time and keeping the parameters c and b fixed.

Table1: Effect of R, α_1 and α_2 on performance measures

(c=90,b=0.7)

R	α_1	α_2	$E(W)$	$V(W)$
-0.4	0.2	1	0.5647	0.3189
-0.2	0.2	1	0.6588	0.4340
0	0.2	1	0.7906	0.6250
0.2	0.2	1	0.9882	0.9766
0.4	0.2	1	1.3176	1.7362
0.4	0.4	1	1.4738	2.1721
0.4	0.6	1	1.6321	2.6639
0.4	0.8	1	1.7896	3.2027
0.4	1	1	1.9427	3.7752
0.4	1	1.2	2.0897	4.3669
0.4	1	1.4	2.2274	4.9613
0.4	1	1.6	2.3544	5.5433
0.4	1	1.8	2.4697	6.0995
0.4	1	2	2.5730	6.6204

From the numerical values computed, the following observations are made:

1. As R increases, the correlation between the inter-decision times increases and hence the mean and variance of the time to recruitment increases.

2. As α_1 increases, the mean numbers of exits in grade A increases and hence the mean and variance of the time to recruitment increase.

3. As α_2 increases, the mean number of exits in grade B increases and hence the mean and variance of the time to recruitment increase.

In the following table the performance measures are calculated by varying the parameters R,β_1 and β_2 one at a time and keeping the parameters c and b fixed.

Table2: Effect of R, β_1 and β_2 on performance measures

(c=90,b=0.7)

R	β_1	β_2	$E(W)$	$V(W)$
-0.4	0.2	0.9	2.6355e+008	6.9461e+016
-0.2	0.2	0.9	3.0748e+008	9.4544e+016
0	0.2	0.9	3.6898e+008	1.3614e+017
0.2	0.2	0.9	4.6122e+008	2.1272e+017
0.4	0.2	0.9	6.1496e+008	3.7818e+017
0.4	0.4	0.9	1.0798e+020	1.1659e+040
0.4	0.6	0.9	7.6129e+035	5.7956e+071
0.4	0.8	0.9	9.4243e+062	8.8817e+125
0.4	0.8	0.7	1.3367e+047	1.7868e+094
0.4	0.8	0.5	1.4443e+027	2.0859e+054
0.4	0.8	0.3	1.0189e+014	1.0381e+028
0.4	0.8	0.1	1.5315e+004	2.3455e+008

From the numerical values computed, the following observations are made:

1. As R increases, the average inter-decision time increases and hence the mean and variance of the time to recruitment increases.

2. As β_1 increases, the mean loss of manpower in grade A decreases and hence the mean and variance of the time to recruitment increase.

3. As β_2 decreases, the mean loss of manpower in grade B increases and hence the mean and variance of the time to recruitment decrease.

In the following table the performance measures are calculated by varying the parameters λ and μ one at a time and keeping the parameters c and b fixed.

Table2: Effect of R, λ and μ on performance measures

(c=90,b=0.7)

R	λ	μ	$E(W)$	$V(W)$
-0.4	0.2	1	3.2830e+007	1.0778e+015
-0.2	0.2	1	3.8302e+007	1.4670e+015
0	0.2	1	4.5962e+007	2.1125e+015
0.2	0.2	1	5.7452e+007	3.3008e+015
0.4	0.2	1	5.0298e+015	2.5299e+031
0.4	0.4	1	3.3025e+023	1.0907e+047
0.4	0.6	1	2.1685e+031	4.7022e+062
0.4	0.8	1	7.1190e+038	5.0680e+077
0.4	1	0.2	7.6603e+007	5.8681e+015
0.4	1	0.3	6.2072e+011	3.8530e+023
0.4	1	0.4	5.0298e+015	2.5299e+031
0.4	1	0.5	4.0757e+019	1.6611e+039

From the above table, we observe the following:

1. As R increases, the correlation between the inter-decision times increases and hence the mean and variance of the time to recruitment increases.

2. As λ increases, the mean numbers of exits in grade A decreases and hence the mean and variance of the time to recruitment increase.

3. As μ increases, the mean number of exits in grade B decreases and hence the mean and variance of the time to recruitment increase.

References

[1] J.B.Esther Clara and A.Srinivasan ,*Mean and variance of the time to recruitment in a two graded manpower system*,Acta Ciencia Indica, Vol.XXXIV M, 2 [2008], 589-593.

[2] J.Gurnald, *Distribution of the maximum of Arithmetic Mean of Correlated Random variables*, Applied Mathematical Statistics , 26,[1955], 294-300.

[3] Karlin Samuel and Taylor,M.Hayward,*A first course in Stochastic Process*, Academic press New York, San Fransis Co, London. ,1981.

[4] K.Kasturri and A.Srinivasan,*Expected time to recruitment for correlated inter- decision time of exits when threshold distribution has SCBZ property*, Acta Ciencia Indica.Vol. XXXI M.,1[2005],277-283.

[5] P.Mariappan and A.Srinivasan, *Expected Time to Recruitment in Manpower planning associated with correlated renewal sequences - a shock model Approach*,Acta ciencia Indica,3[2001], 351-356.

[6] B.Mercy Alice, J.B.Esther Clara and A.Srinivasan, *Mean and variance of the time to recruitment in a two graded manpower system with different renewal process for the inter-decision times*, [2009],To appear in Acta Ciencia Indica.

[7] D.Selvambigai and A.Srinivasan, *Stochastic model for time to recruitment in a two graded organization-I*, Acta Ciencia Indica,Vol.XXXIV M,4[2008], 2041-2044.

[8] D.Selvambigai and A.Srinivasan, *Stochastic model for time to recruitment in a two graded organization-II*, Acta Ciencia Indica,vol.XXXIV M,4[2008],1913-1916.

[9] S.Senthamizhselvi and A.Srinivasan., *Expected time to recruitment in a two graded manpower system associated with correlated interdecisions times when threshold distribution has SCBZ property* , Pure and Applied Mathematika Sciences, Vol.LXVIII,1-2[2008] , 89-96.

[10] S.Senthamizhselvi.,S.Vidhya. and A.Srinivasan , *Expected time to recruitment in a two graded system when threshold distribution has SCBZ property*, Acta Ciencia Indica,Vol.XXXIII M,1[2007], 67-70.

[11] R.Suresh Kumar, G.Gopal. and R.Sathiyamoorthi, *Stochastic models for the expected time to recruitment in an organization with two grades*, International Journal of Management Systems,Vol.22,2[2006],147-164.