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Three-Dimensional Couette Flow in a Composite Channel Partially Filled with a Porous Medium

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Abstract

The couette flow of a viscous fluid in a channel, partially filled with a porous medium has been analyzed. The lower part of the channel is occupied by a porous medium, while the upper part is occupied by a clear fluid. The lower porous plate and the porous medium are fixed and a transverse sinusoidal injection velocity is applied at it, while the upper porous plate moves with a constant velocity and it is also subjected to a constant suction. The effects of the permeability of the porous medium, its thickness, effective viscosity and the injection parameter, on the flow are analyzed.

Keywords: Couette flow, composite channel, Brinkman porous medium, permeability, effective viscosity

1. Introduction

The viscous fluid flow through and across a porous medium is a subject of growing interest. This is because of many important engineering applications of porous media. Such applications can be found in soil mechanics, oil field operations, in transpiration cooling, lubrication of porous bearings and water purification etc., where fluid flow through porous medium plays a fundamental role. The study of viscous fluid flow and heat transfer in channels and ducts filled/partially filled with a porous medium is important because of its many important industrial applications. Analytical study was conducted of such flow problems in channels with a porous medium by several researchers, such as Neale and Nader (1974), Kuznetsov (1996, 2000), Al-Nimr and Khadrawi (2003), Chauhan and Kumar (2009), Chauhan and Agrawal (2010), Chauhan and Rastogi (2010) and many others.

The viscous effects are important in several problems encountered in engineering system. For example, the flow of a lubricant between fast moving parts is laminar, in view of the small dimensions of the parts and of the high viscosity of the lubricating fluid. In these cases large velocity changes occur over short distances and even at moderate velocities, friction becomes important. Such flows in the presence of porous boundaries or porous material in varies geometries have several applications. One of important viscous fluid flow situations in porous media is couette flow in parallel channel, in which one wall attached with a porous layer is stationary and the other is moving in its own plane with a constant velocity. Since in porous layer the viscous forces in the boundary layer near the stationary plate can be significant, to obtain a correct description of the flow, it is necessary to account for the viscous effects by taking Brinkman equation to model the flow in porous layer.

Investigations on couette flow in porous media are limited to the case of a Brinkman extended-Darcy porous medium. Bhargava and Sacheti (1989), Daskalakis (1990), Chauhan and Soni (1994) investigated such flows using Brinkman equation. Nakayama (1992) obtained analytical solutions for different situations of couette flow in a porous medium filled with an inelastic non-Newtonian fluid where the porous medium is described by the Brinkman-Forchheimer extension of the Darcy law. Couette flow of a compressible Newtonian fluid in the presence of a naturally permeable boundary is investigated by Chauhan and Shekhawat (1993), and Chauhan and Vyas (1995). One couette flow situation can occur, for example, when the parallel channel is completely filled with a porous material which is at rest, and there is no gap between the porous medium and the moving plate. However in such case large friction forces developed between the porous matrix and the moving plate can damage the geometry of the porous matrix in practical situations. Therefore it is more suitable if there is a gap, may be very small, between the moving plate and the porous medium. Thus study of flow in a composite channel partially filled with a porous material and partial with a clear fluid is important. However in this geometry, clear fluid is adjacent to porous medium, and so correct boundary conditions at the porous medium-clear fluid interface must be specified for better results. Boundary conditions at the fluid-porous interface are discussed in detail by Beavers and Joseph (1967), Saffman (1971), Kim and Russel (1985), Ochoa-Tapia and Whittaker (1995a, b), and James and Davis (1996).

Suction and injection at the plate also play a fundamental role in the plane couette flow. It remains two-dimensional if the suction and injection applied at the porous parallel plate are uniform, but by the application of the transverse sinusoidal injection at the stationary plate and constant suction at the moving plate, the flow remains three-dimensional as studied by Singh (1999). A similar problem of three dimensional couette flow of dusty viscous fluid was investigated by Govindarajan et al. (2007) with transpiration cooling. Such flow problems are important for studies of transpiration cooling process by investigating associated heat transfer

problems. Kuznetsov (1998) investigated two-dimensional couette flow in a composite channel partially filled with porous medium modeled by Brinkman-Forchheimer-Darcy equation, and associated heat transfer. In his study, he utilized the boundary conditions at the fluid-porous interface suggested by Ochoa-Tapia and Whittaker (1995a, b).
In the present study the three-dimensional couette flow in a composite channel, partially filled with a porous medium has been analyzed to the case of a Brinkman-Darcy porous medium with effective medium considerations.

2. Formulation of the Problem

Steady viscous flow in a composite channel bounded by two infinite parallel porous plates is considered. The lower part of the channel is occupied by a fully saturated porous medium of thickness '*h*' with uniform permeability, while the upper part is occupied by a clear fluid. The lower plate and the porous medium are fixed, while the upper plate moves with a constant velocity 'U_0'. The upper plate is separated from the porous medium by a gap filled with clear fluid of thickness'*d*'. The upper moving porous plate is also subjected to a constant suction velocity V_0 and the lower porous plate to a transverse sinusoidal injection velocity of the form:

$$V = V_0\left(1 + \varepsilon \cos\frac{\pi z}{d}\right). \tag{1}$$

Where ε (<<1) is a positive modulation parameter. Because of this injection velocity the fully developed laminar flow in the channel remains three dimensional. The surface of the porous medium is taken horizontal in $x - z$ plane. The *x*-axis is taken in the flow direction and the y-axis is taken normal to the porous medium interface. Let (u, v, w) and (U, V, W) are the velocity components for the free fluid region and porous layer in the directions (x, y, z) respectively. Since the channel is infinite in the x-direction, all physical quantities will be independent of x.
The governing equations for the free fluid region $(0 \le y \le d)$ are:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\rho\left(v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{3}$$

$$\rho\left(v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right), \tag{4}$$

$$\rho\left(v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right). \tag{5}$$

The governing equations for the porous region $(-h \le y \le 0)$ are:

$$\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0\,, \tag{6}$$

$$0=\frac{\bar{\mu}}{\rho}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)-\frac{\mu}{K_0\rho}U\,, \tag{7}$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\bar{\mu}}{\rho}\left(\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)-\frac{\mu}{K_0\rho}V\,, \tag{8}$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\bar{\mu}}{\rho}\left(\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)-\frac{\mu}{K_0\rho}W\,. \tag{9}$$

The appropriate boundary conditions for the present problem are:

at $\quad y=d;\ u=U_0,\ v=V_0,\ w=0\,,$ (10)

at $\quad y=0;\ u=U,\ v=V,\ w=W,\ p=P,$

$$\mu\left(\frac{\partial u}{\partial y}\right)=\bar{\mu}\left(\frac{\partial U}{\partial y}\right),\ \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\bar{\mu}\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right), \tag{11}$$

at $\quad y=-h;\ U=0,\ V=V_0\left(1+\varepsilon\cos\frac{\pi z}{d}\right),\ W=0\,,$ (12)

where μ, p and $\bar{\mu}, P$ are the viscosity and pressure in the free fluid and porous region respectively ρ, density and K_0, the permeability of the porous layer. Introducing the following non-dimensional quantities in equations (2) to (12),

$$y^*=y/d\,,\ z^*=z/d\,,\ u^*=u/V_0\,,\ v^*=v/V_0\,,\ w^*=w/V_0\,,\ U^*=U/V_0\,,\ V^*=V/V_0\,,$$
$$W^*=W/V_0\,,\ p^*=p/\rho V_0^2\,,\ P^*=P/\rho V_0^2,\ K=K_0/d^2,\ a=h/d\,, \tag{13}$$

we obtain after dropping asterisks for convenience the following:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\,, \tag{14}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{15}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{16}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{17}$$

$$\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0\,, \tag{18}$$

$$\phi\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)-\frac{U}{K}=0\,, \tag{19}$$

$$\phi\left(\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)-\frac{V}{K}=\lambda\frac{\partial P}{\partial y}\,, \tag{20}$$

$$\phi\left(\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)-\frac{W}{K}=\lambda\frac{\partial P}{\partial z}, \tag{21}$$

and the boundary conditions:

at $\quad y=1;\ u=A,\ v=1,\ w=0,$ (22)

at $\quad y=0;\ u=U,\ v=V,\ w=W,\ p=P,$

$$\frac{\partial u}{\partial y}=\phi\frac{\partial U}{\partial y},\ \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\phi\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right), \tag{23}$$

at $\quad y=-a;\ U=0,\ V=\left(1+\varepsilon\cos\pi z\right),\ W=0,$ (24)

where $\phi=\bar{\mu}/\mu,\ \lambda=V_0 d/\upsilon,\ A=U_0/V_0$.

3. Method of Solution

Here the parameter $\varepsilon(<<1)$, i.e. ε is very small. When $\varepsilon=0$ the problem reduces to the two dimensional coupled flow in the channel when there is a constant injection at the bottom and constant suction at the upper moving plate. We assume

$$u(y,z)=u_0(y)+\varepsilon u_1(y,z)+\varepsilon^2 u_2(y,z)+.... \tag{25}$$

Similar expressions hold for other variables *v, w, U, V, W, p* and *P*. In the case when $\varepsilon=0$, equations (14) to (21) reduces to govern the two dimensional coupled flow with the corresponding boundary conditions. The solution of this two dimensional problem is

$$u_0(y)=b_1\exp\left(\lambda y\right)+b_2,\ v_0=1,\ w_0=0,\ p_0=\text{constant},$$

$$U_0(y)=b_3\exp\left(\sqrt{\frac{1}{\phi K}}y\right)+b_4\exp\left(-\sqrt{\frac{1}{\phi K}}y\right),\ V_0=1,$$

$$W_0=0,\ P_0=-\frac{y}{K\lambda}+\text{constant}, \tag{26}$$

where

$$b_1=\frac{A}{b}\sqrt{\frac{\phi}{K}}\left(1+\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right)\right),$$

$$b_2=\frac{A}{b}\left[\lambda\left(1-\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right)\right)-\sqrt{\frac{\phi}{K}}\left(1+\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right)\right)\right],$$

$$b_3=\frac{A\lambda}{b},\ b_4=-b_3\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right),$$

$$b=\lambda\left(1-\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right)\right)-\sqrt{\frac{\phi}{K}}\left(1+\exp\left(-2a\sqrt{\frac{1}{\phi K}}\right)\right)\left(1-\exp(\lambda)\right).$$

When $\varepsilon \neq 0$, substituting (25) into equations (14) to (21) and comparing the coefficients of identical powers of ε, neglecting those of $\varepsilon^2, \varepsilon^3$ etc; we get the following first order equations with the help of (26):

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{27}$$

$$v_1 \frac{\partial u_0}{\partial y} + \frac{\partial u_1}{\partial y} = \frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right), \tag{28}$$

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right), \tag{29}$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right), \tag{30}$$

$$\frac{\partial V_1}{\partial y} + \frac{\partial W_1}{\partial z} = 0, \tag{31}$$

$$\phi\left(\frac{\partial^2 U_1}{\partial y^2} + \frac{\partial^2 U_1}{\partial z^2}\right) - \frac{U_1}{K} = 0, \tag{32}$$

$$\phi\left(\frac{\partial^2 V_1}{\partial y^2} + \frac{\partial^2 V_1}{\partial z^2}\right) - \frac{V_1}{K} = \lambda \frac{\partial P_1}{\partial y}, \tag{33}$$

$$\phi\left(\frac{\partial^2 W_1}{\partial y^2} + \frac{\partial^2 W_1}{\partial z^2}\right) - \frac{W_1}{K} = \lambda \frac{\partial P_1}{\partial z}. \tag{34}$$

The corresponding boundary conditions are:

at $y = 1;\ u_1 = 0,\ v_1 = 0,\ w_1 = 0,$ (35)

at $y = 0;\ u_1 = U_1,\ v_1 = V_1,\ p_1 = P_1,$

$$\frac{\partial u_1}{\partial y} = \phi \frac{\partial U_1}{\partial y},\ \left(\frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial y}\right) = \phi\left(\frac{\partial V_1}{\partial z} + \frac{\partial W_1}{\partial y}\right), \tag{36}$$

at $y = -a;\ U_1 = 0,\ V_1 = \cos \pi z,\ W_1 = 0.$ (37)

These are linear partial differential equations describing the three dimensional coupled flow. We choose expressions of velocity components in such a form so that the equation of continuity is satisfied. We assume

$$u_1(y,z) = u_{11}(y)\cos \pi z, \tag{38}$$

$$v_1(y,z) = v_{11}(y)\cos \pi z, \tag{39}$$

$$w_1(y,z) = -\frac{1}{\pi} v'_{11}(y)\sin \pi z, \tag{40}$$

$$p_1(y,z) = p_{11}(y)\cos \pi z, \tag{41}$$

$$U_1(y,z) = U_{11}(y)\cos \pi z, \tag{42}$$

$$V_1(y,z) = V_{11}(y)\cos \pi z, \tag{43}$$

$$W_1(y,z) = -\frac{1}{\pi} V'_{11}(y)\sin \pi z, \tag{44}$$

$$P_1(y,z) = P_{11}(y)\cos\pi z\ , \tag{45}$$

where the prime denotes the differentiation with respect to *y*. Substituting the expressions (38) to (45) into (27) to (34) and solving under the corresponding transformed boundary conditions, we obtain

$$u_1(y,z) = \begin{bmatrix} L_1 e^{r_1 y} + L_2 e^{r_2 y} \\ +\lambda b_1 \left\{ \dfrac{A_1}{\pi} e^{(\lambda+\pi)y} - \dfrac{A_2}{\pi} e^{(\lambda-\pi)y} + \dfrac{A_3}{2r_1} e^{(\lambda+r_1)y} + \dfrac{A_4}{2r_2} e^{(\lambda+r_2)y} \right\} \end{bmatrix} \cos\pi z, \tag{46}$$

$$v_1(y,z) = \left(A_1 e^{\pi y} + A_2 e^{-\pi y} + A_3 e^{r_1 y} + A_4 e^{r_2 y}\right)\cos\pi z\ , \tag{47}$$

$$w_1(y,z) = -\frac{1}{\pi}\left(\pi A_1 e^{\pi y} - \pi A_2 e^{-\pi y} + A_3 r_1 e^{r_1 y} + A_4 r_2 e^{r_2 y}\right)\sin\pi z\ , \tag{48}$$

$$p_1(y,z) = -\left(A_1 e^{\pi y} + A_2 e^{-\pi y}\right)\cos\pi z\ , \tag{49}$$

$$U_1(y,z) = \left(L_3\, e^{sy} + L_4 e^{-sy}\right) cos\,\pi z\ , \tag{50}$$

$$V_1(y,z) = \left(B_1 e^{\pi y} + B_2 e^{-\pi y} + B_3 e^{sy} + B_4 e^{-sy}\right)\cos\pi z\ , \tag{51}$$

$$W_1(y,z) = -\frac{1}{\pi}\left(\pi B_1 e^{\pi y} - \pi B_2 e^{-\pi y} + sB_3 e^{sy} - sB_4 e^{-sy}\right)\sin\pi z\ , \tag{52}$$

$$P_1(y,z) = -\frac{1}{\lambda\pi K}\left(B_1 e^{\pi y} - B_2 e^{-\pi y}\right)\cos\pi z\ , \tag{53}$$

where,

$$r_1 = \frac{\lambda + \sqrt{\lambda^2 + 4\pi^2}}{2},\quad r_2 = \frac{\lambda - \sqrt{\lambda^2 + 4\pi^2}}{2},\quad s = \sqrt{\pi^2 + \frac{1}{\phi K}}\ .$$

The constants of integration $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, L_1, L_2, L_3, L_4,$ have been obtained using the corresponding boundary conditions and their expressions are not reported here for the sake of brevity.

When there is no porous layer, i.e. *a*→0, the above results are in agreement with Singh [13].

The expression for the shear stress τ_x in the main flow direction in non-dimensional form at the porous interface is given by

$$\tau_x = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_{11}}{dy}\right)_{y=0} \cos\pi z \tag{54}$$

4. Discussion

Figure 1, shows the main velocity distribution in the composite channel for the dimensionless thickness of the porous medium $a = 0.2$, and $z = 0$. The upper plate is a moving plate, and the dimensionless fluid velocity u at $y = 1$ equals $A\left(= U_0/V_0\right)$, which is taken as unity in our computation work. This fluid velocity quickly decreases with distance from the moving plate, since the porous medium creates resistance to the fluid flow. A decrease in permeability of the porous medium leads to a faster velocity decrease in the porous medium and it become zero at the bottom fixed plate. It is evident from the figure that the injection parameter causes decrease in the main flow. Further it is also observed that by increasing viscosities ratio $\phi\left(=\bar{\mu}/\mu\right)$, the velocity u in the channel decreases, because the fluid experiences greater viscosity (effective viscosity $\bar{\mu}$) in porous medium hence offers resistance to the flow. Givler and Altobelli (1994) discussed that the effective viscosity ($\bar{\mu}$) in the porous medium is not the same as the viscosity of the clear fluid in general and demonstrated that the effective viscosity for highly porous foam can be about ten times the clear fluid viscosity. Thus utilizing the results of this study, we plotted the graphs for various values of the viscosity ratio ϕ. Thus the main flow in the composite channel can be controlled by the injection parameter and by the porous material used in the channel of different permeability and for which effective viscosity differs from the clear fluid viscosity.

The cross flow velocity component w, generated because of the transverse sinusoidal injection velocity applied at the bottom porous plate, is plotted in Figure 2. Since there is injection at the bottom stationary plate and suction at the moving plate, this secondary flow component decreases as the injection parameter λ increases inside the porous layer and up to certain distance in the middle of the channel and then increases with the increase in λ in the upper part in the channel. However it increases by increasing ϕ inside the porous layer and up to certain distance in the clear fluid region, then decreases as we move to the upper part in the channel. The results are also compared to those when $a \to 0$ i.e. when there is no porous layer attached to the bottom porous plate. Figure 3, shows the transverse velocity profiles for various values of permeability parameter K and also the case when there is no porous layer. It is interesting to note that by increasing K, w decreases till half porous region then increases up to certain distance in the lower half of the free liquid region and decreases afterwards in the upper part of the channel till the moving plate, where there is a constant withdrawal of the fluid.

There are several techniques for reducing the skin-friction on walls. In general, the attempts to control the flow depend on changes to the wall boundary conditions including variations of longitudinal and transverse surface curvatures, the nature of the surface and mass transfer through the surface. Thus suction/injection and porous medium lining on the wall play an important role in reducing skin friction at the walls of a channel. Figure 4 shows the variations of the skin friction component τ_x in the main flow direction at the porous interface $z = 0$ for various parameters such as, injection parameter λ, permeability parameter K, thickness of the porous layer 'a', and the viscosities ratio ϕ. It is found that τ_x decreases with the increase of λ or the permeability K of the porous medium. It is also clear from this figure that increase in 'a' reduces the skin friction but this change in τ_x is significant only for small 'a', depending on the permeability of the porous medium K. For small permeability, changes occur only up to small changes in 'a', which increases when K increases. This figure also shows that when viscosities ratio ϕ increases, the fluid in the porous medium experiences greater viscosity and the τ_x increases accordingly.

5. Conclusions

A problem of viscous fluid flow in three-dimensional couette flow through a composite channel which is partially filled with a fluid saturated porous medium and partially with a clear fluid is investigated. The flow in the porous medium is modeled by the Brinkman equation and an appropriate set of boundary conditions are applied at the interface of the clear fluid-porous medium, with effective medium considerations. Analytical solutions for the velocity profiles and the shear stress in the main flow direction at the porous interface are obtained.

It is observed that by the increase in permeability of the porous medium the main flow velocity increases in the channel, whereas the injection parameter and the effective viscosity of the porous medium decrease the main flow. Thus the main flow in the composite channel can be controlled by these parameters. It is also found that the shear stress in the main flow direction is reduced by the introduction of the permeability or by increasing the injection parameter λ. As well as increase in the thickness of porous layer reduces τ_x. Thus these results can be used for reducing the skin friction on walls.

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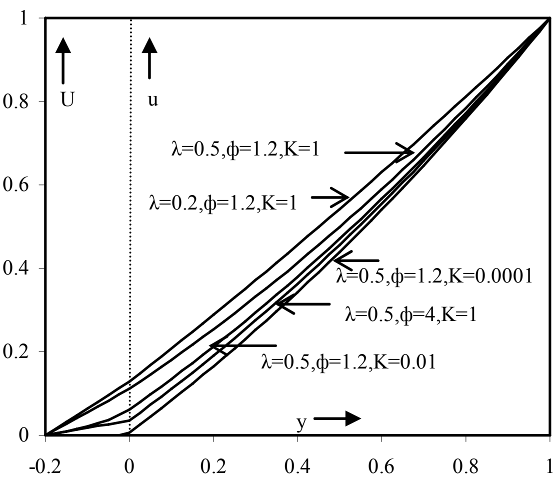

Fig. [1] Main flow velocity u vs y, for $a = 0.2, z = 0$

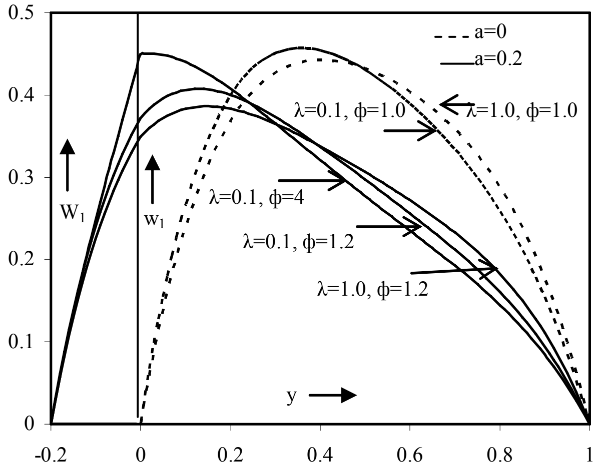

Fig. [2] Transverse velocity w vs y for $K = 1$ and $z = 0.5$.

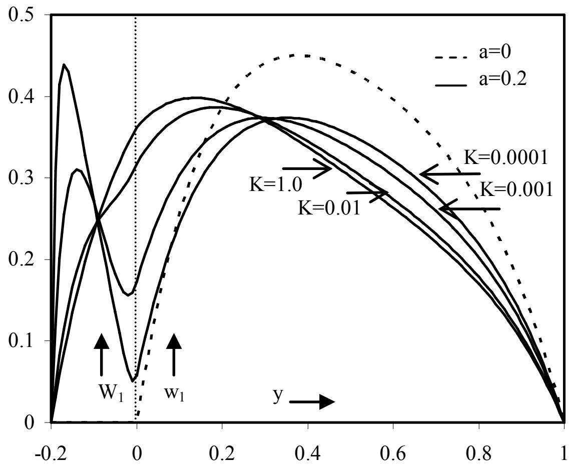

Fig. [3] Transverse velocity w vs y for $\lambda = 0.5$, $\phi = 1.2$ and $z = 0.5$

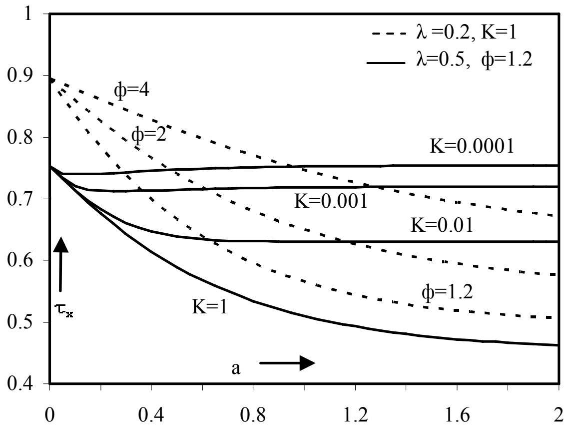

Fig. [4] Shear stress τ_x vs a for $z = 0$.

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