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A New Algorithm for Minimum Path in a Network

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Abstract

A new algorithm namely, path labeling algorithm for finding the minimum path from a specified node to other nodes in a network having crisp or imprecise weights is introduced. The proposed algorithm is simple and easy to understand and apply. The path labeling algorithm is illustrated with help of numerical examples. The minimum paths obtained from one node to each node of a network can be helpful to decision-makers as they make decision to use minimum number of nodes.

Mathematics Subject Classification: 90C27, 90C35, 90B10, 94C15

Keywords: Network optimization; Shortest path; Minimum path; Path labeling algorithm.

1 Introduction

Network optimization is a very popular and frequently applied field among the well studied areas of Operations Research. Many practical problems arising in real life situations can be formulated as network models. The main aim of network models emerging from various application domains as transportation networks, communication networks and planning networks is to optimize the performance with respect to predefined objectives. Many real life situations dealing with the networks require the computation of the best or shortest path from one node to another node and they are called shortest path problem (SPP) of a network. The objectives used in finding out the shortest path in such networks may be one or more, features such as minimization of cost, minimization of time, maximization of stability, maximization of reliability etc. When only one objective is considered in the SPP network, it is called a single objective shortest path problem. A multiple objective shortest path problem of a network consists of more than one objective functions.

In network optimization, a large number of shortest path algorithms [9,8,12,16,18,19] have been worked out more thoroughly than any other algorithm. Some of these algorithms are better than others, some are more suited for a particular structure than others and some are only minor variations of earlier algorithms. Some algorithms like the Dijkstra's algorithm [5] can solve shortest path problems where there are no negative weights. Other algorithms such as the Ford-Moore-Bellman algorithm [10] can handle negative weight edges of shortest path problems as long as they don't appear in cycles. In network, negative weights make the problem harder and negative cycles make the problem intractable. The algorithms given by Bellman[2], Dijkstra [5] and Dreyfus [6] are referred to as the standard shortest path algorithms.

The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There is the possibility of vaguely formulated information due to some uncontrollable factors. Therefore, fuzziness can be introduced into a network through edge capacities, edge weights and / or arc lengths. The fuzzy shortest path problem was first analyzed in [7] and solved using Floyd's algorithm and Ford's algorithm. Klein [14] proposed a dynamical programming recursion-based fuzzy shortest path algorithm. Lin and Chen [17] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada and Soper [21] proposed a fuzzy shortest path algorithm based on multiple labeling methods. Chuang and Kung [3] found fuzzy shortest path length among all possible paths in a network using a fuzzy shortest path length procedure. Kung and Chuang [15] proposed a new algorithm to deal shortest path problems with discrete fuzzy arc lengths. Nayeem and Pal [20] proposed the shortest path algorithm in a network with its arc lengths as interval numbers or triangular fuzzy numbers based on the acceptability index.

Sengupta and Pal [22] have given a single fuzzy shortest path or a guideline in a network for choosing the best fuzzy shortest path according to the decision-maker's viewpoint. Yu and Wei [13] have solved a fuzzy shortest path problem by a linear multiple objective programming approach. Hernandes et al. [10] have proposed a generic algorithm to find a fuzzy shortest path problem on networks. Their algorithm can be implemented using any fuzzy numbers ranking index chosen by the decision-maker, but also it can work correctly with crisp numbers. Iraj Mahdavi et al. [11] have proposed an iterative algorithm based on dynamic programming for fuzzy shortest chain problems.

In this paper, we propose a new algorithm namely, path labeling algorithm to find the minimum path from a specified node to every node in a network having crisp or imprecise weights. The proposed algorithm is simple and easy to understand and apply. The path labeling algorithm is illustrated with the help of numerical examples in each type of network. A number of nondominated paths derived from the network may be too large for a decision maker to choose a preferable path. The proposed method provides minimum path from a specified node to every node using the proposed algorithm such paths can be helpful to decision-makers as they make decision to avoid or use more intermediate nodes between the two nodes.

The main advantage of the proposed algorithm is that it is possible easily find all the intermediate nodes in the shortest path / minimum path between any two nodes immediately without moving backward in the network. In each iterations of the proposed algorithm, the label moves from a node to its adjacent node having less path weight, not less edge weight. In the label formation, we consider each path connecting two nodes in the network at most one time.

2 Preliminaries

A network represented by $G=(V,E,d)$ consists of the set V of n nodes, the set $E \subseteq V \times V$ of edges (arcs) and each edge $(v_i, v_j) \in E$ is associated with a weight d_{ij} which is a number, an interval, a fuzzy number or a vector. If each edge of a network has direction, then it is called directed network.

A path $p(v_i, v_j)$ from the node v_i to the node v_j in the network is a sequence of continuous edges $v_i, ..., v_j$ that connects the nodes v_i and v_j . If the sequence of nodes is finite, then the path is known as finite, otherwise it is called infinite. The length of a path is the number of edges constituting the path and the weight of a path is the sum of the weights of the edges constituting the path. A shortest path from a node u to another node v in a network is a path from u to v with the property that no other such path has a lower weight. A path which starts and ends at the same node is called a cycle. If there is a cycle in a network, it is called cyclic network. Otherwise, it is called acyclic network. If the weight of a cycle in a network is negative, then the cycle is called a negative cycle.

Definition 1: A minimum path from a node $\boldsymbol{u}$ to another node $\boldsymbol{v}$ in a network is a shortest path from $\boldsymbol{u}$ to v with the property that no other such path has a lower length.

Remark 1: The weight of the shortest path is the same as the weight of the minimum path.

Remark 2: If there is only one shortest path from a node $\boldsymbol{u}$ to a node $\boldsymbol{v}$, then the minimum path from $\boldsymbol{u}$ to $\boldsymbol{v}$ is the shortest path.

We need the following definition of the addition and partial ordering on closed bounded intervals which can be found in [4].

Definition 2: Let $A=[a,b]$ and $B=[c,d]$ be two intervals on the real line *R*. Then, $A \oplus B=[a+c, b+d]$.

Definition 3: Let $A=[a,b]$ and $B=[c,d]$ be two intervals on the real line *R*. Then, (i) $A \le B$ if $a \le c$ and $b \le d$;
(ii) $A \ge B$ if $B \le A$, that is, $a \ge c$ and $b \ge d$ and
(iii) $A = B$ if $A \le B$ and $B \le A$, that is, $a = c$ and $b = d$.

We use the following definition of the addition on fuzzy triangular numbers and partial ordering which can be found in [1] .

Definition 4 : Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then , $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1+b_1, a_2+b_2, a_3+b_3)$.

Definition 5: The magnitude of the triangular fuzzy number $\tilde{u}=(a,b,c)$ is given by

$$Mag(\tilde{u}) = \frac{a+10b+c}{12} .$$

Definition 6: Let $\tilde{u}$ and $\tilde{v}$ be two triangular fuzzy numbers. The ranking of $\tilde{u}$ and $\tilde{v}$ by the $Mag(.)$ on the set of triangular fuzzy numbers is defined as follows:

(i) $\tilde{u} \succ \tilde{v}$ if and only if $Mag(\tilde{u}) > Mag(\tilde{v})$;
(ii) $\tilde{u} \prec \tilde{v}$ if and only if $Mag(\tilde{u}) < Mag(\tilde{v})$ and
(iii) $\tilde{u} \approx \tilde{v}$ if and only if $Mag(\tilde{u}) = Mag(\tilde{v})$.

3 Path labeling algorithm

In a network, a label at a node n_t from n_0 can be represented as follows

$$(k\,; p(n_0, n_t))$$

where $p(n_0, n_t)$ is a path from n_0 to n_t and k is the weight of the path $p(n_0, n_t)$ which may be a number, interval or fuzzy number.

Let $(k_{\mathrm{i}}; p_{\mathrm{i}}(n_0, n_{\mathrm{t}}))$, i = *1,2,3,…, m* be *m* labels. The label $(k_e; p_e(n_0, n_t))$, $e \in \{1,2,...,m\}$ is said to be a minimum label at the node n_t from n_0 if $k_e \le k_{\mathrm{i}}$ for all i. Clearly, the path $p_e(n_0, n_t)$ is the shortest path to n_t from n_0.

Result 1: From the definition of minimum label, if $(k; p(n_0, n_{\mathrm{t}}))$ is a minimum label at the node n_t from n_0 and $(m\,; p(n_{\mathrm{t}}, n_s))$ is a minimum label at the node n_s from n_t , then $(k+m\,; p(n_0, n_{\mathrm{t}}) + p(n_t, n_s))$ is a minimum label at the node n_s

from n_0 where $p(n_0, n_t) + p(n_t, n_s)$ is the path joining the two paths $p(n_0, n_t)$ and $p(n_t, n_s)$ at the node n_t.

Now, we propose the following new algorithm namely, path labeling algorithm for finding a minimum path/ shortest path from a specified node to every node in a network.

Let n_0 be the specified node in the given network. Our aim is to compute a shortest path and a minimum path from n_0 to every node in the network.

The proposed algorithm proceeds as follows:

Algorithm

Step 1: Find the collection of nodes S_1 in the network which is adjacent to n_0. If S_1 is empty, then go to the Step 7.. Otherwise go to the Step 2.

Step 2: Compute the minimum label /shortest path / minimum path at each node of S_1 from n_0.

Step 3: Find the collection of nodes S_2 in the network which is adjacent to S_1. If S_2 is empty, then go to the Step 7.. Otherwise go to the Step 4.

Step 4: Compute the minimum label / shortest path / minimum path at each node of S_2 from n_0 using the Result 1.. If the shortest path between n_0 and any one node of S_2 with negative weight is a cycle, go to the Step 7.. Otherwise go to the Step 5..

Step 5: Repeat the Step 1 to the Step 4 until to obtain the set of collection of nodes in the network which are adjacent to each of the minimum label nodes is empty.

Step 6: Compute the shortest path / minimum path from n_0 to each of nodes in the network in Step 5.. Stop.

Step 7: There is no path from the node n_0 to the label node. Stop.

Algorithm Information:

Each iteration of the proposed algorithm moves from a set of nodes to another set of nodes. In the case of a non-existence of negative circuit, the proposed algorithm converges in a maximum of $k-1$ iterations. Note that the number k is less than the number of nodes and S_k is empty. The maximum number of edges of a path between a node to its adjacent node is $E_{\max}$. The maximum number of paths between a node and its adjacent node is $P_{\max}$. Therefore, the maximum number of additions to calculate the cost of the path between a node and its adjacent node is

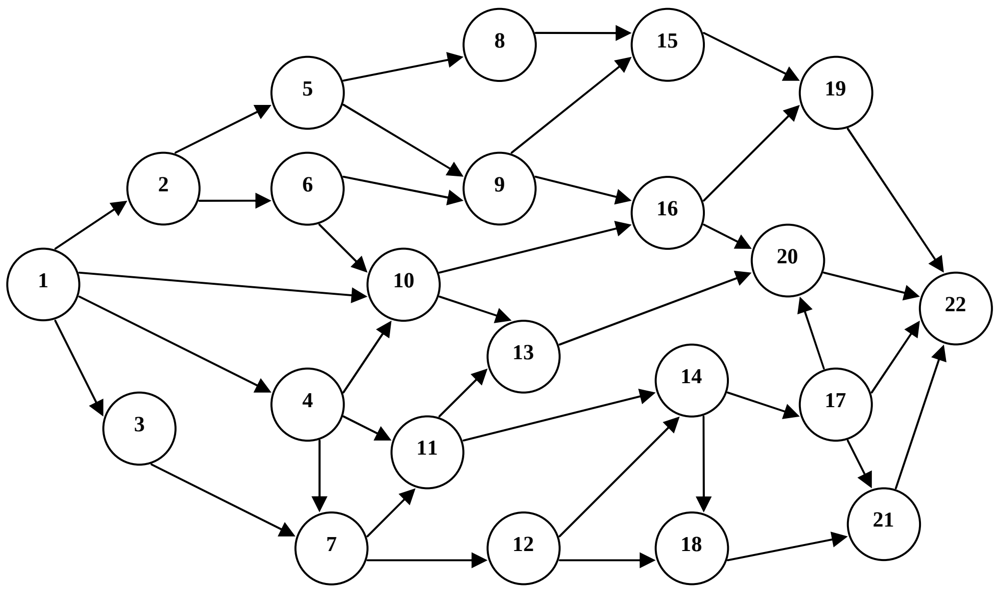
$E_{\max} P_{\max}$. The number of comparisons of the paths between a node and its adjacent node is $P_{\max} - 1$.

Let $S_{\max}$ = Maximum { $|S_i|, i = 1,2,...,k\}$. For each iteration, we have the complexity of $O(S_{\max} E_{\max} P_{\max} + P_{\max} - 1)$. Since the proposed algorithm executes in maximum of $k-1$ iterations, it has a complexity of $O((k-1)(S_{\max} E_{\max} P_{\max} + P_{\max} - 1))$.

4 Numerical Examples

The proposed method is illustrated with the following examples in each type of networks.

Example 1 : Consider the following network



with the following deterministic weights

Edge	Weight		Edge	Weight		Edge	Weight
(1,2)	12		(7,11)	4		(15,19)	9
(1,3)	9		(7,12)	3		(16,19)	7
(1,4)	7		(8,15)	6		(16,20)	6
(1,10)	14		(9,15)	12		(17,20)	15
(2,5)	5		(9,16)	15		(17,21)	3
(2,6)	6		(10,13)	8		(17,22)	5
(3,7)	10		(10,16)	6		(18,21)	6
(4,7)	6		(11,13)	13		(19,22)	13
(4,10)	7		(11,14)	9		(20,22)	12
(4,11)	10		(12,14)	10		(21,22)	4
(5,8)	6		(12,18)	17			
(5,9)	10		(13,20)	11			
(6,9)	9		(14,17)	8			
(6,10)	6		(14,18)	5			

To find the minimum path from the node 1 to each of the other nodes in the network.

Now, $S_1 = \{2,10,4,3\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following :

End Node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
2	(12 ; 1-2)	1-2	1-2
10	(14;1-10) (14;1-4-10)	1-10 1-4-10	1-10
4	(7 ;1-4)	1-4	1-4
3	(9 ;1-3)	1-3	1-3

Now, $S_2 = \{5,6,16,13,11,7\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following :

End Node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
5	(17;1-2-5)	1-2-5	1-2-5
6	(18 ; 1-2-6)	1-2-6	1-2-6
16	(20;1-10-16) (20;1-4-10-16)	1-10-16 1-4-10-16	1-10-16
13	(22; 1-10-13) (22;1-4-10-13)	1-10-13 1-4-10-13	1-10-13
11	(17 ; 1-4-11)	1-4-11	1-4-11
7	(13 ; 1-4-7)	1-4-7	1-4-7

Now, $S_3 = \{8,9,19,20,14,12\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following:

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
8	(23; 1-2-5-8)	1-2-5-8	1-2-5-8
9	(27 ; 1-2-5-9) (27; 1-2-6-9)	1-2-5-9 1-2-6-9	1-2-5-9 1-2-6-9
19	(27 ; 1-10-16-19) (27 ; 1-4-10-16-19)	1-10-16-19 1-4-10-16-19	1-10-16-19
20	(26 ; 1-10-16-20) (26 ; 1-4-10-16-20)	1-10-16-20 1-4-10-16-20	1-10-16-20
14	(26 ; 1-4-7-12-14) (26; 1-4-11-14)	1-4-7-12-14 1-4-11-14	1-4-11-14
12	(16 ; 1-4-7-12)	1-4-7-12	1-4-7-12

Now, $S_4 = \{15,22,17,18\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following :

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
15	(29 ; 1-2-5-8-15)	1-2-5-8-15	1-2-5-8-15
22	(38 ; 1-10-13-20-22) (38; 1-4-10-13-20-22)	1-10-13-20-22 1-4-10-13-20-22	1-10-13-20-22
17	(34 ; 1-4-7-12-14-17) (34 ; 1-4-11-14-17)	1-4-7-12-14-17 1-4-11-14-17	1-4-11-14-17
18	(31; 1-4-7-12-14-18) (31 ; 1-4-11-14-18)	1-4-11-14-18	1-4-11-14-18

Now, $S_5 = \{21\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following:

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
21	(37 ; 1-4-7-12-14-18-21) (37; 1-4-11-14-17-21) (37; 1-4-11-14-18-21) (37; 1-4-7-12-14-17-21)	1-4-7-12-14-18-21 1-4-11-14-17-21 1-4-11-14-18-21 1-4-7-12-14-17-21	1-4-11-14-17-21 1-4-11-14-18-21

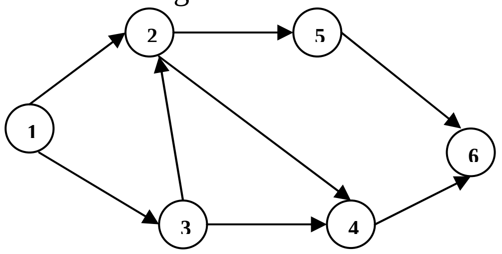
Now, $S_6 = \{\}$. We stop the computation.

By the proposed method, the shortest path and the minimum path from the node 1 to each other nodes is given below.

Node	Shortest Path	Weight of the shortest path	Minimum path
2	1-2	12	1-2
3	1-3	9	1-3
4	1-4	7	1-4
5	1-2-5	17	1-2-5
6	1-2-6	18	1-2-6

7	1-4-7	13	1-4-7
8	1-2-5-8	23	1-2-5-8
9	1-2-5-9 1-2-6-9	27	1-2-5-9 1-2-6-9
10	1-10 1-4-10	14	1-10
11	1-4-11	17	1-4-11
12	1-4-7-12	16	1-4-7-11
13	1-10-13 1-4-10-13	22	1-10-13
14	1-4-7-12-14 1-4-11-14	26	1-4-11-14
15	1-2-5-8-15	29	1-2-5-8-15
16	1-10-16 1-4-10-16	20	1-10-16
17	1-4-7-12-14-17 1-4-11-14-17	34	1-4-11-14-17
18	1-4-7-12-14-18 1-4-11-14-18	31	1-4-11-14-18
19	1-10-16-19 1-4-10-16-19	27	1-10-16-19
20	1-10-16-20 1-4-10-16-20	26	1-10-16-20
21	1-4-7-12-14-18-21 1-4-11-14-17-21 1-4-11-14-18-21 1-4-7-12-14-17-21	37	1-4-11-14-17-21 1-4-11-14-18-21
22	1-10-13-20-22 1-4-10-13-20-22	38	1-10-13-20-22

Example 2: Consider the following network



with the following interval weights

Edge	1-2	1-3	3-2	2-4	2-5	3-4	4-6	5-6
Weight	[30,80]	[10,30]	[40,40]	[8,10]	[10,20]	[20,60]	[60,100]	[55,87]

To find the minimum path form 1 to each of the other nodes in the network.

Now, $S_1 = \{2,3\}$ and using the Step 1 to the Step 3 of the proposed algorithm, we have the following:

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
2	([30,80]; 1-2) ([50,70];1-3-2)	1-2 1-3-2	1-2
3	([10,30];1-3)	1-3	1-3

Now, $S_2 = \{4,5\}$ and using the Step 4 to the Step 6. of the proposed algorithm, we have following:

End node	Minimum Label From the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
4	([30,90];1-3-4) ([58,80];1-3-2-4)	1-3-4 1-3-2-4	1-3-4
5	([40,100]; 1-2-5) ([60,90];1-3-2-5)	1-2-5 1-3-2-5	1-2-5

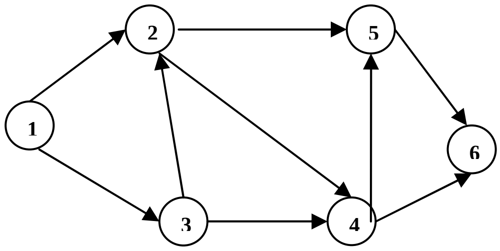
Now, $S_3 = \{6\}$ and using the Step 4 to the Step 6. of the proposed algorithm, we have following

End node	Minimum Label From the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
6	([90,190];1-3-4-6) ([95,187]; 1-2-5-6) ([115,177];1-3-2-5-6)	1-3-4-6 1-2-5-6 1-3-2-5-6	1-3-4-6 1-2-5-6

Now, $S_4 = \{\,\}$. We stop the computation.
By the proposed method, the shortest path and the minimum path form the node 1 to each other nodes is given below using the proposed method.

Node	Minimum path	Shortest path	Weight of the shortest path
2	1-2	1-2 1-3-2	[30,80] [50,70]
3	1-3	1-3	[10,30]
4	1-3-4	1-3-4 1-3-2-4	[30,90] [58,80]
5	1-2-5	1-2-5 1-3-2-5	[40,100] [60,900]
6	1-3-4-6 1-2-5-6	1-3-4-6 1-2-5-6 1-3-2-5-6	[90,190] [95,187] [115,177]

Example 3: Consider the following network



with the following fuzzy weights

Edge	1-2	1-3	2-3	2-4	2-5	3-4	4-5
Weight	(1,2,3)	(5,7,9)	(3,5,7)	(10,11,12)	(5,6,7)	(8,9,10)	(-9,-8,-7)

Edge	4-6	5-6
Weight	(11,13,14)	(-2,1,4)

To find the minimum path form 1 to each of the other nodes in the network.

Now, $S_1 = \{2,3\}$ using the Step 1 to the Step 3 of the proposed algorithm, we have the following :

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
2	((1,2,3);1-2)	1-2	1-2
3	((5,7,9);1-3) ((4,7,10);1-2-3)	1-3 1-2-3	1-3

Now, $S_2 = \{5,4\}$ and using the Step 4 to the Step 6. of the proposed algorithm, we have following:

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
5	((2,5,8);1-2-4-5)	1-2-4-5	1-2-4-5
4	((11,13,15);1-2-4)	1-2-4	1-2-4

Now, $S_3 = \{6\}$. Using the Step 4 to the Step 6. of the proposed algorithm, we have following:

End node	Minimum Label from the node 1	Shortest path between the node 1 and the end node	Minimum path between the node 1 and the end node
6	((10,14,18);1-2-4-5-6) ((9,14,19);1-2-4-6)	1-2-4-5-6 1-2-4-6	1-2-4-6

Now, $S_4 = \{\}$. We stop the computation.
By the proposed method, the shortest path and the minimum path from the node 1 to each other nodes is given below.

End Node	Shortest path	Weight of the shortest path	Minimum path
2	1-2	(1,2,3)	1-2
3	1-3 1-2-3	(5,7,9) (4,7,10)	1-3 1-2-3
4	1-2-4	(11,13,15)	1-2-4
5	1-2-4-5	(2,5,8)	1-2-4-5
6	1-2-4-5-6 1-2-4-6	(10,14,18) (9,14,19)	1-2-4-6

Remark 3: We use the path labeling algorithm for finding a minimum path between two specified nodes with minor modifications.

Let n_0 and n_t be two nodes in the given network. A shortest path and a minimum path between n_0 and n_t is obtained from the path labeling algorithm with the following modifications:

1. Replace the Step 1 of the path labeling algorithm by the following new step:

 Step 1: Find the collection of nodes S_1 in the network which is adjacent to n_0. If S_1 is empty or $n_t \in S_1$, then go to the Step 7. Otherwise go to the Step 2.

2. Use the Step 2 to the Step 5 of the path labeling algorithm.

3. Replace the Step 6 of the labeling algorithm by the following new step

 Step 6: If S_1 is empty, then there is no path between the n_0 and the node n_t. If $n_t \in S_1$, then the shortest path / minimum path between the node n_0 to the n_t is the path corresponding to the minimum label at the node n_t. Stop.

5 Conclusion

The shortest path problem is a classical and important network optimization problem appearing in many real life applications. In this paper, we provide a new algorithm namely, path labeling algorithm for solving shortest path problems as well as minimum path problems on a network. In the proposed method, we are able to obtain all non-dominated paths from the specified node to every other node and any two specified nodes. A number of non-dominated paths derived from a network may be too large for a decision maker to choose a preferable path. We provide minimum paths from a specified node to every node and also, any two specified nodes using the path labeling algorithm. The minimum paths obtained from a network can be helpful to decision-makers deciding on the number of nodes.

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