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Optimization of Air Lift Operation under Fuzzy Environment

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Abstract

The intention of this paper is to illustrate a diligence of fuzzy transportation model in airlift process contrived according to the atmospheric conditions throughout rainy seasons. This paper formulates a procedure to deduce the fuzzy objective value of the fuzzy transportation problem in that the quantum of consignments, requirements and airlifts are fuzzy numbers. To illustrate the procedure, a numerical instance is given.

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1. Introduction

Transportation nourishes economic and social activity and is cardinal to operations research and management science. When operation research egresses as a structured field during World War II, some of the first problems inquired arose from the need to optimize military logistics and transportation activities. Airlifts became practical during World War II as aircraft became prominent and sophiscated enough to handle large cargo postulates. After the war ended, the scope of operation research diligences broadened but transportation problems always resided a primal place. It is now widely recognized that some of the most successful diligences of operations research are bumped in transportation, most significantly in the airline industry where it underlies almost every aspect of strategic, tactical and operational planning.

In our paper we are deliberating the craggy terrain of North East India that feels a high oftenness of adverse atmospheric condition. It is very difficult for Army to transport their unconstipated requirements to forward to border areas through roads on mounds because of atmosphere and road conditions throughout rainy seasons. Thus, one of the basal tasks of Air force in NE India is to furnish airlift support for Army from Foundation or Forward Bestriding Foundations (FBF) to forward or mete areas. Air force usually transports the loads from Foundation or FBF's by dropping sallies at Dropping Partitions (DP) and landing sallies at some Advanced Downing Ground (ADG). The view of the meteorological department was assayed to ascertain the numerate of salutary weather days and partial salutary weather days at the Foundations/FBF's from which the airlift process would originate. It was desirable to render more loads from Foundations/FBF with salutary weather conditions in a month. With partially salutary weather conditions the airlift processes were to be carried out with quashed load. No process was contrived in speculative weather conditions as there was possibility that the aircrafts might turn back to the Foundation/FBF under these circumstances. Different types of aircrafts were usable for the airlift process depending upon the potentiality of the Foundation/FBF from which it operated and the weather conditions of the day. The process of certain aircraft was not possible for certain foundations. The count of airlift process was also different on a day with salutary weather and partially salutary weather. It was the normal practice that an aircraft after airlifting from a Foundation/FBF to an ADG/DP would turn back to its original position. Here, the capacity per airlift process for different aircrafts varies and the monetary value of airlift process depended upon the distance in km from Foundation/FBF to termini and the fuel consumption in appropriate units for aircraft to carry one tonne for 100 km.

In transportation problems quantum of consignments, requirements, airlifts, salutary weather days and partially salutary weather days may not be known in precise manner. For example; salutary weather days and partially salutary weather days may vary in a month during the rainy seasons. The quantum consignments, requirements and airlifts may be uncertain due to atmospheric conditions. By using fuzzy transportation, it is a sensible endeavor to find peculiar solutions for hazardous material transportation because of the possibility of enforcing the affirmative and bearish concepts into account.
Transportation problem was first posed by Ferguson and Dantzig [6] in a paper discussing the allocation of aircraft to routes. Formal methods of solving the transportation problem are presented in standard texts, for example, Charnes and Cooper [1], Dantzig [11] as well as some theoretical insights into the structure of the problem.

The roots of the present paper lie in four main sections. We start in Section 2 by introducing some preliminaries on fuzzy set theory together with the concept of α - level set of fuzzy numbers. In section 3 we state the fuzzy transportation problem formally. Section 4 provides a numerical instance to illustrate the theory. Finally, the paper is concluded in Section 5.

2. Fuzzy Preliminaries

L.A. Zadeh advanced the concepts of fuzzy theory in 1965. The theory indicates a mathematical technique for handling with imprecise concepts and problems which have many possible solutions. The general idea of fuzzy mathematical scheduling on a general level for the first time intended by Tanaka et al (1974) in the theoretical account of the fuzzy conclusion of Zadeh .L.A and Bellman [12]. Now, we present some necessary definitions that come from [8].

2.1. Definition

A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{a}}(x)$ is given beneath

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \,, \\ \dfrac{x - a_3}{a_2 - a_3} & a_2 \le x \le a_3 \,, \\ 0 & otherwise \,. \end{cases}$$

2.2. Definition (α-cut)

The α - level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level $\alpha \in [0,1]$. The formula $L_\alpha(\tilde{a})$ is given as

$$L_\alpha(\tilde{a}) = \{a \in R^m \,/\, \mu_{\tilde{a}}(a_i) \ge \alpha; i = 1,2,...,m\}$$

3. Problem Formulation

Given k types of aircrafts were available for the airlift process depending upon the capability of m types of Foundation/FBF to the n types of termini from which it operated and the atmospheric conditions of the day, so as to minimize the total operating monetary value.

Indices:

i = 1, 2,…,m refers to type of Foundations
j = 1, 2,…,n refers to type of termini
k = 1, 2, 3 refers to type of aircraft

Notations:

R_k = Number of available aircrafts of type k.

M_k = Maximum capacity of aircrafts of type k

$\tilde{G}_i$ = Number of salutary weather days in different Foundation i in a tending Calendar month

$\tilde{O}_i$ = Number of partially salutary weather days in different Foundation i in a tending Calendar month.

$\tilde{S}_{ijk}$ = Number of airlifts possible on salutary weather day from foundation i to termini j By aircraft k.

$\tilde{P}_{ijk}$ = Number of airlifts possible on partially salutary weather day from Foundation i to termini j by aircraft k.

$\tilde{x}_{ijk}$ = Number of consignments from type i to type j by aircraft k

C_{ijk} = Cost of carrying one tonne from Foundation i to termini j by aircraft k

$\tilde{a}_i$ = fuzzy quantum consignments.

$\tilde{b}_j$ = fuzzy requirements

The fuzzy transportation problem (FTP) can be formulated mathematically as follows:

$$\text{Minimize } z = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ijk}\tilde{x}_{ijk}$$

subject to

$$\sum_{j\in J}\sum_{k\in K}\tilde{x}_{ijk} \le \tilde{a}_i \quad ; \; i \in I$$

$$\sum_{i\in I}\sum_{k\in K}\tilde{x}_{ijk} \geq \tilde{b}_j \quad ; \; j \in J$$

$$\tilde{x}_{ijk} \geq 0 \quad \text{for all } i \in I\,,\; j \in J\,,\; k \in K$$

where $\tilde{a}_i = (a_1, a_2, a_3)$ and $\tilde{b}_j = (b_1, b_2, b_3)$ represents fuzzy parameter involved in the constraints, along with their membership function $\mu_{\tilde{a}}(x)$. For a certain degree of α, together with the concept of α - level set of the fuzzy number $\tilde{a}_i$, the FTP can be understood by the following α - Fuzzy Transportation Problem.

Minimize z = $\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ijk}\tilde{x}_{ijk}$

subject to

$$\sum_{j\in J}\sum_{k\in K}\tilde{x}_{ijk} \leq a_i \quad ; \; i \in I$$

$$a_i \in L_\alpha\,(\tilde{a}_i)$$

$$\sum_{i\in I}\sum_{k\in K}\tilde{x}_{ijk} \geq b_j \quad ; \; j \in J$$

$$b_j \in L_\alpha\,(\tilde{b}_j)$$

$$\tilde{x}_{ijk} \geq 0 \quad \text{for all } i \in I\,,\; j \in J\,,\; k \in K$$

where $L_\alpha\,(\tilde{a}_i)$ and $L_\alpha\,(\tilde{b}_j)$ is the α - level set of fuzzy numbers $\tilde{a}_i$ and $\tilde{b}_j$. Based on the α - level set of the fuzzy numbers, the concept of α - optimal solution is defined below.

α - FTP can be rewritten in the following equivalent form:

Minimize Z = $\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ijk}\tilde{x}_{ijk}$

subject to

$$\sum_{j\in J}\sum_{k\in K}\tilde{x}_{ijk} \leq a_i \quad ; \; i \in I$$

$$h_i \leq a_i \leq H_i$$

$$\sum_{i \in I} \sum_{k \in K} \tilde{x}_{ijk} \geq b_j \quad ; \; j \in J$$
$$h_j \leq b_j \leq H_j$$
$$\tilde{x}_{ijk} \geq 0 \quad \text{for all } i \in I, \; j \in J, \; k \in K.$$

It should be noted that the constraints $a_i \in L_\alpha(\tilde{a}_i)$ and $b_j \in L_\alpha(\tilde{b}_j)$ has been replaced by the constraints $h_i \leq a_i \leq H_i$ and $h_j \leq b_j \leq H_j$ where i = 1,2,…,m ; j = 1,2,…,n ; H_i and h_i are upper and lower bounds on a_i and b_j respectively.

4. An Illustrative Instance

Consider the fuzzy transportation model in airlift process contrived according to the atmospheric conditions throughout rainy seasons specified by the following data:

Salutary weather and the partially salutary weather days in various Foundation / FBF in the tending calendar month.

Foundation	1	2	3	4	5	6
Salutary weather days	(8,10,13)	(6,8,11)	(6,8,9)	(6,7,8)	(4,5,6)	(4,6,8)
Partially Salutary Weather days	(5,6,7)	(2,4,5)	(3,4,5)	(2,3,4)	(0,2,3)	(1,2,3)

Aircraft Operation Capability

Base	1	2	3	4	5	6
Types of aircrafts which can be operated	1,2	1	1	2,3	2	3
No. of airlifts possible on a salutary weather day	(4,5,6) (3,5,6)	(3,4,5)	(2,3,4)	(2,3,4) (1,2,3)	(1,2,3)	(0,1,2)
No. of airlifts possible on a partially salutary weather day	(2,3,4) (0,1,2)	(1,2,3)	(0,1,2)	(0,1,2) (0,1,2)	(0,1,2)	(0,1,2)

Table -1 Requirement at Regions for the tending calendar month (Tonnes)

Region	1	2	3	4	5	6
Requirements (Tonnes)	(100,200,300)	(90,140,200)	(100,140,180)	(100,210,300)	(50,100,200)	(40,70,100)

The maximum capacities per airlift operation for the three aircrafts were 5 tonnes, 2.5 tonnes and 2 tonnes respectively. The cost of carrying one tonne from Foundation i to Termini j by aircraft k is proportional to C_{ijk}, where

$$C_{ijk} = \frac{F_{ij} R_k}{100}, \quad i = 1,2,\ldots,6,\ j = 1,2,\ldots,6,\ k = 1,2,3.$$

Table -2 Distances between Various Foundations/FBF and Termini (in km)

Foundation/FBF (i)	Termini (j) 1	2	3	4	5	6
1	219	44	82	427	442	441
2	48	177	135	392	448	439
3	*	273	226	492	560	557
4	278	463	416	*	705	712
5	262	348	324	195	280	294
6	146	150	129	324	366	*

Here, * indicates that the aircraft cannot be operated in the route.

The quantum of consignment which can be airlifted from various Foundation/FBF in three types of aircrafts in the month taking into account both salutary and partially salutary weather days can be calculated as:

Quantum of Consignment $= \left\{ \left(\tilde{S}_{ijk} \times \tilde{G}_i \right) + \left(\tilde{P}_{ijk} \times \tilde{O}_i \right) \right\} M_k$; i = 1,2,…,6 ; j = 1,2,…,6; k = 1,2,3

Foundation/FBF	1	2	3	4	5	6
Aircraft Type	1,2	1	1	2,3	2	3
Quantum of Consignments (Tonnes)	(210,340,530) (60,140,230)	(100,200,350)	(60,140,230)	(30,60,100) (12,34,64)	(10,30,60)	(0,16,44)

The maximum consignment which could be airlifted is (482, 960, 1608) tonnes while the total requirement from Table – 1 was (480, 860, 1280) tonnes.
If we consider the α - level set to be $\alpha = 0.5$, we get

Requirement as,

$150 \le x \le 250$, $115 \le x \le 170$, $120 \le x \le 160$, $155 \le x \le 255$, $75 \le x \le 150$, $55 \le x \le 85$

Quantum of Consignment as,

$275 \le x \le 435$, $100 \le x \le 185$, $150 \le x \le 275$, $100 \le x \le 185$, $45 \le x \le 80$, $23 \le x \le 49$, $20 \le x \le 45$, $8 \le x \le 30$

Thus, the problem can be formulated as a FTP with 8 origins and 7 termini and the unit transportation monetary value corresponding to each route can be obtained from Table 2.

The following results are obtained using TORA optimization software for optimality by solving the above nonfuzzy problem.

The optimum objective function for the Lower bound value is Z = 1555.64 and the decision variables are found:

$$x_{12} = 15,\ x_{13} = 112,\ x_{14} = 18,\ x_{15} = 75,\ x_{16} = 55,\ x_{22} = 100,\ x_{31} = 133,$$
$$x_{34} = 17,\ x_{44} = 100,\ x_{51} = 17,\ x_{57} = 28,\ x_{67} = 23,\ x_{74} = 20,\ x_{83} = 8.$$
with
$$a_1 = 275,\ a_2 = 100,\ a_3 = 150,\ a_4 = 100,\ a_5 = 45,\ a_6 = 23,\ a_7 = 20,\ a_8 = 8,$$
$$b_1 = 150,\ b_2 = 115,\ b_3 = 120,\ b_4 = 155,\ b_5 = 75,\ b_6 = 55.$$

Also, the optimum objective function for the Upper bound value is Z = 2471.15 and the decision variables are found:

$$x_{13} = 145,\ x_{14} = 55,\ x_{15} = 150,\ x_{16} = 85,\ x_{22} = 170,\ x_{23} = 15,\ x_{31} = 250,$$
$$x_{34} = 25,\ x_{44} = 130,\ x_{47} = 55,\ x_{57} = 80,\ x_{67} = 49,\ x_{74} = 45,\ x_{87} = 30.$$
with
$$a_1 = 435,\ a_2 = 185,\ a_3 = 275,\ a_4 = 185,\ a_5 = 80,\ a_6 = 49,\ a_7 = 45,\ a_8 = 30,$$
$$b_1 = 250,\ b_2 = 170,\ b_3 = 160,\ b_4 = 255,\ b_5 = 150,\ b_6 = 85.$$

5. Conclusion

In this paper, we have described the diligence of fuzzy transportation model in airlift process contrived according to the atmospheric conditions in the tending calendar month. The method is based on α - level representation of fuzzy numbers. An illustrative instance has been given to clarify the theory and the allocation procedure.

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