

Cost-Benefit Analysis of a Two-Unit Parallel System Subject to Degradation after Repair

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Abstract

The purpose of this paper is to carryout the cost-benefit analysis of a two-unit system in which units work in parallel and becomes degraded after repair. There is a single repairman who visits the system immediately to do inspection and repair of the units. The repairman inspects the degraded unit when it fails to see the feasibility of repair. If repair of the unit is not feasible, it is replaced by new one. The system is considered in up-state if any one or two of new and/or degraded units are operative. The failure time of the unit is exponentially distributed while the distributions of inspection and repair times are taken as arbitrary. The expressions for some reliability and economic measures are derived using semi-Markov process and regenerative point technique. The numerical results for a particular case are also evaluated for these measures to show their behavior graphically.

Mathematics Subject Classification: Primary90 B25 and Secondary 60K10

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Introduction

Earlier reliability engineers and scholars have shown a keen interest in the analysis of two component parallel systems owing to their practical utility in modern industrial and technological set-ups. A two-identical unit parallel system with geometric failure and repair time distributions discussed by [5]. But no

attention was paid to reliability evaluation of parallel system due to degradation after failure. Since the working capacity and efficiency of a repaired unit depend more or less on the standard of the repair mechanism exercised. If the unit is repaired by an ordinary server, it may not make to work with full efficiency and so becomes degraded unit. The system subject to degradation was studied by [2]. This was improved by [6] and [4] with a more general system in which inspection can be done to find out the feasibility of repair. Also, due to the excessive use, repair of the degraded unit neither possible nor economical to the system. In such cases inspection can be done to find out the feasibility of repair. If repair of the unit is not feasible, it is replaced by new one in order to avoid the unnecessary expanses on repair.

While incorporating the concepts of degradation and inspection, this paper has been designed to carryout the cost-benefit analysis of a redundant system in which two identical units work in parallel and the unit becomes degraded after repair. There is a single repairman who visits the system immediately. The repairman inspects the degraded unit when it fails to see the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one in order to avoid the unnecessary expanses on repair. The system is considered in up-state if any one or two of new and/or degraded units are operative. The random variables are assumed as independent and uncorrelated. The distribution of failure time of the unit follows negative exponential while that of inspection and repair times are taken as arbitrary. The switch devices are perfect. The system is observed at suitable regenerative epochs by using semi-Markov process and regenerative point technique to obtain various measures of system effectiveness such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period for server and expected number of visits by the server. The expression for profit function is also derived. The numerical results for a particular case are obtained for these measures to depict their behavior graphically.

The system of power supply and engine system of the aeroplane can be cited as good examples of parallel-unit systems.

Notations

E	:	Set of regenerative states
N_0	:	The unit is new and operative
D_0	:	The unit is degraded and operative
p/q	:	Probability that repair of degraded unit is feasible/not feasible
λ/λ_1	:	Constant failure rate of new /degraded unit
$g(t)/G(t), g_1(t)/G_1(t)$:	pdf/cdf of repair time for new/degraded unit
$h(t)/H(t)$:	pdf/cdf of inspection time of the degraded unit
$NF_{ur}/NF_{UR}/$ NF_{wr}/NF_{WR}	:	New unit is failed and under repair/ under continuous repair from previous state/waiting for repair/ continuously waiting for repair from previous state
DF_{ur}/DF_{wr}	:	Degraded unit is failed and under repair/waiting for repair.

$DF_{ui}/DF_{wi}/$ DF_{UI} /DF_{WI}	:	Degraded unit is failed and is under inspection / waiting for inspection/under inspection continuously from the previous state/waiting for inspection continuously.
$q_{ij}(t), Q_{ij}(t)$:	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0,t]$.
$q_{ij.kr}(t), Q_{ij.kr}(t)$:	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in $(0,t]$.
$M_i(t)$:	P [system up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state]
$W_i(t)$:	P [server is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states]
m_{ij}	:	Contribution to mean sojourn time in state $S_i \in E$ and non regenerative state if occurs before transition to $S_j \in E$.
\otimes/\odot	:	Symbols for Stieltjes convolution/Laplace convolution
$\sim *$:	Symbols for Laplace Stieltjes Transform (LST)/Laplace Transform (LT)
'(desh)	:	Symbol for derivative of the function
$S_i (i=0-16)$:	The possible transition states

The states $S_0, S_1, S_3, S_4, S_6, S_7, S_8, S_9$ and S_{11} are regenerative states while the remaining states are non-regenerative states. Thus $E = \{S_0, S_1, S_3, S_4, S_6, S_7, S_8, S_9, S_{11}\}$. The possible transition between states along with transition rates for the system model is shown in figure 1.

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$\begin{aligned}
 p_{ij} &= Q_{ij}(\infty) = \int q_{ij}(t) dt \text{ as} \\
 p_{01} &= p_{68}, \quad p_{12} = 1-g^*(\lambda) = p_{1,4,2}, \quad p_{13} = g^*(\lambda), \quad p_{34} = \frac{\lambda}{\lambda + \lambda_1}, \\
 p_{37} &= \frac{\lambda_1}{\lambda + \lambda_1}, \quad p_{46} = g^*(\lambda_1), \quad p_{4,8,5} = 1 - g^*(\lambda_1) = p_{45}, p_{7,0} = qh^*(\lambda), \\
 p_{7,9} &= p h^*(\lambda), \quad p_{7,10} = 1 - h^*(\lambda), \quad p_{7,1,10} = [1 - h^*(\lambda)]q, \quad p_{7,4,10,16} = p[1 - h^*(\lambda_1)], \\
 p_{8,3} &= h^*(\lambda_1), \quad p_{8,11} = ph^*(\lambda_1), \quad p_{8,14} = 1 - h^*(\lambda_1), \quad p_{8,7,14} = [1 - h^*(\lambda_1)]q \\
 p_{8,8,14,15} &= p[1 - h^*(\lambda_1)], \quad p_{9,3} = g_1^*(\lambda), \quad p_{9,13} = 1 - g_1^*(\lambda) = p_{9,4,13}, \\
 p_{11,6} &= g_1^*(\lambda_1), \quad p_{11,12} = 1 - g_1^*(\lambda_1) = p_{11,8,12} \tag{1}
 \end{aligned}$$

For these transition probabilities, it can be verified that

$$\begin{aligned} p_{01} &= p_{68} = p_{12} + p_{13} = p_{1,4,2} + p_{13} = p_{34} + p_{37} = p_{45} + p_{46} = p_{46} + p_{4,8,5} = p_{7,0} + p_{7,10} + p_{7,9} \\ &= p_{7,0} + p_{7,9} + p_{7,1,10} + p_{7,4,10,16} = p_{83} + p_{8,11} + p_{8,14} = p_{83} + p_{8,11} + p_{8,7,14} + p_{8,8,14,15} \\ &= p_{9,3} + p_{9,13} = p_{9,3} + p_{9,4,13} = p_{11,12} + p_{11,6} = p_{11,6} + p_{11,8,12} = 1 \end{aligned} \quad (2)$$

The mean sojourn times μ_i in state S_i are given by

$$\begin{aligned} \mu_0 &= \frac{1}{2\lambda}, & \mu_1 &= \frac{1}{\lambda} [1 - g^*(\lambda)], & \mu_3 &= \frac{1}{\lambda + \lambda_1}, & \mu_4 &= \frac{1}{\lambda_1} [1 - g^*(\lambda_1)], \\ \mu_6 &= \frac{1}{2\lambda_1}, & \mu_7 &= \frac{1}{\lambda} [1 - h^*(\lambda)], & \mu_8 &= \frac{1}{\lambda_1} [1 - h^*(\lambda_1)], & \mu_9 &= \frac{1}{\lambda} [1 - g_1^*(\lambda)], \\ \mu_{11} &= \frac{1}{\lambda_1} [1 - g^*(\lambda_1)] \end{aligned} \quad (3)$$

The unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j is stated as:

$$m_{ij} = \int_0^{\infty} t dQ_{ij}(t) = -q_{ij}^{*'}(0) \text{ and } \mu_i = E(T) = \int_0^{\infty} P(T > t) dt = \sum_j m_{ij} \quad (4)$$

where T denotes the time to system failure.

Thus

$$\begin{aligned} m_{01} &= \mu_0, & m_{12} + m_{13} &= \mu_1, & m_{34} + m_{37} &= \mu_3, \\ m_{45} + m_{46} &= \mu_4, & m_{68} &= \mu_6, & m_{7,0} + m_{7,10} + m_{7,9} &= \mu_7, \\ m_{83} + m_{8,11} + m_{8,14} &= \mu_8, & m_{9,3} + m_{9,13} &= \mu_9, & m_{11,12} + m_{11,6} &= \mu_{11} \end{aligned} \quad (5)$$

Mean Time to System Failure

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) \otimes \phi_j(t) + \sum_k Q_{i,k}(t) \quad (6)$$

where j is an operative regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly.

Taking L.S.T. of relations (6) and solving for $\tilde{\phi}_0(s)$.

Using this, we have

$$R^*(s) = (1 - \tilde{\phi}_0(s))/s \quad (7)$$

The reliability $R(t)$ can be obtained by taking Laplace inverse transform of (7).

The mean time to system failure can be given by

$$MTSF(T_1) = \lim_{s \rightarrow 0} R^*(s) = \frac{N_{11}}{D_{11}}, \quad (8)$$

Where

$$\begin{aligned}
 N_{11} = & -(m_{68}p_{8,11}p_{11,6} + m_{8,11}p_{11,6} + m_{11,6}p_{8,11})[1 - p_{37}(p_{13}p_{70} + p_{93}p_{79}) - \{p_{12}(1 - p_{93}p_{37}p_{79}) \\
 & + p_{13}p_{34}p_{45} + p_{13}p_{37}(p_{7,10} + p_{79}p_{9,13})\}] + (1 - p_{8,11}p_{11,6})[m_{37}(p_{13}p_{70} + p_{93}p_{79}) + p_{37}(m_{01} \\
 & p_{13}p_{7,0} + m_{13}p_{70} + m_{70}p_{13} + m_{93}p_{79} + m_{79}p_{93}) + m_{01}\{p_{12}(1 - p_{93}p_{37}p_{79}) + p_{13}p_{34}p_{45} + p_{13} \\
 & p_{37}(p_{7,10} + p_{79}p_{9,13})\} + \{m_{12}(1 - p_{93}p_{37}p_{79}) - p_{12}(m_{93}p_{37}p_{79} + m_{37}p_{93}p_{79} + m_{79}p_{93}p_{37}) \\
 & + m_{13}p_{34}p_{45} + m_{34}p_{13}p_{45} + m_{45}p_{13}p_{34} + (m_{13}p_{37} + m_{37}p_{13})(p_{7,10} + p_{79}p_{9,13}) + p_{13}p_{37}(m_{7,10} \\
 & + m_{79}p_{9,13} + m_{9,13}p_{79})\}] + (m_{34}p_{46} + m_{46}p_{34} + m_{68}p_{34}p_{46})\{p_{83}p_{13} + p_{13}(p_{8,14} + p_{8,11}p_{11,12})\} \\
 & + p_{34}p_{46}\{m_{83}p_{13} - p_{83}(m_{01}p_{12} + m_{12}) + (m_{01}p_{13} + m_{13})(p_{8,14} + p_{11,12}) + p_{13}(m_{8,14} \\
 & + m_{8,11}p_{11,12} + m_{11,12}p_{8,11})\} \\
 D_{11} = & \{1 - p_{37}(p_{13}p_{70} + p_{93}p_{79})\}(1 - p_{8,11}p_{11,6}) - p_{83}p_{34}p_{46}
 \end{aligned}$$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at $t=0$. The recursive relations for $A_i(t)$ are given by :

$$A_i(t) = M_i(t) + \sum q_{i,j}^{(n)}(t) \odot A_j(t) \tag{9}$$

Where j is any successive regenerative state to which the regenerative state i can transit through $n \geq 1$ (natural number) transitions. We have,

$$\begin{aligned}
 M_0(t) = e^{-\lambda t}, \quad M_1(t) = e^{-\lambda t} \bar{G}(t), \quad M_3(t) = e^{-(\lambda+\lambda_1)t}, \quad M_4(t) = e^{-\lambda_1 t} \bar{G}(t), \quad M_6(t) = e^{-2\lambda_1 t}, \\
 M_7(t) = e^{-\lambda t} \bar{H}(t), \quad M_8(t) = e^{-\lambda_1 t} \bar{H}(t), \quad M_9(t) = e^{-\lambda t} \bar{G}_1(t), \quad M_{11}(t) = e^{-\lambda_1 t} \bar{G}_1(t)
 \end{aligned} \tag{10}$$

Taking LT of relations (9) and solving for $A_0^*(s)$.

The steady-state availability of the system can be given by

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_{12}}{D_{12}}, \tag{11}$$

Where

$$\begin{aligned}
 N_{12} = & -[p_{34}p_{83} + (p_{7,4,10,16} + p_{79}p_{9,4,13})(p_{83}p_{37} + p_{8,7,14}) + p_{93}p_{79}p_{34}p_{8,7,14}](\mu_0 + \mu_1) \\
 & + (1 - p_{8,8,14,15} - p_{8,11})[(1 - p_{93}p_{37}p_{79})(\mu_0 + \mu_1) - \mu_0 p_{37}p_{13}p_{7,1,10} + \mu_3 p_{13} \\
 & + p_{13}p_{37}(\mu_7 + \mu_9 p_{79})] + ((p_{13}p_{34} + p_{14,2})p_{8,7,14} + p_{1,4,2}p_{37}p_{83})(\mu_7 + \mu_9 p_{79}) \\
 & - p_{7,1,10}\mu_0) + [p_{13}\{p_{34} + p_{37}(p_{7,4,10,16} + p_{79}p_{9,4,13})\} + p_{1,4,2}(1 - p_{37}p_{79}p_{93})] \\
 & [(1 - p_{8,8,14,15} - p_{8,11}p_{11,8,12})(p_{46}\mu_6 + \mu_4) + p_{8,11}p_{11,6}(p_{4,8,5}\mu_6 - \mu_4) + (\mu_8 \\
 & + p_{8,11}\mu_{11})] + \mu_3\{p_{1,4,2}(p_{83} + p_{93}p_{79}p_{8,7,14}) - p_{13}p_{8,7,14}(p_{7,4,10,16} + p_{79}p_{9,4,13})\}
 \end{aligned}$$

$$\begin{aligned}
 D_{12} = & (m_{46} + m_{68}p_{46} + m_{4,8,5})[p_{34}p_{83} + (p_{7,4,10,16} + p_{79}p_{9,4,13})(p_{83}p_{37} + p_{8,7,14}) \\
 & + (p_{7,1,10} + p_{7,0})\{(p_{13}p_{34} + p_{1,4,2})p_{8,7,14} + p_{1,4,2}p_{37}p_{83}\} + p_{93}p_{79}p_{34}p_{8,7,14}] \\
 & + [m_{34}p_{83} + m_{83}p_{34} + (m_{7,4,10,16} + m_{79}p_{9,4,13} + m_{9,4,13}p_{79})(p_{83}p_{37} + p_{8,7,14}) \\
 & + (m_{83}p_{37} + m_{37}p_{83} + m_{8,7,14})(p_{7,4,10,16} + p_{79}p_{9,4,13}) + (m_{7,1,10} + m_{01}p_{70} + m_{70}) \\
 & \{(p_{13}p_{34} + p_{1,4,2})p_{8,7,14} + p_{1,4,2}p_{37}p_{83}\} + (p_{7,1,10} + p_{70})\{(m_{13}p_{34} + m_{34}p_{13} \\
 & + m_{1,4,2})p_{8,7,14} + (p_{13}p_{34} + p_{1,4,2})m_{8,7,14} + m_{1,4,2}p_{37}p_{83} + m_{37}p_{83}p_{1,4,2} + m_{83}p_{37} \\
 & p_{1,4,2}\} + m_{93}p_{79}p_{34}p_{8,7,14} + m_{79}p_{93}p_{34}p_{8,7,14} + m_{34}p_{93}p_{79}p_{8,7,14} + m_{8,7,14}p_{93}p_{79} \\
 & p_{34}] - [m_{8,8,14,15} + m_{8,11} + p_{8,11}(m_{11,8,12} + m_{68}p_{11,6} + m_{11,6})][p_{13}p_{37}(p_{7,1,10} + p_{70}) \\
 & + p_{37}p_{79}p_{93} - 1] + [1 - p_{8,8,14,15} - p_{8,11}][m_{13}p_{37} + m_{37}p_{13}(p_{7,1,10} + p_{70}) + p_{13}p_{37} \\
 & (m_{7,1,10} + m_{01}p_{70} + m_{70}) + m_{93}p_{37}p_{79} + m_{37}p_{93}p_{79} + m_{79}p_{93}p_{37}]
 \end{aligned}$$

Busy Period Analysis for Server

Let $B_i(t)$ be the probability that the server is busy at an instant t given that the system entered regenerative state i at $t = 0$. The following are the recursive relations for $B_i(t)$

$$B_i(t) = W_i(t) + \sum_{q_{i,j}^{(n)}(t)} \odot B_j(t) \tag{12}$$

where j is a subsequent regenerative state to which state i transits through $n \geq 1$ (natural number) transitions.

State Transition Diagram

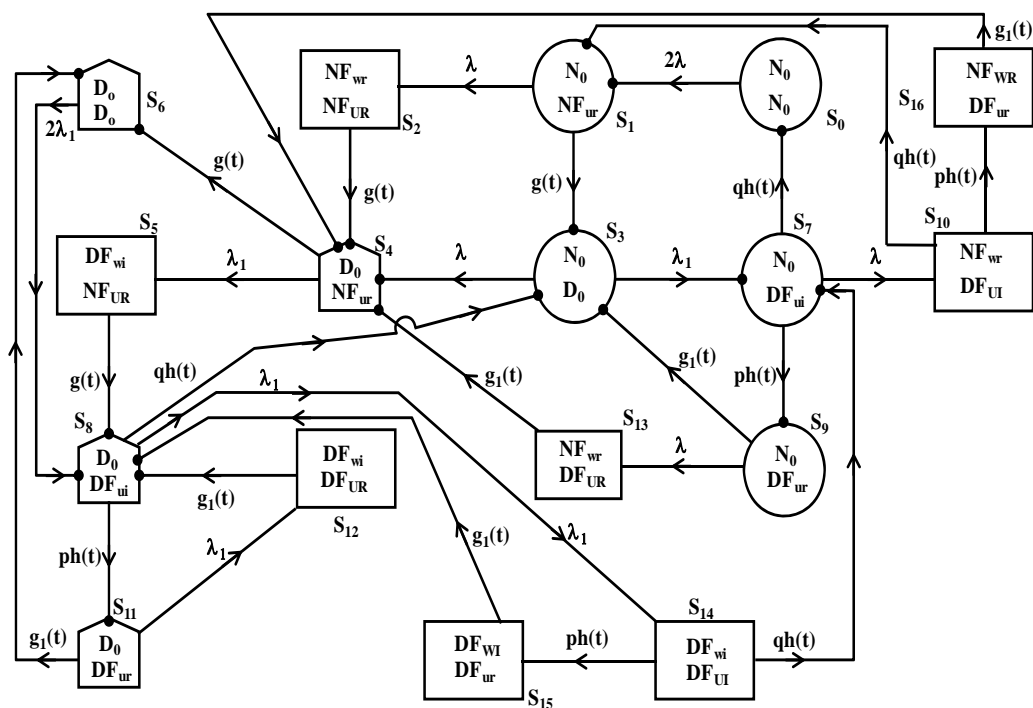


Fig.1

- : Transition point
- : Up-State
- ◑ : Degraded-State
- ◻ : Failed-State

We have,

$$W_1(t) = [e^{-\lambda t} + (\lambda e^{-\lambda t} \odot 1)] \bar{G}(t),$$

$$W_4(t) = [e^{-\lambda_1 t} + (\lambda_1 e^{-\lambda_1 t} \odot 1)] \bar{G}(t),$$

$$\begin{aligned}
 W_7(t) &= e^{-\lambda t} \bar{H}(t) + (\lambda e^{-\lambda t} \odot 1) \bar{H}(t) + (\lambda e^{-\lambda t} \odot \text{ph}(t) \odot 1) \bar{G}_1(t), \\
 W_8(t) &= e^{-\lambda_1 t} \bar{H}(t) + (\lambda_1 e^{-\lambda_1 t} \odot 1) \bar{H}(t) + (\lambda_1 e^{-\lambda_1 t} \odot \text{ph}(t) \odot 1) \bar{G}_1(t), \\
 W_9(t) &= [e^{-\lambda t} + (\lambda e^{-\lambda t} \odot 1)] \bar{G}_1(t), \quad W_{11}(t) = [e^{-\lambda_1 t} + (\lambda_1 e^{-\lambda_1 t} \odot 1)] \bar{G}_1(t) \quad (13)
 \end{aligned}$$

Taking LT of relations (12) and solving for $B_0^*(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_{13}}{D_{12}} \quad (14)$$

$$\begin{aligned}
 N_{13} = & -w_1 [p_{34} p_{83} + (p_{7,4,10,16} + p_{79} p_{9,4,13})(p_{83} p_{37} + p_{8,7,14}) + p_{93} p_{79} p_{34} p_{8,7,14}] \\
 & + (1 - p_{8,8,14,15} - p_{8,11}) [(1 - p_{93} p_{37} p_{79}) w_1 + p_{13} p_{37} (w_7 + p_{79} w_9)] + [(p_{13} p_{34} \\
 & + p_{1,4,2}) p_{8,7,14} + p_{1,4,2} p_{37} p_{83}] [w_7 + p_{79} w_9] + [p_{13} \{p_{34} + p_{37} (p_{7,4,10,16} + p_{79} p_{9,4,13})\} \\
 & + p_{1,4,2} (1 - p_{37} p_{79} p_{93})] [(1 - p_{8,8,14,15} - p_{8,11}) w_4 + w_8 + p_{8,11} w_{11}]
 \end{aligned}$$

And D_{12} is already mentioned.

Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0,t]$ given that the system entered the regenerative state i at $t=0$. We have the following recursive relations for $N_i(t)$:

$$N_i(t) = \sum_j Q_{i,j}(t) \otimes [\delta_j + N_j(t)] \quad (15)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_i=1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_i=0$.

Taking LST of relations (15) and solving for $\tilde{N}_0(s)$.

The expected number of visits per unit time are given by

$$N_0 = \lim_{s \rightarrow 0} s \tilde{N}_0(s) = \frac{N_{14}}{D_{12}} \quad (16)$$

$$\begin{aligned}
 N_{14} = & -[p_{34} p_{83} + (p_{7,4,10,16} + p_{79} p_{9,4,13})(p_{83} p_{37} + p_{8,7,14}) + p_{93} p_{79} p_{34} p_{8,7,14}] \\
 & + (1 - p_{8,8,14,15} - p_{8,11}) [(1 - p_{93} p_{37} p_{79}) - p_{37} p_{13} p_{7,1,10} + p_{13}] - p_{7,1,10} [p_{8,7,14} (p_{13} p_{34} \\
 & + p_{1,4,2}) + p_{1,4,2} p_{37} p_{83}] + [p_{13} \{p_{34} + p_{37} (p_{7,4,10,16} + p_{79} p_{9,4,13})\}] \\
 & + p_{1,4,2} (1 - p_{37} p_{79} p_{93}) [(1 - p_{8,8,14,15} - p_{8,11} p_{11,8,12}) p_{46} + p_{8,11} p_{11,6} p_{4,8,5}] \\
 & + [p_{1,4,2} (p_{83} + p_{93} p_{79} p_{8,7,14}) - p_{13} p_{8,7,14} (p_{7,4,10,16} + p_{79} p_{9,4,13})]
 \end{aligned}$$

Cost-Benefit Analysis

Profit incurred to the system model in steady state is given by

$$P_1 = K_0 A_0 - K_1 B_0 - K_2 N_0$$

Where K_0 = Revenue per unit up time of the system

K_1 = Cost per unit time for which server is busy

K_2 = Cost per visit by the server

Particular Case

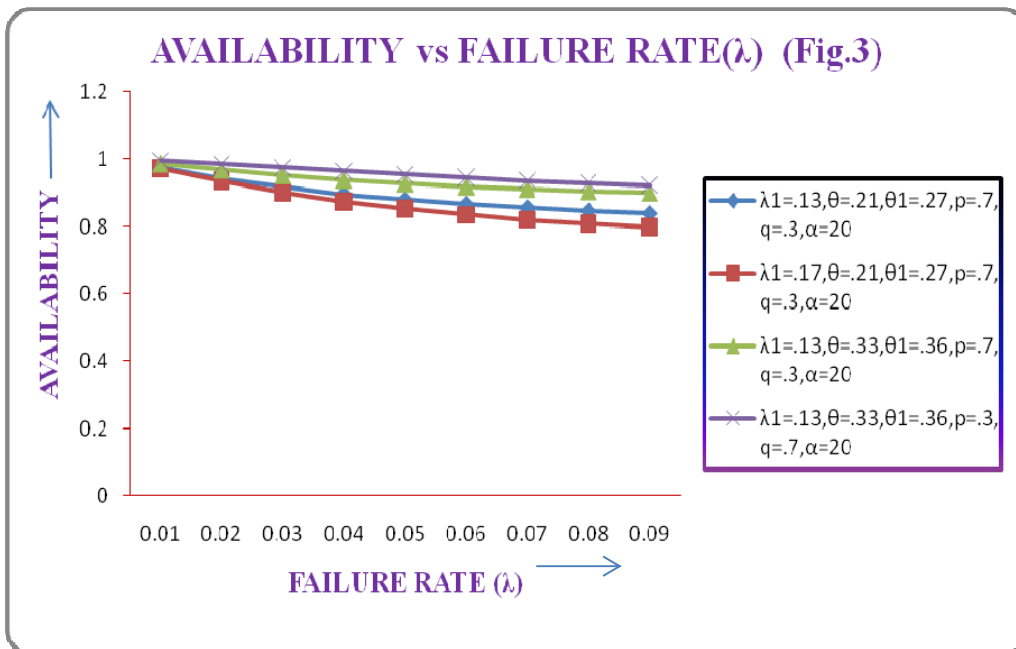
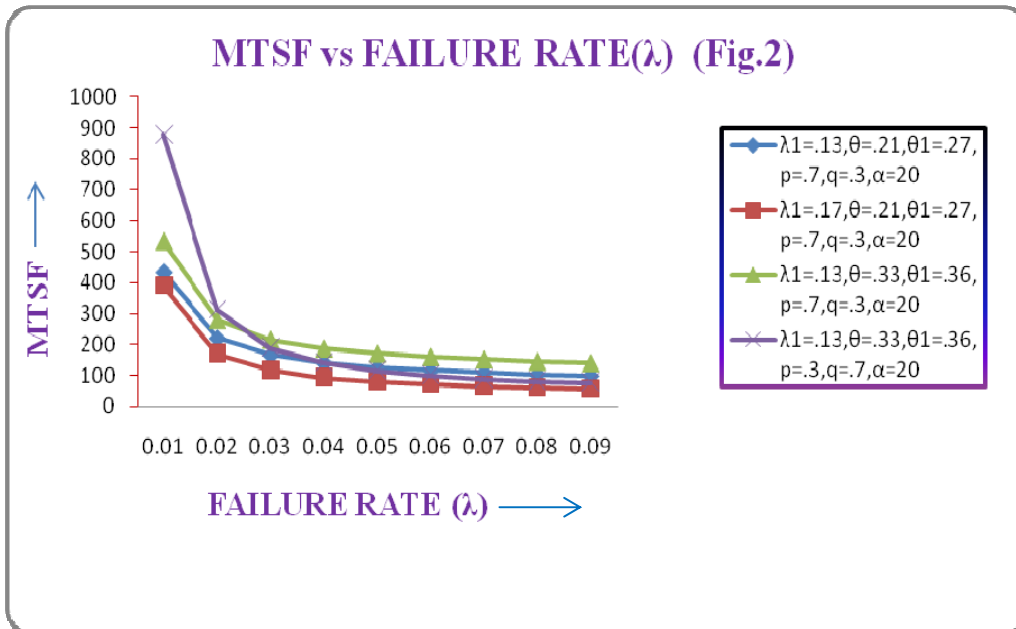
By taking $g(t)=\theta e^{-\theta t}$, $g_1(t)=\theta_1 e^{-\theta_1 t}$ and $h(t)=\alpha e^{-\alpha t}$ in equation 8,11,14 and 16, we can obtain MTSF, availability and profit to this model which are shown graphically by figure 2,3 and 4 respectively.

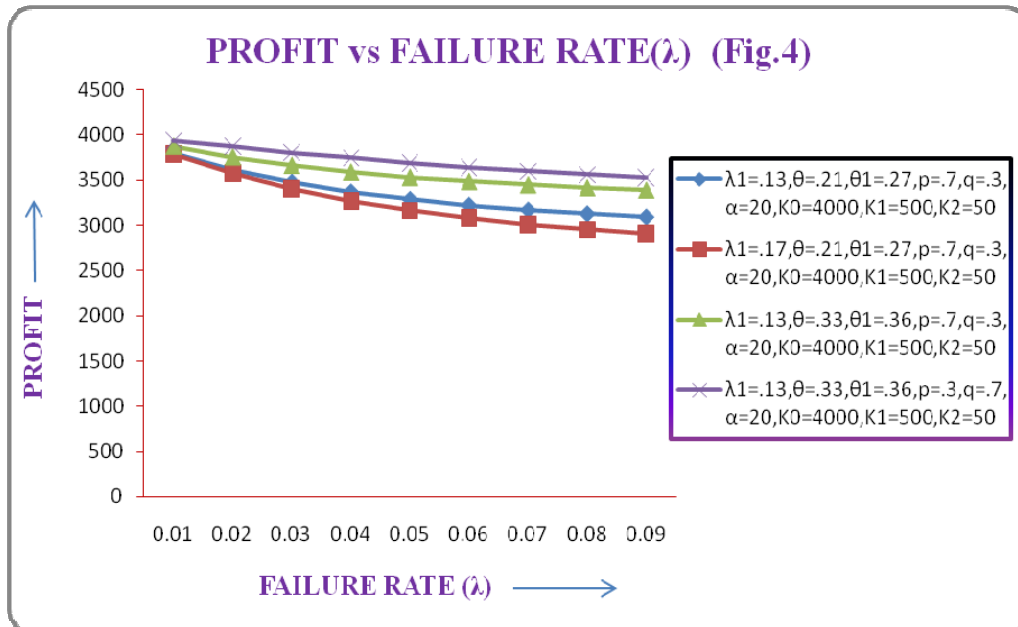
Conclusion

The mean time to system failure (MTSF) and availability of the system model decrease more rapidly with the increase of failure rates λ and λ_1 for fixed values of other parameters as shown in figure 2 and 3. However, their values increase as repair rates θ and θ_1 increase for fixed values of other parameters. The behavior of profit of the system model is shown in figure 4. It is analyzed that the values of profit decreases with the increase of failure rates but increase as and when repair rates increase. Also, if we interchange p and q the availability and the profit of the system increase. Hence on the basis of the results obtained for a particular case, it is concluded that the system can be made reliable and profitable to use if the degraded unit is replaced by new one when it fails.

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