Relativistic graphene ratchet on semidisk Galton board

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(Dated: October 21, 2010)

Using extensive Monte Carlo simulations we study numerically and analytically a photogalvanic effect, or ratchet, of directed electron transport induced by a microwave radiation on a semidisk Galton board of antidots. A comparison between usual two-dimensional electron gas (2DEG) and electrons in graphene shows an enhancement of ratchet current in graphene by two to three orders of magnitude. This enhancement opens promising possibilities for room temperature graphene based sensitive photogalvanic detectors of microwave and terahertz radiation.

PACS numbers: 81.05.ue, 72.40.+w, 73.50.-h, 05.45.Ac

Graphene [1] is a new two-dimensional material with a variety of fascinating physical properties (see e.g. [2]). One of them is a relativistic dispersion law for electron dynamics with an effective "light" velocity $s \approx 10^8 cm/s$ [3], another is a high mobility at room temperature which in suspended graphene reaches $\mu \approx 200,000 cm^2/Vs$ [4–6]. Our theoretical studies predict that these properties lead to emergence of a giant photogalvanic effect induced by radiation at room temperature in a semidisk antidot array placed in graphene plane. Thus, such samples can function as a new type of room temperature sensors of microwave and terahertz radiation.

Ratchet transport, induced in asymmetric systems by ac-driving with zero mean force, attracted a significant interest of scientific community in view of various biological applications of Brownian motors [7–9]. Experimental observations of electron ratchet transport in asymmetric antidot arrays in semiconductor heterostructures have been reported in [10–13]. A detailed theory of ratchet transport of 2DEG on semidisk Galton board was developed in [14, 15] for noninteracting electrons, and it was shown that the effect remains even in presence of strong interactions [16]. The theoretical predictions on polarization dependence have been confirmed in recent experiments [13]. According to theory [14, 15] the velocity of ratchet flow, induced by monochromatic linear polarized microwave force $\mathbf{f}\cos\omega t$ with a frequency ω, has a polarization dependence:

$$(\bar{v}_x, \bar{v}_y) = C_F V_F (f r_d/E_F)^2 (-\cos(2\theta), \sin(2\theta)) \;, \quad (1)$$

where θ is an angle between the polarization direction and $x-$axis of semidisk array shown in Fig. 1, f is amplitude of microwave force, V_F, E_F are Fermi velocity and energy, and C_F is a numerical factor depending on the ratio of periodic cell size R to semidisk radius r_d. In the limit of low density of semidisks with $R \gg r_d$ the theory [15] gives $C_F \propto (\ell_s/r_d)^2/(1+(\omega\ell_s/V_F)^2)$, with $\ell_s \sim R^2/r_d \gg \ell_i$, where ℓ_i is the mean free path related to impurity scattering. The typical parameters of experiment [13] are $f/e \sim 1V/cm$, $r_d \sim 1\mu m$, $R/r_d \approx 4$, electron density $n_e \approx 2.5 \cdot 10^{11} cm^{-2}$ with $V_F \approx 2.2 \cdot 10^7 cm/s$, $E_F \approx 100K$. For these conditions

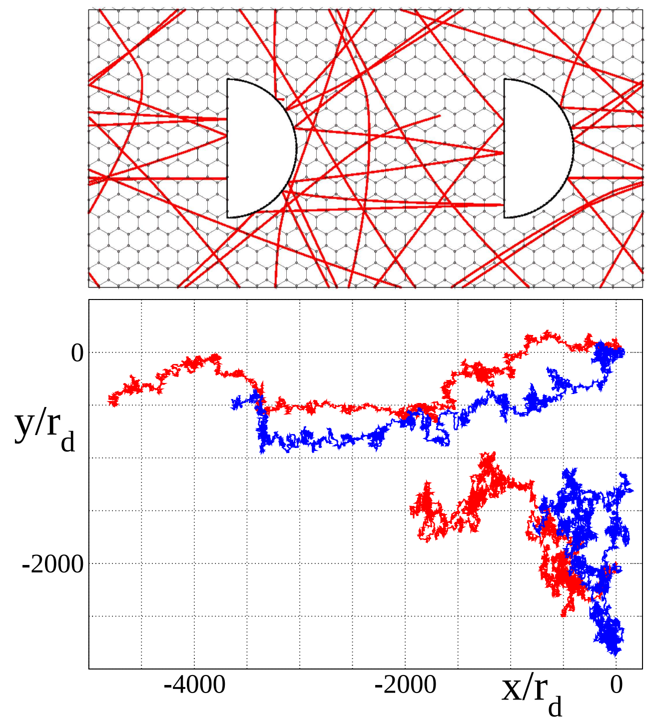

FIG. 1: (Color online) Top panel shows one electron trajectory on semidisk Galton square board (with periodic boundary conditions for two cells, no impurities, time $t \approx 100 r_d/s$ and $f r_d/E_F = 0.05$), graphene structure is shown in a schematic way. Bottom panel shows two ratchet trajectories on a longer time for graphene (with initial conditions $(x,y) = (0,0)$) and 2DEG (with $(x/r_d, y/r_d) = (0, -2000)$) both at the same p_F and $T/E_F = 0.02$; here $R/r_d = 4$, $\theta = 0$, $\omega r_d/s = 0.5$, $V_F/s = 2$, $t \approx 10^5 r_d/s$ and $f r_d/E_f = 0.1$ (red/gray curve), 0.025 (blue/black curve); impurity parameters are $s\tau_i/r_d = 5$, $\alpha_i = \pi/10$.

we have $f r_d/E_F \approx 1/100$, $C_F \approx 0.4$, so that the velocity of ratchet flow remains relatively small but well visible experimentally. Experimental data confirm the linear dependence of ratchet current on microwave power [13]. The relation (1) assumes that $\omega r_d/V_F < 1$ and that the mean free path ℓ_i remains larger semidisk size. For $\ell_i < r_d$ the array asymmetry disappears due to disorder scattering and ratchet velocity goes to zero.

The theory (1) assumes the usual quadratic dispersion law for electron dynamics $E = p^2/2m$, while graphene has a linear relativistic dispersion $E = sp$, so that such a case requires a separate analysis. Recently rectification and photon drag in graphene started to attract experimental [19] and theoretical interest [20–22]. We note that a case of relativistic dispersion appears for a flux quantum in long annular Josephson junction, which has been studied theoretically and experimentally in [23, 24], but there the dynamics takes place in 1D while for graphene it is essentially 2D. In addition, experiments with accelerator beams in crystals [25] show that a crystallographic potential creates an efficient channeling of relativistic particles. This gives some indication on a possible enhancement of ratchet transport of electrons in graphene with a periodic array of asymmetric antidots.

The dynamics of electron on a 2D semidisk Galton board, shown in Fig. 1, is described by the Newton equations

$$d\mathbf{p}/dt = \mathbf{f}\cos\omega t + \mathbf{F}_s + \mathbf{F}_i \ , \qquad (2)$$
$$d\mathbf{r}/dt = s\mathbf{p}/|\mathbf{p}| \ (graphene) \ ; \ \ d\mathbf{r}/dt = \mathbf{p}/m \ (2DEG) \ ,$$

where the second equation corresponds to a relativistic case (l.h.s.) or to a non-relativistic case with an effective mass m (r.h.s.) . Here the force $\mathbf{F}_s$ describes elastic collisions with semidisks and $\mathbf{F}_i$ models impurity scattering on a random angle ϕ_i ($|\phi_i| \leq \alpha_i$) with an effective scattering time τ_i. Following [14], we use the Monte Carlo simulations with the Metropolis algorithm which keeps noninteracting electrons at the Fermi-Dirac distribution with fixed Fermi energy E_F and temperature T. As discussed in [14–16], in a wide range, a variation of energy equilibrium relaxation time τ_{rel} does not influence the ratchet velocity $\bar{\mathbf{v}}$. The later is computed along one or few very long trajectories with times up to $t = 10^8 r_d/s$.

We note that the triangular lattice of disks had been used already by Galton [17] to demonstrate an emergence of statistical laws in deterministic systems. According to the mathematical results of Sinai [18] the dynamics is fully chaotic also for semidisk lattice used here (in absence of impurities, microwave driving and Metropolis thermostat).

A typical example of trajectory for graphene is shown in Fig. 1. The dynamics is clearly chaotic on one cell scale, while on large scale it shows diffusion and ratchet transport directed along x-axis. The ratchet displacement is significantly larger for graphene compared to usual 2DEG with approximately the same parameters. Even more striking a decrease of the driving force by a factor 4 gives a significantly smaller reduction of the ratchet displacement compared to factor 16 expected from the theory for 2DEG (1). Thus the relativistic graphene ratchet has a strong enhancement compared to the usual 2DEG case studied before.

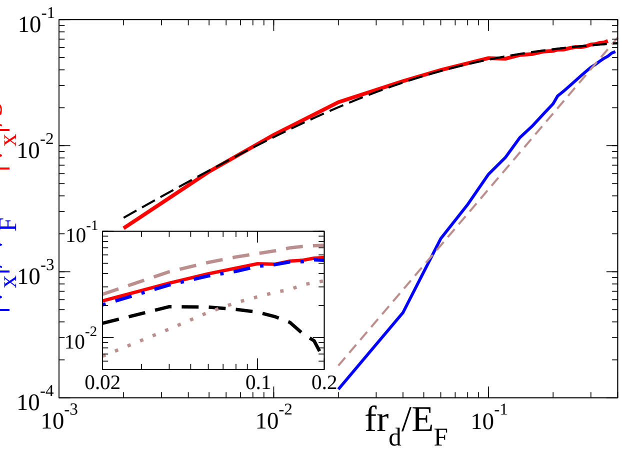

FIG. 2: (Color online) Rescaled ratchet velocity $|\bar{v}_x|$ as a function of rescaled force fr_d/E_F at polarization $\theta = 0$ for graphene (red/gray curve) and 2DEG (blue/black curve); when f is changing the system parameters are kept as Fig. 1. The bottom straight dashed line shows the fit dependence for 2DEG (3), and the top dashed curve shows the fit dependence for graphene (4). Inset shows zoom of graphene data with $\omega r_d/s = 0.25$, $T/E_F = 0.05$, $\alpha_i = \pi/10$ (dashed brown/gray); $\omega r_d/s = 0.5$, $T/E_F = 0.05$, $\alpha_i = \pi/10$ (red/gray, main panel case); $\omega r_d/s = 0.5$, $T/E_F = 0.3$, $\alpha_i = \pi/10$ (dotted-dashed blue/black); $\omega r_d/s = 0.5$, $T/E_F = 0.05$, $\alpha_i = \pi$ (dotted brown/gray); $\omega r_d/s = 0.1$, $T/E_F = 0.05$, $\alpha_i = \pi/10$ (dashed black) (curves from top to bottom at $fr_d/E_F = 0.2$).

Detailed analysis of this enhancement and comparison of ratchet transport for graphene and 2DEG, as a function of microwave driving force, are shown in Fig. 2. We fix $V_F/s = 2$ choosing p_F to be the same for graphene and 2DEG that corresponds to the same electron density n_e. For 2DEG the velocity of ratchet drops quadratically with force and is well described by the dependence

$$|\bar{v}_x|/V_F = C_F(fr_d/E_F)^2 \ , \ \ C_F = 0.45 \ \ , \qquad (3)$$

thus being in a good agreement with numerical data and theory presented in [14–16]. In a case of graphene the field dependence is strikingly different and can be approximately described by the equation

$$|\bar{v}_x|/s = C_{g1} fr_d/(E_F + C_{g2} fr_d) \ , \ C_{g1} = 1.39, C_{g2} = 18.87. \qquad (4)$$

According to Fig. 2 and Eqs. (3),(4) we have the enhancement factor for graphene of approximately 100 and 10^3 at $fr_d/E_F = 0.02$ and 0.002 respectively.

Data presented for graphene in the inset of Fig. 2 show that the ratchet velocity is only weakly affected by increase of temperature T, which can become comparable with E_F, if the rate of impurity scattering is kept fixed. This is in agreement with the known results for 2DEG [14, 15]. An increase of impurity scattering (increase of α_i) gives a reduction of $\bar{v}_x$ but still the dependence on f remains approximately linear for $fr_d << E_F$. A decrease of frequency gives only a slight increase of $\bar{v}_x$ for

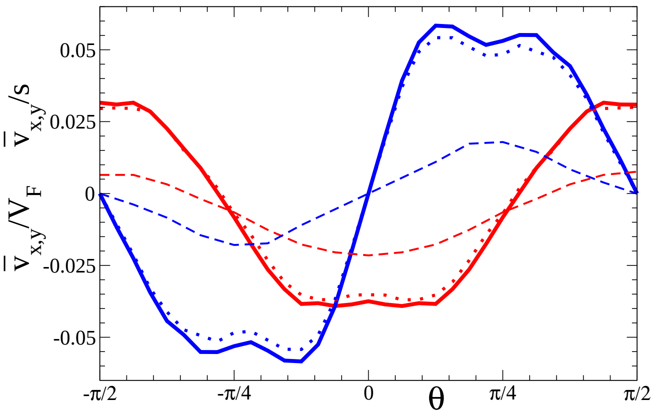

FIG. 3: (Color online) Polarization dependence of ratchet velocity in x (red/gray curves) and y (blue/black curves) directions. Data for graphene are taken at $fr_d/E_F = 0.05$ and $T/E_F = 0.02$ (full curves), 0.3 (dotted curves). Data for 2DEG are shown for $fr_d/E_F = 0.2$ and $T/E_F = 0.02$ (dashed curves). Other parameters are as Fig. 1.

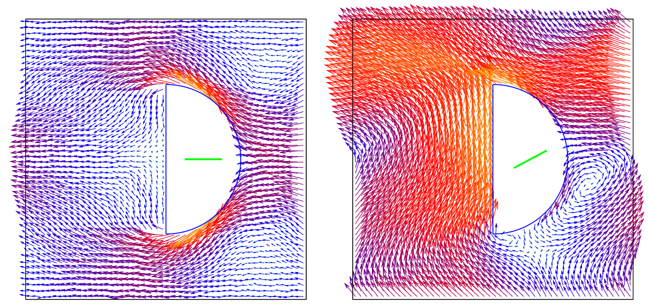

FIG. 4: (Color online) Map of local averaged velocities inside one cell in plane (x, y) for graphene. Parameters are the same as in Fig.3 with $\theta = 0$ (left panel) and $\theta = 5\pi/32$ (right panel), polarization is shown by bar inside semidisk. The velocities are shown by arrows which size is proportional to the velocity amplitude, which is also indicated by color (from large (yellow/gray) to small (blue/black) amplitudes).

$\omega r_d/s \leq 0.5$ while for $\omega r_d/s \geq 1$ we start to see a drop of $\bar{v}_x$ with ω. Such a dependence is in agreement with general theory [15] according to which the ratchet velocity is independent of frequency for $\omega r_d/s \ll 1$ and drops with frequency for $\omega r_d/s \gg 1$. It is interesting to note that at $\omega r_d/s = 1$ the velocity $\bar{v}_x$ starts to decrease at large f. We will discuss this point later.

The polarization dependence of ratchet flow is shown in Fig. 3 for 2DEG and graphene. For 2DEG the dependence is close to those of Eq. (1) being in agreement with previous studies [14–16]. In contrast to that for graphene the dependence on θ is not purely harmonic and appearance of flat domains near velocity maxima is well visible. In part the origin of this flattering can be understood from the picture of average flow inside one periodic cell shown in Fig. 4. While for $\theta = 0$ the flow is relatively regular, for $\theta = 5\pi/32$ there is emergence of vortexes that may be at the origin of flattering in Fig. 3. It is clear that a theoretical description of such a vortex flow is a challenging task for further studies. We note that in agreement with theory [15] the polarization dependence on temperature is rather weak.

However, the main theoretical task is to understand the origin of a slow decrease of ratchet velocity with f. We argue that this is due to the linear dispersion law for graphene which drastically modify the dependence of velocity $\mathbf{v}$ on momentum and force. Indeed, in absence of collisions and impurities we have $\mathbf{v} = (\mathbf{p}_0 + \mathbf{f}/\omega \sin \omega t)/m$ where $\mathbf{p}_0$ is initial momentum. Thus a small force gives only a small oscillating component of velocity and thus the ratchet flow appears only in a second order perturbation theory being proportional only to f^2 [15]. The situation is strongly different for graphene. In absence of collisions and impurities we obtain from (1) $\mathbf{v} = s(\mathbf{p}_0 + \mathbf{f}/\omega \sin \omega t)/|(\mathbf{p}_0 + \mathbf{f}/\omega \sin \omega t)|$. Thus even for small force we have a large variation of velocity direction being of the order of radian for $|\mathbf{p}_0| \sim f/\omega$. These direction variations have many frequency harmonics in contrast to 2DEG. In presence of semidisk asymmetry and relaxation such oscillations should create a directed transport with ratchet velocity $\bar{v} \sim s$. However, in the Fermi-Dirac distribution the fraction w of electrons with such small momenta $p_0 < f/\omega$ is $w \sim fs/(\omega E_F) \sim fr_d/E_F$, where we took into account that in the case of small ω the collisions with semidisks will restrict the length s/ω by a length proportional to r_d. These arguments lead to the ratchet velocity $\bar{v} \sim sw \sim sfr_d/E_F$ being compatible with the dependence (4) found numerically. According to them a physical origin of slow decrease of $\bar{v}$ with f is related to linear dispersion and Dirac singularity in graphene. In such a case electrons with momentum $p \ll p_F$ still have a large velocity s and give a large contribution to the ratchet flow in contract to 2DEG.

It is interesting to note that at high frequencies ω only few collisions happen during a period of oscillations so that in average we have $< |\mathbf{p}| > \approx \sqrt{p_0^2 + f^2/2\omega^2}$ that looks like an appearance of effective mass at large f. This mass effectively drives the system to a situation similar to 2DEG given a reduction of $\bar{v}$ at large f (see black dashed curve in Fig. 2 inset).

The electron current produced by ratchet flow is given by relation $j = en_e\bar{v}$. According to (3),(4) we have for 2DEG

$$j_F \approx 0.4en_eV_F(fr_d/E_F)^2 \sim J_F(n_0/n_e)^{1/2}(fr_d/1K)^2 \ , \ J_F \approx 4 \cdot 10^{-5} A/cm \ ; \quad (5)$$

and for graphene

$$j_g \approx 1.4en_es(fr_d/E_F) \sim J_g(n_e/n_0)^{1/2}(fr_d/1K) \ , \ J_g \approx 6 \cdot 10^{-3} A/cm \ ; \quad (6)$$

where we normalize data in respect to typical parameters of 2DEG in [13] with $n_0 = 2.5 \cdot 10^{11} cm^{-2}$, $m = 0.067 m_e$, $f/e = 1V/cm$, $r_d = 1\mu m$, $fr_d \approx 1K$, $E_F \approx 100K$, $V_F \approx 2.2 \cdot 10^7 cm/s$ and usual parameters of graphene $s \approx 10^8 cm/s$, $E_F \approx 900K$ at same $n_e = n_0$ and p_F [3, 5]. Thus at these parameters we have enhancement factor $\kappa = j_g/j_F = n_e J_g / n_0 J_F \times (1K/fr_d) \approx 100$ for graphene ratchet current. As high electron densities as $n_e = 3 \cdot 10^{12} cm^{-2}$ are well accessible for graphene [3] that should allow to reach very strong ratchet currents of $j_g \approx 0.02 A/cm$ with $\kappa \approx 10^3$ at rather moderate microwave field of $fr_d \sim 1K$. Of course, the above Eqs. (4),(6) assume that the mean free path in graphene is larger than the semidisk radius $r_d \sim 1\mu m$, but according to experiments [5, 6] this condition can be fulfilled for suspended monolayer graphene. At present it is possible to realize large size samples with epitaxial graphene [26, 27] and chemical vapor deposition [28] with room temperature mobility of $20000 cm^2/Vs$.

In conclusion, our theoretical studies show that ratchet transport in asymmetric antidot arrays in graphene has giant enhancement compared to analogous case of 2DEG. This shows that such graphene structures can have future promising applications for simple room temperature sensors of microwave and terahertz radiation. A decrease of antidot size can make such structures to be sensitive to infrared radiation with possible photovoltaic applications.

We thank A.D.Chepelianskii for useful discussions and for pointing to us promising properties of graphene [4–6] for ratchet transport. This work is supported in part by ANR PNANO project NANOTERRA.

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