On the Capacity of Multiple-Access-Z-Interference Channels

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***Abstract*—The capacity of a network in which a multiple access channel (MAC) generates interference to a single-user channel is studied. An achievable rate region based on superposition coding and joint decoding is established for the discrete case. If the interference is strong, the capacity region is obtained for both the discrete memoryless channel and Gaussian channel. A boundary point of the capacity region is determined for a subclass of Gaussian channels with mixed interference.**

I. Introduction

In a cellular system, co-channel interference is often ignored as the co-channel cells are strategically placed so that interference is kept at a minimum. As such, the downlink transmission is typically modeled as a broadcast channel (BC) while uplink transmission is modeled as a multiple access channel (MAC), both free of interference. This effectively isolates each cell from all the other co-channel cells and allows application of the capacity regions for the Gaussian BC and general MAC, which have been completely determined (see [1]).

However, as the need for spectrum reuse increases, various frequency reuse schemes have been proposed in recent years and it is no longer realistic to ignore co-channel interference in both dowlink and uplink transmissions. Recently, the Gaussian broadcast-interference channel model has been studied in [2] and [3] with an emphasis on a one-sided interference model. The capacity regions for very strong and slightly strong interference, and some boundary points on the capacity regions of moderate and weak interference were determined. It was shown that the capacity is achieved by fully decoding the interference when it is strong, partially decoding the interference when it is moderate, and treating the interference as noise when it is weak.

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In this paper, we consider an uplink model with interference, namely the multiple access-interference channel. As with [2] and [3], we focus on the MAC with one-sided interference as is illustrated in Fig. 1. Mobile users TX_1 and TX_2 belong to cell 1 while TX_3 belongs to cell 2 and the transmissions of TX_1 and TX_2 cause interference at RX_2, the basestation at cell 2. The interference from TX_3 to RX_1, on the other hand, is assumed to be negligible.

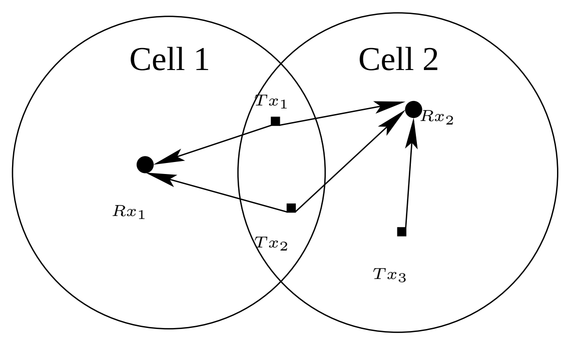

Fig. 1. Two-cell uplink transmission.

Fig. 2 is an abstract model of the above network. Transmitters 1 and 2 and receiver 1 form a MAC. Transmitter 3 and receiver 2 form a single-user channel and receiver 2 receives interference from transmitters 1 and 2. This channel model is specified below.

$$Y_1 = X_1 + X_2 + Z_1, \quad (1)$$
$$Y_2 = \sqrt{a}X_1 + \sqrt{b}X_2 + X_3 + Z_2, \quad (2)$$

where X_i and Y_j are the transmitted and received signals of transmitter i and receiver j, respectively, for $i = 1, 2, 3$ and $j = 1, 2$. The channel coefficients a and b are fixed and known at both the transmitters and the receivers. Without loss of generality, we assume $a, b > 0$, i.e., they are strictly positive. For each j, Z_j is Gaussian noise with zero mean and unit variance and we assume all the noise terms are independent of each other and over time. For transmitter i, the user/channel input sequence $X_{i1}, X_{i2}, \cdots, X_{in}$ is subject to a block power constraint $\sum_{k=1}^{n} P_{ik} \le nP_i$. We denote the rates for messages

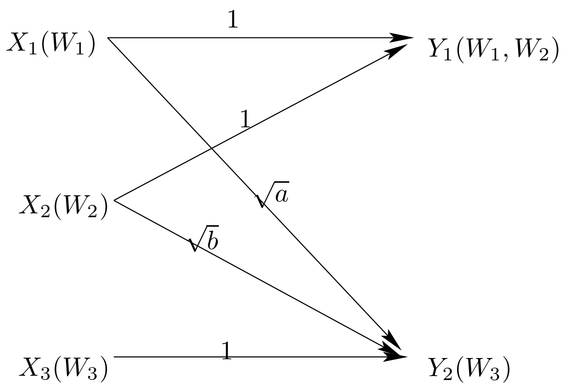

Fig. 2. The Multiple-Access-Z-interference Channel model

W_1, W_2 and W_3 by R_1, R_2 and R_3, respectively. The channel defined here is referred to as a Multiple-Access-Z-Interference channel (MAZIC). Our goal is to obtain capacity results for the strong and mixed interference cases for the MAZIC.

The rest of the paper is organized as follows. We give the problem formulation in Section II. Section III gives an achievable rate region for the discrete memoryless MAZIC and the result is extended to the Gaussian case. Capacity results for strong interference and mixed interference cases are derived in Section IV and V respectively. Section VI concludes the paper.

II. Preliminaries

A discrete memoryless MAZIC is defined by $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, p, \mathcal{Y}_1, \mathcal{Y}_2)$, where $\mathcal{X}_1, \mathcal{X}_2$ and $\mathcal{X}_3$ are finite input alphabet sets; $\mathcal{Y}_1$ and $\mathcal{Y}_2$ are finite output alphabet sets; and $p(y_1y_2|x_1x_2x_3)$ is the channel transition probability. As the receivers do not cooperate, the capacity depends only on the marginal channel transition probabilities thus we can assume

$$p(y_1y_2|x_1x_2x_3) = p(y_1|x_1x_2)p(y_2|x_1x_2x_3). \quad (3)$$

We consider only memoryless channels, so that the channel transition probability satisfies

$$p(y_1^n y_2^n|x_1^n x_2^n x_3^n) = \prod_{i=1}^{n} p(y_{1i}y_{2i}|x_{1i}x_{2i}x_{3i}), \quad (4)$$

where $x_i^n = [x_{i1}, x_{i2}, \cdots, x_{in}]$ and $y_j^n = [y_{j1}, y_{j2}, \cdots, y_{jn}]$, for $i = 1, 2$, and $j = 1, 2, 3$. The message W_i for transmitter i is generated from an integer set $\{1, 2, \cdots, 2^{nR_i}\}$, $i = 1, 2, 3$. A code $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ consists of 3 encoders:

$$\begin{aligned} f_1 &: \{1, 2, \cdots, 2^{nR_1}\} \to \mathcal{X}_1^n, \\ f_2 &: \{1, 2, \cdots, 2^{nR_2}\} \to \mathcal{X}_2^n, \\ f_3 &: \{1, 2, \cdots, 2^{nR_3}\} \to \mathcal{X}_3^n, \end{aligned}$$

and 2 decoders:

$$\begin{aligned} g_1 &: \mathcal{Y}_1^n \to \{1, 2, \cdots, 2^{nR_1}\} \times \{1, 2, \cdots, 2^{nR_2}\}, \\ g_2 &: \mathcal{Y}_2^n \to \{1, 2, \cdots, 2^{nR_3}\}. \end{aligned}$$

The error probability is defined as

$$P_e = \Pr\{g_1(Y_1^n) \neq (w_1, w_2), \text{ or } g_2(Y_2^n) \neq w_3 \\ |W_1W_2W_3 = w_1w_2w_3\}.$$

Assuming W_1, W_2 and W_3 are all uniformly distributed, a rate triple (R_1, R_2, R_3) is achievable if there exist a sequence of codes $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ for n sufficiently large such that $P_e \to 0$ when $n \to \infty$. Throughout this paper, we make the assumption that all the transmitters implement deterministic encoders instead of stochastic encoders as one can easily prove, following the same approach as that of [4], that stochastic encoders do not increase the capacity for a MAZIC. Before proceeding, we introduce some notation which will be used throughout the paper.

- $p_X(x)$ or $p(x)$ is the probability mass function of a discrete random variable X, or a probability density function of a continuous random variable X.
- $A_\epsilon^{(n)}(X)$ denotes the set of length-n ϵ-typical sequences of X.
- $I(\cdot;\cdot)$, $H(\cdot)$ and $h(\cdot)$ are respectively the mutual information, discrete entropy and differential entropy.
- ϕ denotes the empty set.
- $\bar{x} = 1 - x$.
- $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{S})$ means that $\mathbf{x}$ has a Gaussian distribution with zero mean and covariance matrix $\mathbf{S}$.

III. Achievable Region for the General MAZIC

We use superposition coding and joint decoding to derive an achievable rate region. Consider the independent messages W_1 and W_2 generated by transmitters 1 and 2 respectively. We split them into

$$\begin{aligned} W_1 &= [W_{1c}, W_{1p}], \\ W_2 &= [W_{2c}, W_{2p}], \end{aligned}$$

where W_{1c} and W_{2c} denote the common messages that are to be decoded at receiver 2; and where W_{1p} and W_{2p} represent the private messages that are to be decoded at receiver 1.

We first introduce the auxiliary random variables Q, U_1, and U_2, where Q is a time-sharing random variable, U_1 and U_2 contain the information W_{1c} and W_{2c} respectively. The input distribution is

$$\begin{aligned} & p(qu_1u_2x_1x_2x_3) \\ = \; & p(q)p(u_1|q)p(x_1|u_1,q)p(u_2|q)p(x_2|u_2,q)p(x_3|q). \end{aligned} \tag{5}$$

We can obtain the following achievable rate region.

Theorem 1: For a discrete memoryless MAZIC, an achievable rate region is given by the set of all nonnegative rate triples (R_1, R_2, R_3) that satisfy

$$R_1 \le I(X_1;Y_1|X_2Q), \tag{6}$$
$$R_2 \le I(X_2;Y_1|X_1Q), \tag{7}$$
$$R_3 \le I(X_3;Y_2|U_1U_2Q), \tag{8}$$
$$R_1+R_2 \le I(X_1X_2;Y_1|Q), \tag{9}$$
$$R_1+R_3 \le I(X_1;Y_1|U_1X_2Q)+I(U_1X_3;Y_2|U_2Q), \tag{10}$$
$$R_2+R_3 \le I(X_2;Y_1|U_2X_1Q)+I(U_2X_3;Y_2|U_1Q), \tag{11}$$
$$\begin{aligned} R_1+R_2+R_3 \le\; & I(X_1X_2;Y_1|U_1U_2Q) \\ & +I(U_1U_2X_3;Y_2|Q), \end{aligned} \tag{12}$$
$$R_1+R_2+R_3 \le I(X_1X_2;Y_1|U_1Q)+I(U_1X_3;Y_2|U_2Q), \tag{13}$$
$$R_1+R_2+R_3 \le I(X_1X_2;Y_1|U_2Q)+I(U_2X_3;Y_2|U_1Q), \tag{14}$$
$$\begin{aligned} R_1+2R_2+R_3 \le\; & I(X_2;Y_1|U_2X_1Q)+I(X_1X_2;Y_1|U_1Q) \\ & +I(U_1U_2X_3;Y_2|Q), \end{aligned} \tag{15}$$
$$\begin{aligned} 2R_1+R_2+R_3 \le\; & I(X_1;Y_1|U_1X_2Q)+I(X_1X_2;Y_1|U_2Q) \\ & +I(U_1U_2X_3;Y_2|Q), \end{aligned} \tag{16}$$

where the input distribution factors as (5). Furthermore, the region remains the same if we impose the constraints $\|\mathcal{Q}\| \le 12$, $\|U_1\| \le \|X_1\|+5$, $\|U_2\| \le \|X_2\|+5$.

The proof of Theorem 1 is omitted due to space limitations. The MAC and the Z-interference channel (ZIC) are two special cases of a MAZIC. On setting $X_3U_1U_2=\phi$, we obtain the capacity region for the MAC:

$$\begin{aligned} R_1 &\le I(X_1;Y_1|X_2Q), \\ R_2 &\le I(X_2;Y_1|X_1Q), \\ R_1+R_2 &\le I(X_1X_2;Y_1|Q). \end{aligned}$$

Alternatively, on setting $U_2X_2=\phi$, we obtain Han and Kobayashi's achievable rate region for the ZIC [5] [6] [7]:

$$\begin{aligned} R_1 &\le I(X_1;Y_1|Q), \\ R_3 &\le I(X_3;Y_2|U_1Q), \\ R_1+R_3 &\le I(X_1;Y_1|U_1Q)+I(U_1X_3;Y_2|Q). \end{aligned}$$

Theorem 1 allows us to obtain a computable achievable region for Gaussian MAZICs.

Corollary 1: For any nonnegative pair $[\alpha,\beta] \in [0,1]$, the nonnegative rate triples (R_1,R_2,R_3) satisfying the conditions (17)-(27)(see the top of the next page) are achievable for a Gaussian MAZIC.

Proof: Corollary 1 follows directly from Theorem 1 by choosing $\|\mathcal{Q}\|=1$, $X_1 \sim \mathcal{N}(0,P_1)$, $X_2 \sim \mathcal{N}(0,P_2)$, and $X_1=U_1+V_1$, $X_2=U_2+V_2$, where U_1, U_2, V_1 and V_2 are independent random variables with $U_1 \sim \mathcal{N}(0,\alpha P_1)$, $U_2 \sim \mathcal{N}(0,\beta P_2)$, $V_1 \sim \mathcal{N}(0,\bar{\alpha}P_1)$ and $V_2 \sim \mathcal{N}(0,\bar{\beta}P_2)$. ■

In the following, we discuss capacity results for different interference regimes for MAZICs.

IV. MAZICs with Strong Interference

A. Discrete Case

Similar to [8], the discrete MAZIC with strong interference is defined as a discrete memoryless MAZIC satisfying

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2|X_2X_3), \tag{28}$$
$$I(X_2;Y_1|X_1) \le I(X_2;Y_2|X_1X_3), \tag{29}$$
$$I(X_1X_2;Y_1) \le I(X_1X_2;Y_2|X_3), \tag{30}$$

for all product distributions on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$.

The above single letter conditions imply multi-letter conditions as stated below.

Lemma 1: For a discrete memoryless interference channel, if (28)-(30) are satisfied for all product probability distributions on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$, then

$$I(\mathbf{X_1};\mathbf{Y_1}|\mathbf{X_2}) \le I(\mathbf{X_1};\mathbf{Y_2}|\mathbf{X_2X_3}), \tag{31}$$
$$I(\mathbf{X_2};\mathbf{Y_1}|\mathbf{X_1}) \le I(\mathbf{X_2};\mathbf{Y_2}|\mathbf{X_1X_3}), \tag{32}$$
$$I(\mathbf{X_1X_2};\mathbf{Y_1}) \le I(\mathbf{X_1X_2};\mathbf{Y_2}|\mathbf{X_3}). \tag{33}$$

Proof: From the channel model,

$$\begin{aligned} I(\mathbf{X_1};\mathbf{Y_1}|\mathbf{X_2X_3}) &= I(\mathbf{X_1};\mathbf{Y_1}|\mathbf{X_2}), \\ I(\mathbf{X_2};\mathbf{Y_1}|\mathbf{X_1X_3}) &= I(\mathbf{X_2};\mathbf{Y_1}|\mathbf{X_1}), \\ I(\mathbf{X_1X_2};\mathbf{Y_1}|\mathbf{X_3}) &= I(\mathbf{X_1X_2};\mathbf{Y_1}). \end{aligned}$$

The rest of the proof can be established using similar techniques as those in [8]. ■

$$
\begin{aligned}
R_1 &\leq \frac{1}{2}\log(1+P_1), && (17)\\
R_2 &\leq \frac{1}{2}\log(1+P_2), && (18)\\
R_3 &\leq \frac{1}{2}\log\left(1+\frac{P_3}{1+a\alpha P_1+b\beta P_2}\right), && (19)\\
R_1+R_2 &\leq \frac{1}{2}\log\left(1+P_1+P_2\right), && (20)\\
R_1+R_3 &\leq \frac{1}{2}\log\left(1+\alpha P_1\right)+\frac{1}{2}\log\left(1+\frac{a\bar{\alpha}P_1+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (21)\\
R_2+R_3 &\leq \frac{1}{2}\log\left(1+\beta P_2\right)+\frac{1}{2}\log\left(1+\frac{b\bar{\beta}P_2+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (22)\\
R_1+R_2+R_3 &\leq \frac{1}{2}\log\left(1+\alpha P_1+\beta P_2\right)+\frac{1}{2}\log\left(1+\frac{a\bar{\alpha}P_1+b\bar{\beta}P_2+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (23)\\
R_1+R_2+R_3 &\leq \frac{1}{2}\log\left(1+\alpha P_1+P_2\right)+\frac{1}{2}\log\left(1+\frac{a\bar{\alpha}P_1+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (24)\\
R_1+R_2+R_3 &\leq \frac{1}{2}\log\left(1+P_1+\beta P_2\right)+\frac{1}{2}\log\left(1+\frac{b\bar{\beta}P_2+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (25)\\
R_1+2R_2+R_3 &\leq \frac{1}{2}\log\left(1+\beta P_2\right)+\frac{1}{2}\log\left(1+\alpha P_1+P_2\right)+\frac{1}{2}\log\left(1+\frac{a\bar{\alpha}P_1+b\bar{\beta}P_2+P_3}{1+a\alpha P_1+b\beta P_2}\right), && (26)\\
2R_1+R_2+R_3 &\leq \frac{1}{2}\log\left(1+\alpha P_1\right)+\frac{1}{2}\log\left(1+P_1+\beta P_2\right)+\frac{1}{2}\log\left(1+\frac{a\bar{\alpha}P_1+b\bar{\beta}P_2+P_3}{1+a\alpha P_1+b\beta P_2}\right). && (27)
\end{aligned}
$$

The above lemma leads to the following theorem.

Theorem 2: For a discrete memoryless MAZIC with conditions (28)-(30) for all product probability distributions on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3$, the capacity region is given by the set of all the nonnegative rate triples (R_1, R_2, R_3) that satisfy

$$
\begin{aligned}
R_1 &\leq I(X_1;Y_1|X_2Q), && (34)\\
R_2 &\leq I(X_2;Y_1|X_1Q), && (35)\\
R_3 &\leq I(X_3;Y_2|X_1X_2Q), && (36)\\
R_1+R_2 &\leq I(X_1X_2;Y_1|Q), && (37)\\
R_2+R_3 &\leq I(X_2X_3;Y_2|X_1Q), && (38)\\
R_1+R_3 &\leq I(X_1X_3;Y_2|X_2Q), && (39)\\
R_1+R_2+R_3 &\leq I(X_1X_2X_3;Y_2|Q), && (40)
\end{aligned}
$$

where the input distribution factors as

$$
p(qx_1x_2x_3) = p(q)p(x_1|q)p(x_2|q)p(x_3|q). \qquad (41)
$$

Furthermore, the region remains invariant if we impose the constraints $\|\mathcal{Q}\| \leq 8$.

B. Gaussian Case

For a one-sided Gaussian interference channel, the strong interference defined in (28)-(30) implies $a \geq 1$ and $b \geq 1$. While Theorem 1 still applies, a better rate splitting strategy can be devised for this case. If (R_1, R_2, R_3) is an achievable rate triple, then receiver 2 can reliably recover X_1 and X_2 at these rates. Therefore, receiver 2 can decode whatever receiver 1 can decode. Thus, if we choose the private message sets for user 1 and 2 to be empty, i.e., $\alpha = \beta = 0$, we can obtain the following achievable rate region, which is indeed the capacity region for Gaussian MAZIC with strong interference.

Corollary 2: For a Gaussian MAZIC with conditions $a, b \geq 1$, the capacity region is given by the set of all the nonnegative rate triples (R_1, R_2, R_3) that satisfy

$$
\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(1+P_1\right), && (42)\\
R_2 &\leq \frac{1}{2}\log\left(1+P_2\right), && (43)\\
R_3 &\leq \frac{1}{2}\log\left(1+P_3\right), && (44)\\
R_1+R_2 &\leq \frac{1}{2}\log\left(1+P_1+P_2\right), && (45)
\end{aligned}
$$

$$R_2 + R_3 \leq \frac{1}{2}\log\left(1 + bP_2 + P_3\right), \quad (46)$$
$$R_1 + R_3 \leq \frac{1}{2}\log\left(1 + aP_1 + P_3\right), \quad (47)$$
$$R_1 + R_2 + R_3 \leq \frac{1}{2}\log\left(1 + aP_1 + bP_2 + P_3\right). \quad (48)$$

One can immediately see that when the interference is strong, the capacity region is achieved by fully decoding the interference.

V. GAUSSIAN MAZICS WITH MIXED INTERFERENCE

The mixed interference case corresponds to the condition $a \leq 1, b \geq 1$ or $a \geq 1, b \leq 1$. In the following, we consider a subclass of Gaussian MAZICs with mixed interference, and we determine some boundary points of the capacity region.

Lemma 2: For a Gaussian MAZIC with conditions $a \leq 1, b \geq 1 + aP_1 + P_3$, an achievable rate region is given by the set of all nonnegative rate triples (R_1, R_2, R_3) that satisfy

$$R_1 \leq \frac{1}{2}\log\left(1 + P_1\right), \quad (49)$$
$$R_2 \leq \frac{1}{2}\log\left(1 + P_2\right), \quad (50)$$
$$R_3 \leq \frac{1}{2}\log\left(1 + \frac{P_3}{1 + a\alpha P_1}\right), \quad (51)$$
$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + P_1 + P_2\right), \quad (52)$$
$$R_1 + R_3 \leq \frac{1}{2}\log\left(1 + \alpha P_1\right) + \frac{1}{2}\log\left(1 + \frac{a\bar{\alpha}P_1 + P_3}{1 + a\alpha P_1}\right), \quad (53)$$
$$R_1 + R_2 + R_3 \leq \frac{1}{2}\log\left(1 + \alpha P_1 + P_2\right) \quad (54)$$
$$+ \frac{1}{2}\log\left(1 + \frac{a\bar{\alpha}P_1 + P_3}{1 + a\alpha P_1}\right), \quad (55)$$

for $\alpha \in [0, 1]$.

Proof: If $b \geq 1 + aP_1 + P_3$, we know that receiver 2 can decode user 2's message by treating its own signal as well as the interference from user 1 as noise. Therefore, there is no need to do rate splitting for user 2, i.e, $\beta = 0$. On applying Corollary 1 and remove all the redundant inequalities, we get Lemma 2. ■

Remark: $\frac{1}{2}\log\left(1 + \alpha P_1 + P_2\right) + \frac{1}{2}\log\left(1 + \frac{a\bar{\alpha}P_1 + P_3}{1 + a\alpha P_1}\right)$ is an increasing function of α if $a(1+P_2) \leq 1$. Thus, the maximal achievable sum rate for the above achievable rate region is attained when $\alpha = 1$, which equals $R_s = \frac{1}{2}\log(1 + P_1 + P_2) + \frac{1}{2}\log\left(1 + \frac{P_3}{1+aP_1}\right)$. However, it is easy to see that, if one simply uses time-sharing, the maximum achievable sum rate will be larger than R_s.

Nevertheless, we can still determine a boundary point of the capacity region via outer bounding the capacity region.

Theorem 3: For a Gaussian MAZIC with conditions $a \leq 1, b \geq 1 + aP_1 + P_3$, an outer bound to the capacity region is given by the set of all nonnegative rate triples (R_1, R_2, R_3) that satisfy

$$R_1 \leq \frac{1}{2}\log(1 + P_1), \quad (56)$$
$$R_2 \leq \frac{1}{2}\log(1 + P_2), \quad (57)$$
$$R_3 \leq \frac{1}{2}\log(1 + P_3), \quad (58)$$
$$R_1 + R_2 \leq \frac{1}{2}\log(1 + P_1 + P_2), \quad (59)$$
$$R_1 + R_3 \leq \frac{1}{2}\log(1 + P_1) + \frac{1}{2}\log(1 + \frac{P_3}{1 + aP_1}). \quad (60)$$

Proof: (56), (57) and (59) form an outer bound to the capacity region of a MAC, and (58) is a natural bound on R_3. Therefore, we only need to prove (60). We have

$$\begin{aligned}
& n(R_1 + R_3) \\
&= H(W_1) + H(W_3) \\
&\overset{(a)}{\leq} I(X_1^n; Y_1^n) + I(X_3^n; Y_2^n) + n\epsilon \\
&\overset{(b)}{\leq} I(X_1^n; Y_1^n | X_2^n) + I(X_3^n; Y_2^n | X_2^n) + n\epsilon \\
&= I(X_1^n; X_1^n + Z_1^n) + I(X_3^n; \sqrt{a}X_1^n + X_3^n + Z_2^n) + n\epsilon,
\end{aligned}$$

where (a) is from Fano's Inequality; and (b) is because of the mutual independence among X_1^n, X_2^n and X_3^n. From the last line, it is easy to see that the sum of the first two terms is bounded by the sum rate capacity of a two-user one-sided interference channel with weak interference ($a \leq 1$), where the outputs of the channel take the form:

$$\begin{aligned}
\tilde{Y}_1 &= X_1 + Z_1, \\
\tilde{Y}_2 &= \sqrt{a}X_1 + X_3 + Z_2.
\end{aligned}$$

Thus we can obtain (60), where the right hand side is the sum rate capacity of the above interference channel. ■

From Lemma 2 and Theorem 3, we can directly get a boundary point on the capacity region.

Corollary 3: For a Gaussian MAZIC with $a \leq 1$ and $b \geq 1 + aP_1 + P_3$, the rate triple (R_1^*, R_2^*, R_3^*) is on the boundary of the capacity region, where

$$R_1^* = \frac{1}{2}\log(1+P_1), \tag{61}$$

$$R_2^* = \frac{1}{2}\log\left(1+\frac{P_2}{1+P_1}\right), \tag{62}$$

$$R_3^* = \frac{1}{2}\log\left(1+\frac{P_3}{1+aP_1}\right). \tag{63}$$

It is easy to see that this boundary point is achieved by fully decoding the interference from transmitter 2 and treating the interference from transmitter 1 as noise.

VI. Conclusion

In this paper we have studied the capacity of an uplink network with co-channel interference. By modeling such networks using a multiple access interference channel with one-sided interference, we have obtained an inner bound on the capacity region for both the discrete memoryless case and the Gaussian case. The capacity region for such channel models with strong interference has been established and we have also determined a boundary point of the capacity region for a subclass with mixed interference. Furthermore, the result can be easily extended to the multi-user case.

Appendix

A. Proof of Theorem 2

The achievability part follows directly from Theorem 1 by setting $U_1 = U_2 = \phi$. For the converse, (34), (35) and (37) form an outer bound on the capacity region of a MAC. Moreover, (36) is a natural bound on R_3. Therefore, we need to prove only (38)-(40). First,

$$\begin{aligned}
& n(R_2+R_3) \\
= \;& H(W_2)+H(W_3) \\
\overset{(a)}{\leq} \;& I(X_2^n;Y_1^n)+I(X_3^n;Y_2^n)+n\epsilon \\
\overset{(b)}{\leq} \;& I(X_2^n;Y_1^n|X_1^n)+I(X_3^n;Y_2^n|X_1^n)+n\epsilon \\
\overset{(c)}{\leq} \;& I(X_2^n;Y_2^n|X_1^nX_3^n)+I(X_3^n;Y_2^n|X_1^n) \\
& +n\epsilon \\
= \;& I(X_2^nX_3^n;Y_2^n|X_1^n)+n\epsilon \\
= \;& H(Y_2^n|X_1^n)-H(Y_2^n|X_1^nX_2^nX_3^n)+n\epsilon \\
= \;& \sum_{i=1}^n \left\{H(Y_{2i}|Y_2^{i-1}X_1^n)-H(Y_{2i}|Y_2^{i-1}X_1^nX_2^nX_3^n)\right\} \\
& +n\epsilon \\
\overset{(d)}{\leq} \;& \sum_{i=1}^n \{H(Y_{2i}|X_{1i})-H(Y_{2i}|X_{1i}X_{2i}X_{3i})\}+n\epsilon \\
= \;& I(X_{2i}X_{3i};Y_{2i}|X_{1i})+n\epsilon,
\end{aligned}$$

where (a) is from Fano's inequality; (b) is because of the mutual independence among X_1^n, X_2^n and X_3^n; (c) is due to (32); and (d) uses the fact that conditioning reduces entropy and the memoryless property. Similarly, we can prove the bound on R_1+R_3. We further have

$$\begin{aligned}
& n(R_1+R_2+R_3) \\
= \;& H(W_1,W_2)+H(W_3) \\
\overset{(a)}{\leq} \;& I(X_1^nX_2^n;Y_1^n)+I(X_3^n;Y_2^n)+n\epsilon \\
\overset{(b)}{\leq} \;& I(X_1^nX_2^n;Y_1^n)+I(X_3^n;Y_2^n|X_1^nX_2^n)+n\epsilon \\
\overset{(c)}{\leq} \;& I(X_1^nX_2^n;Y_2^n|X_3^n)+I(X_3^n;Y_2^n|X_1^nX_2^n) \\
& +n\epsilon \\
= \;& I(X_1^nX_2^nX_3^n;Y_2^n)+n\epsilon \\
= \;& H(Y_2^n)-H(Y_2^n|X_1^nX_2^nX_3^n)+n\epsilon \\
= \;& \sum_{i=1}^n \left\{H(Y_{2i}|Y_2^{i-1})-H(Y_{2i}|Y_2^{i-1}X_1^nX_2^nX_3^n)\right\} \\
& +n\epsilon \\
\overset{(d)}{\leq} \;& \sum_{i=1}^n \{H(Y_{2i}-H(Y_{2i}|X_{1i}X_{2i}X_{3i}))\}+n\epsilon \\
= \;& I(X_{1i}X_{2i}X_{3i};Y_{2i})+n\epsilon.
\end{aligned}$$

By introducing a time-sharing random variable Q as in the proof of the converse of the capacity region of a MAC [1, Pg. 402], we obtain Theorem 2. The cardinality of $\mathcal{Q}$ can be verified using Caratheodory theorem.

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