Holography and Entanglement in Flat Spacetime

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We propose a holographic correspondence of the flat spacetime based on the behavior of the entanglement entropy and the correlation functions. The holographic dual theory turns out to be highly non-local. We argue that after most part of the space is traced out, the reduced density matrix gives the maximal entropy and the correlation functions become trivial. We present a toy model for this holographic dual using a non-local scalar field theory that reproduces the same property of the entanglement entropy. Our conjecture is consistent with the entropy of Schwarzschild black holes in asymptotically flat spacetimes.

I. INTRODUCTION

One of the most powerful tools to study quantum gravity is the holographic duality conjecture: the quantum gravity in spacetime $\mathcal{M}$ is equivalent to a quantum field theory living on the boundary $\partial\mathcal{M}$ [1]. In particular, the understanding of the quantum gravity in anti-de Sitter space has been well-developed owing to the AdS/CFT correspondence [2] from the string theory. However, to understand our current universe and its creation, we need to study quantum gravity in other spacetimes such as the flat space, the big bang spacetime, and de Sitter space. The purpose of this paper is to investigate holography in the flat spacetime and to present a consistent outline of its basic properties and mechanism. We will focus on the Euclidean flat spacetime $\mathbb{R}^{d+1}$ since the Euclidean formulation is often simpler and better defined than the Lorentzian version, as in the case of AdS/CFT [3].

In polar coordinates, the metric of the Euclidean spacetime $\mathbb{R}^{d+1}$ is

$$ds^2_{\mathbb{R}^{d+1}} = d\rho^2 + \rho^2 ds^2_{S^d} \tag{1}$$

The holographic principle dictates that the boundary dual theory of the gravity in $\mathbb{R}^{d+1}$ lives on the sphere S^d at $\rho = \rho_\infty$, where ρ_∞ is the bulk cut-off radius and is related to the UV cut-off in the boundary field theory (as in AdS/CFT); and we take the limit $\rho_\infty \to \infty$.

The assumptions we adopt in this paper are: 1) the dual field theory allows a path-integral formulation even if it is non-local; and 2) the bulk-to-boundary correspondence [3] holds, which implies that the partition function of gravity in $\mathbb{R}^{d+1}$ equals that of holographic dual theory on S^d. In the Lorentzian version of the holography, S^d is replaced by the d-dimensional de Sitter space (see [4, 5] for an earlier study of the holography in the de Sitter patch). See also [6–8] for different viewpoints of holography in flat spacetime.

II. ENTANGLEMENT ENTROPY

When a quantum system is divided into two subsystems: A and its complement B, the von Neumann entropy $S_A = -\mathrm{Tr}\rho_A \log\rho_A$ (where ρ_A is the reduced density matrix after tracing out B) is called the entanglement entropy of the subsystem A. The scaling behavior and certain universal coefficients of the entanglement entropy encode important information on the degrees of freedom and non-local correlations of the system.

More importantly, the entanglement entropy is a general-purpose quantity since it can be defined in any quantum many-body system that allows a path-integral formalism — even in non-local field theories, as will be shown later. Thus the entanglement entropy is particularly useful when we know little else about the holographic dual of a given gravity theory, such as the gravity in the flat space considered in this paper.

On the gravity side, there is a general prescription to compute the entanglement entropy holographically: when the d-dimensional boundary system is divided into two parts A and B, the holographic dual of the entanglement entropy of the subsystem A is given by the following area formula [9]

$$S_A^{hol.} = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{(d+1)}}, \tag{2}$$

where $\mathrm{Area}(\gamma_A)$ is the area of the *minimal surface* γ_A that lies inside the $(d+1)$-dimensional bulk and borders on the boundary ∂_A of the subsystem A; $G_N^{(d+1)}$ is the $(d+1)$-dimensional Newton's constant.

Now we apply (2) to compute the holographic entanglement entropy of a Euclidean field theory living on the boundary of $\mathbb{R}^{d+1}$. The Lorentzian version is a QFT in d-dimensional de Sitter space. The metric of the boundary sphere S^d is $ds^2_{S^d} = d\tau^2 + \cos^2\tau d\Omega^2_{d-1}$, where $\tau \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is regarded as the Euclidean time, and the spatial slice of constant τ is S^{d-1}, whose metric can be written as $d\Omega^2_{d-1} = d\theta^2 + \sin^2\theta d\Omega^2_{d-2}$. We divide the spatial slice S^{d-1} at $\tau = \tau_0$ into two spherical caps A and B using a subsphere S^{d-2} given by $\theta = \theta_0$. The radius of this S^{d-2} in $\mathbb{R}^{d+1}$ is $\rho_\infty \cos\tau_0 \sin\theta_0 \equiv \rho_\infty \sin\frac{\alpha}{2}$, where $\alpha \in [0, 2\pi]$ and $\rho_\infty\alpha$ is the geodesic distance in S^d of antipodal points of S^{d-2} (see Fig.1). The holographic entanglement entropy is

$$S_A^{hol.} = S_B^{hol.} = \mathcal{V}_{d-1} \cdot \frac{\rho_\infty^{d-1}\left(\sin\frac{\alpha}{2}\right)^{d-1}}{4G_N^{(d+1)}}, \tag{3}$$

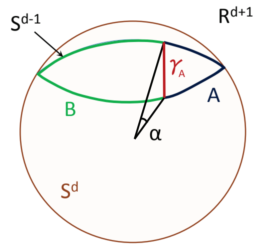

FIG. 1. The geometric computation of the entanglement entropy S_A in the flat space holography. The constant-time slice S^{d-1} of the boundary S^d is divided into A and B. The entanglement entropy S_A is proportional to the area of the minimal surface γ_A, whose size is measured by the angle α.

where $\mathcal{V}_{d-1} = \frac{\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d+1}{2})}$ is the volume of the unit $(d-1)$-ball.

First we notice that for a small subsystem A ($\alpha \ll 1$), (3) approaches A's volume instead of its area. Moreover, S_A with generic α is extensive since it is proportional to the spatial volume of the full boundary system (see also [10] for an earlier study). These two facts are in sharp contrast with the behavior of S_A in local field theories [11] and with the holographic entanglement entropy in AdS spaces [9]. In a local field theory at its ground state, the leading divergence of the entanglement entropy is always proportional to the surface area of the subsystem, hence the so-called "area law" [11]. The entropy becomes extensive only when the system is in highly excited states with energy around the UV cut-off [12, 13]. However, the holographic dual of the gravity in flat space should not be restricted to any particular type of states in the boundary theory since this "volume law" applies to the holographic entanglement entropy of any asymptotically flat space. Taking all these into account, it is natural to conjecture that the holographic dual is described by a certain non-local field theory. Below we will construct one such example based on the scalar field theory.

Also note that although our total system is in a pure state as evidenced by $S_A = S_B$, because S_A satisfies the volume law and moreover saturates the holographic bound [14] (here given by the volume of A in S^d) in the $\alpha \to 0$ limit, the reduced density matrix ρ_A in the $\alpha \to 0$ limit approaches the one with the maximal entropy (=$\log dim\mathcal{H}_\mathcal{A}$ where $\mathcal{H}_\mathcal{A}$ is the Hilbert space of the subsystem A). In other words, the subsystem A is maximally entangled with B in the $\alpha \to 0$ limit.

Let us consider a generic (not necessarily local) free scalar field theory on S^d defined by the action

$$S_{boundary} = \int d^d x \sqrt{g}\,[\phi \cdot f(-\Delta) \cdot \phi]\,, \tag{4}$$

where Δ is the Laplacian on S^d and $f(x)$ is an arbitrary smooth function (see [15] for an analogous computation for $\mathbb{R}^d$). When $f(x) = x$, (4) reduces to the standard free massless scalar action.

To see the extensive behavior of the entanglement entropy, it suffices to consider the simplest configuration with $\alpha = \pi$. In this case, S_A can be expressed as follows (this is very similar to the geometric entropy introduced in [16]).

$$S_A = \frac{\partial}{\partial N} \log \frac{Z_N}{(Z_1)^{1/N}}\bigg|_{N=1}, \tag{5}$$

where Z_N is the partition function of the scalar theory on the orbifold $S^d/\mathbb{Z}_N$; the $\mathbb{Z}_N$ action is defined by a $\frac{2\pi}{N}$ rotation of S^d: writing the S^d metric as a series of spherical suspension (i.e., $d\Omega_n^2 = d\varphi_n^2 + \sin^2\varphi_n d\Omega_{n-1}^2$ and $d\Omega_1^2 = d\varphi_1^2$), $\mathbb{Z}_N$ acts as $\varphi_1 \to \varphi_1 + \frac{2\pi}{N}$. The partition function is evaluated using the Schwinger representation

$$\log Z_N = \frac{1}{2}\int_\epsilon^\infty \frac{ds}{s} \mathrm{Tr}_{(N)} e^{-sf(-\Delta)}, \tag{6}$$

where the cut-off ϵ is related to the UV cut-off in the field theory.

Spherical harmonics on S^d are labeled by angular momentum quantum numbers $(l, m_1, \ldots, m_{d-1})$, which range as $l \geq m_1 \geq \ldots \geq m_{d-2} \geq 0$ and $m_{d-2} \geq |m_{d-1}|$. The eigenvalues of the Laplacian Δ are $-l(l+d-1)$. The $\mathbb{Z}_N$ orbifolding acts by multiplying a phase factor $e^{\frac{2\pi i}{N} m_{d-1}}$. The relevant trace of the kernel is then:

$$\begin{aligned}&\mathrm{Tr}_{(N)} e^{-sf(-\Delta)} - \frac{1}{N}\mathrm{Tr}_{(1)} e^{-sf(-\Delta)} \\ &= \frac{1}{N}\sum_{k=1}^{N-1}\sum_l e^{-sf(l(l+d-1))} g\left(l, d, \frac{k}{N}\right),\end{aligned} \tag{7}$$

where $g(l, d, \frac{k}{N}) \equiv \sum_{\{m_i\}} e^{2\pi i \frac{k}{N} m_{d-1}}$ incorporates the sum over all magnetic quantum momenta m_i.

The relation $g(l, d, \frac{k}{N}) = g(l-1, d, \frac{k}{N}) + g(l, d-1, \frac{k}{N})$ suggests us to solve $g(l, d, \frac{k}{N})$ by first obtaining its generating function:

$$\begin{aligned}G(x, y, a) &\equiv \sum_{l=0}^\infty \sum_{d=2}^\infty g(l, d, \frac{k}{N}) \cdot x^l \cdot y^{d-2} \\ &= \frac{1}{1-x-y} \cdot \frac{1-x^2}{(1-\frac{x}{\xi})(1-\xi x)},\end{aligned} \tag{8}$$

where $\xi \equiv e^{2\pi i \frac{k}{N}}$. Then expanding $G(x, y, a)$ in terms of x and y gives

$$g\left(l, d, \frac{k}{N}\right) = \sum_{n=0}^l \binom{n+d-3}{d-3} \sum_{m=-(l-n)}^{l-n} e^{2\pi i \frac{k}{N} m}, \tag{9}$$

for $d = 2$, the binomial is $\delta_{n,0}$.

Lower dimensional spheres have more compact results: $g(l, 2, \frac{k}{N}) = \frac{\sin[\frac{\pi k}{N}(2l+1)]}{\sin(\frac{\pi k}{N})}$ and $g(l, 3, \frac{k}{N}) = \left(\frac{\sin[\frac{\pi k}{N}(l+1)]}{\sin(\frac{\pi k}{N})}\right)^2$ and need to be treated separately.

For higher dimensional spheres ($d \geq 4$), $g(l, d, \frac{k}{N})$ is a (pseudo-)polynomial of degree-$(d-3)$ with respect to l; its leading term is

$$g(l, d \geq 4, \frac{k}{N}) = \frac{l^{d-3}}{2(d-3)! \cdot \sin^2 \frac{\pi k}{N}} + O(l^{d-4}) \qquad (10)$$

Summing over all twisted sectors using $\sum_{k=1}^{N-1} \frac{1}{\sin^2 \frac{\pi k}{N}} = \frac{1}{3}(N^2 - 1)$ and then applying $\lim_{N \to 1} \frac{\partial}{\partial N}$, we obtain the leading divergence of the entanglement entropy:

$$S_A^{d \geq 4} = \frac{1}{6} \int_\epsilon^\infty \frac{ds}{s} \sum_{l=0}^{\infty} \frac{l^{d-3}}{(d-3)!} e^{-sf[l(l+d-1)]} + \ldots . \qquad (11)$$

Now we impose the UV cutoff. For S^d with radius L and lattice spacing a, the azimuthal angular momentum l has an upper bound given by $l_{max} = \frac{L}{a}$. This translates into a lower bound on the integration parameter s: $s \geq \epsilon = \frac{1}{f(l_{max}^2)}$.

First, let's look at actions with $f(x) = x^p$. Theories of this type are always local; in particular, $p = 1$ corresponds to the standard massless scalar. The leading divergence of the entanglement entropy is

$$S_A = \frac{\Gamma(\frac{d-2}{2p})}{6 \cdot (d-2)!} \left(\frac{L}{a}\right)^{d-2}. \qquad (12)$$

Although this result is obtained for $d \geq 4$, an exact computation for $d = 2, 3$ shows that it actually holds for all $d \geq 2$. In particular for $d = 2$, (12) gives $S_A = \frac{p}{3} \log \frac{L}{a}$ (after an infinite constant term is dropped). Therefore theories with $f(x) = x^p$ always obey "area law".

Now we make the theory non-local by choosing $f(x) = e^{x^q}$. For all S^d with $d \geq 2$, the leading divergence becomes

$$S_A = \frac{2q}{6 \cdot (d-2)! \cdot (d-2+2q)} \left(\frac{L}{a}\right)^{d-2+2q}. \qquad (13)$$

In particular for $d = 2$, $S_A = \frac{1}{6} \left(\frac{L}{a}\right)^{2q}$. Therefore S_A obeys the volume law when $q = \frac{1}{2}$, in any dimension d. To summarize, we find that a non-local scalar field theory defined by the action

$$S_{boundary} = \int d^d x \sqrt{g} \left[\phi(x) \cdot e^{\sqrt{-\Delta}} \cdot \phi(x)\right], \qquad (14)$$

gives rise to an entanglement entropy that exhibits the volume law.

It is tempting to speculate that the holographic dual of the flat spacetime is given by the non-local generalization of a non-abelian gauge theory on S^d: the theory now has a non-local kinetic term like (14). Indeed, a similar non-local structure is known to appear in open string field theory (see e.g.[17]). Moreover, the unconventional kinetic term in (14) is natural when we rewrite our flat space metric into $ds^2_{\mathbb{R}^{d+1}} = \frac{dr^2}{r^2} + (\log r)^2 d\Omega_d^2$ and draw a parallel with AdS metric $ds^2_{AdS_{d+1}} = \frac{dr^2}{r^2} + r^2 d\vec{x}^2$: the boundary kinetic terms of these two spaces scale the same since $e^{\partial_\Omega} \sim r \sim \partial_x$. This comparison also shows that $\rho = \log r$ should be regarded as the energy scale, thus ρ_∞ is the UV energy cut-off — this corresponds to the UV cut-off from the viewpoint of open string theory [18]. This argument can be viewed as a logarithmic generalization of the holographic duals of Lifshitz-like fixed points introduced recently in [19].

III. CORRELATION FUNCTIONS

Another important quantity in establishing the holography is the correlation function. Here we will compute holographic correlation functions assuming a natural extension of the Euclidean bulk-to-boundary relation [3] to our flat space (1). We impose the UV cut-off at $\rho = \rho_\infty$ as before.

Consider a massless scalar in the bulk:

$$S_{bulk} = \frac{1}{32\pi G_N^{(d+1)}} \int d^{d+1}x \sqrt{g} \partial_\mu \phi \partial^\mu \phi, \qquad (15)$$

which is normalized similarly to Einstein-Hilbert action. Now we follow the bulk-to-boundary procedure to compute the boundary two-point functions. We impose the Dirichlet boundary condition: $\phi(\rho_\infty, \Omega_d) = \phi_\infty(\Omega_d)$. The bulk-to-boundary propagator K defined by

$$\phi(\rho, \Omega_d) = \lim_{\rho' \to \rho_\infty} \int K(\rho, \Omega_d; \rho', \Omega'_d) \cdot \phi(\rho', \Omega'_d) \cdot \rho'^d \, d\Omega'_d, \qquad (16)$$

is given by the derivative of the bulk Green's function under Dirichlet boundary condition ($G(\rho, \Omega_d; \rho', \Omega'_d) = 0$ at $\rho = \rho_\infty$ or $\rho' = \rho_\infty$):

$$K(\rho, \Omega_d; \rho', \Omega'_d) = \frac{\partial}{\partial \rho'} \Big[\frac{1}{(\rho^2 + \rho'^2 - 2\rho\rho' \cos\Delta\theta)^{\frac{d-1}{2}}} - \frac{1}{((\frac{\rho\rho'}{\rho_\infty})^2 + \rho_\infty^2 - 2\rho\rho' \cos\Delta\theta)^{\frac{d-1}{2}}} \Big] \cdot \frac{1}{(d-1)\mathcal{A}_d}, \qquad (17)$$

where $\cos\Delta\theta = \vec{\Omega}_d \cdot \vec{\Omega}'_d$ and $\mathcal{A}_d$ is the surface area of the unit sphere S^d. Then the on-shell action is:

$$\begin{aligned} S_{bulk}^{on-shell} &= \lim_{\rho \to \rho_\infty} \frac{1}{32\pi G_N^{(d+1)}} \int \rho^d d\Omega_d \phi(\rho, \Omega_d) \partial_\rho \phi(\rho, \Omega_d) \\ &= \mathcal{N} \cdot \frac{\rho_\infty^{d-1}}{G_N^{(d+1)}} \int d\Omega_d d\Omega'_d \, \frac{\phi_\infty(\Omega_d)\phi_\infty(\Omega'_d)}{(1-\cos\Delta\theta)^{\frac{d+1}{2}}}, \end{aligned} \qquad (18)$$

where $\mathcal{N} \equiv (2^{\frac{d+9}{2}} \pi \mathcal{A}_d)^{-1}$. Therefore the boundary two-point function is

$$\langle \hat{O}(\Omega_d) \hat{O}(\Omega'_d) \rangle = \mathcal{N} \cdot \frac{\rho_\infty^{d-1}}{G_N^{(d+1)}} \frac{1}{(1-\cos\Delta\theta)^{\frac{d+1}{2}}}. \qquad (19)$$

This agrees with the analysis in [20], though our interpretations are slightly different. Note that the above result contains only a divergent term. Therefore we argue that after adding non-local boundary counter-terms to cancel the divergence, the physical two-point function is actually vanishing. Non-local counter-terms have also been used in the holography of NS5-branes [4, 21].

Next consider a massive scalar defined by the action on $\mathbb{R}^{d+1}$

$$S_{bulk} = \frac{1}{32\pi G_N^{(d+1)}} \int d^{d+1}x\sqrt{g}(\partial_\mu\phi\partial^\mu\phi + M^2\phi^2). \quad (20)$$

We impose Dirichlet boundary condition again and decompose the boundary scalar field as: $\phi_\infty(\Omega_d) = \sum_{l,\vec{m}} c_{l,\vec{m}} Y_{l,\vec{m}}(\Omega_d)$, where $Y_{l,\vec{m}}(\Omega_d)$ are the orthonormal spherical harmonics on S^d. Under this boundary condition, the equation of motion

$$(\Delta - M^2)\phi = \frac{1}{\rho^d}\partial_\rho(\rho^d\partial_\rho\phi) - \left(M^2 + \frac{l(l+d-1)}{\rho^2}\right)\phi = 0, \quad (21)$$

is solved as follows

$$\phi(\rho,\Omega_d) = \sum_{l,\vec{m}} c_{l,\vec{m}} \frac{\rho^{\frac{1-d}{2}} I_{l+\frac{d-1}{2}}(M\rho)}{\rho_\infty^{\frac{1-d}{2}} I_{l+\frac{d-1}{2}}(M\rho_\infty)} Y_{l,\vec{m}}(\Omega_d), \quad (22)$$

where the modified Bessel function of the first kind $I_\nu(z)$ was chosen to avoid singularity at $\rho = 0$. Then the same bulk-to-boundary procedure produces the boundary two-point function:

$$\langle\hat{O}(\Omega_d)\hat{O}(\Omega'_d)\rangle = \frac{\rho_\infty^d}{32\pi G_N^{(d+1)}} \sum_{l,\vec{m}} F_l(\rho_\infty)\bar{Y}_{l,\vec{m}}(\Omega_d)Y_{l,\vec{m}}(\Omega'_d) \quad (23)$$

where $F_l(\rho) \equiv \partial_\rho \log\left(\rho^{\frac{1-d}{2}} I_{l+\frac{d-1}{2}}(M\rho)\right)$

First, (23) correctly reduces to the massless case (19) when $M = 0$ since $\lim_{\rho_\infty\to\infty}[\rho_\infty F_l(\rho_\infty)] \mid_{M=0} = l$. For the massive scalar, we find that in the limit $\rho_\infty \to \infty$, $\rho_\infty^d F_l(\rho_\infty)$ takes a non-zero finite value after counter-terms are added to cancel the divergence. Using the asymptotic expansion of Bessel function we see that this finite term is given by a degree-$[\frac{d}{2}]$ polynomial of $l(l+d-1)$, which is the eigenvalue of $-\Delta$. For example, when $d = 2$,

$$\rho_\infty^2 F_l(\rho_\infty) \simeq M\rho_\infty^2 - \rho_\infty + \frac{l(l+1)}{2M} + O(\rho_\infty^{-1}). \quad (24)$$

Then using the identity $\sum_{l,\vec{m}} \bar{Y}_{lm}(\Omega_1)Y_{lm}(\Omega_2) = \delta(\Omega_1 - \Omega_2)$, we conclude that the two-point functions consist of δ-functions and their derivatives by Laplacian. Therefore, we argue that the holographic correlation functions for a massive scalar are essentially zero.

Next one could explicitly compute higher-point functions following the bulk-to-boundary principle. However, assuming the dilaton-type massless scalar Lagrangian of the form $\mathcal{L} = (\partial_\mu\phi)^2 P(\phi)$ where $P(\phi)$ is a polynomial, it is easy to see that the results always scale as ρ_∞^{d-1}. Following the same argument as before, they can all be eliminated by adding boundary counter-terms.

In summary, we argue that all n-point correlation functions vanish after counter-terms are added to cancel the divergences. This seems surprising until one recalls our previous observation from the holographic entanglement entropy: A is maximally entangled with B when the size of A approaches zero. Define an infinitesimally small subsystem A as the sum of the n points in the correlation function: $A = x_1 \cup \ldots \cup x_n$. Our previous result implies that in this case the entropy S_A for ρ_A becomes maximal, therefore the matrix ρ_A factorizes into a direct product $\rho_A = \rho_{x_1} \otimes \ldots \otimes \rho_{x_n}$, as in a system at an infinitely high temperature. Thus all correlation functions vanish:

$$\langle\hat{O}(x_1)\ldots\hat{O}(x_n)\rangle \equiv \mathrm{Tr}[\rho_A\hat{O}(x_1)\ldots\hat{O}(x_n)] = 0. \quad (25)$$

IV. DISCUSSION

In the Lorentzian version of (1) given by $ds^2 = d\rho^2 + \rho^2(-dt^2 + \cosh^2 t\ d\Omega_{d-1}^2)$, a static observer at $\rho = \rho_0$ detects a thermal state at the Unruh temperature $T_U = \frac{1}{2\pi\rho_0}$. From this viewpoint we can rewrite the entanglement entropy S_A for maximal size A ($\alpha = \pi$) as follows

$$S_A = \frac{2^{-d-1}\pi^{\frac{1-d}{2}}}{\Gamma(\frac{d+1}{2}) \cdot G_N^{(d+1)} \cdot T_U^{d-1}}, \quad (26)$$

The UV cut-off is shifted to a finite value now. Since S_A measures the amount of information hidden in the subsystem B, which is inaccessible to an observer in A, one expects that it is closely related to the entropy of Schwarzschild black hole with temperature $T_{BH} = T_U$. Indeed, the entropy of the $(d+1)$-dimensional Schwarzschild black hole is

$$S_{BH} = \frac{2^{-2d+1}\pi^{\frac{2-d}{2}}(d-2)^{d-1}}{\Gamma(\frac{d}{2}) \cdot G_N^{(d+1)} \cdot T_{BH}^{d-1}},$$

which agrees with S_A up to a numerical constant. Thus our holographic interpretation is consistent with black hole entropies. This consideration also suggests a string theory interpretation of our holography. In AdS/CFT [2], the holographic dual theory comes from the D-branes that originally sit at the horizon $r = 0$. In our flat spacetime, in the limit of $\rho \to 0$, the Unruh temperature T_U becomes infinitely large and the corresponding observer will detect pair creations of many D-branes. Therefore it is tempting to speculate that the open string theory for them is the non-local field theory that we conjecture to be the holographic dual of the flat spacetime.

Finally, let us examine the connection between UV cut-off in the field theory and the cut-off radius of the bulk more closely. Matching the entanglement entropy obtained from the holographic computation ((3) with $\alpha = \pi$) and the field theory one ((13) with $q = \frac{1}{2}$), we see that if we switch to dimensionless coordinates defined by

$\tilde{\rho} \equiv \frac{\rho}{R}$ where R is a length unit, and accordingly consider the boundary theory on a unit sphere S^d with dimensionless lattice spacing $\tilde{a} = \frac{a}{R}$, we can identify:

$$\tilde{a} = \frac{1}{\tilde{\rho}_\infty}. \tag{27}$$

Now S_A, interpreted as the entanglement entropy for the holographic dual on the unit sphere S^d, scales as

$$S_A \sim \frac{R^{d-1}}{G_N^{(d+1)}} \cdot (\tilde{\rho}_\infty)^{d-1} \sim \frac{n}{\tilde{a}^{d-1}}, \tag{28}$$

where the dimensionless number $n \sim \frac{R^{d-1}}{G_N^{(d+1)}}$ counts the number of fields in the holographic dual. Since the bulk metric $ds^2 = R^2(d\tilde{\rho}^2 + \tilde{\rho}^2 d\Omega_d^2)$ is invariant under the rescaling $(R, \tilde{\rho}) \to (R\lambda, \tilde{\rho}/\lambda)$ for arbitrary λ, there exists a corresponding symmetry in the holographic dual theory:

$$(n, \tilde{a}) \to (\lambda^{d-1} n, \lambda \tilde{a}). \tag{29}$$

Note that the total number of degrees of freedom in the boundary field theory is proportional to $\frac{n}{\tilde{a}^{d-1}}$ therefore remains unchanged. This symmetry suggests that the theory is highly non-local and entangled and will be useful when we go on to identify the precise holographic dual. We leave these questions for future study.

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