Controlled directed particle current despite time-reversal symmetry

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Quantum ratchets are systems that exhibit asymptotic currents when they are driven by a time-periodic potential of zero mean if the proper spatio-temporal symmetries are broken. The current arises from the desymmetrization of the Floquet cyclic states, eigenstates of the unitary evolution operator for one period of the driving force. However, there has been recent debate on whether directed currents may arise even with potentials which do not break these symmetries. We show here that crossed terms in the Floquet basis can induce long-lasting directed currents in the presence of structural degeneracies in the quasienergy spectrum even if the time reversal symmetry is not broken. We present a highly controllable model which can be realized with ultracold atoms in optical lattices, which shows directed currents with amplitudes tunable over a large range and where the time scale of the current reversal can be controlled. We show that one can not only control the momentum generation but also that the kinetic energy of the system can be decreased in a controlled way.

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Brownian motors or ratchets are spatially periodic systems with noise and/or dissipation in which a directed current of particles can emerge from an unbiased zero-mean external force [1, 2]. Models for biological engines that transform chemical energy into unidirectional mechanical motion behave as Brownian motors [3]. Extensive studies of the ratchet effect in classical systems [4] stated the relation between symmetry breaking potentials and the existence of the asymptotic current. Directed transport is only possible if one breaks the following symmetries of the classical trajectories of the system $S_a : (x, p, t) \to (-x, -p, t + T/2)$ where T is the period of the driving potential and the time-reversal symmetry $S_b : (x, p, t) \to (x, -p, -t)$ [5]. Lately there has been an increasing interest in the coherent ratchet effect in Hamiltonian quantum systems [6]. It has been shown that the same symmetry requirements apply to them [7], i.e. if the Hamiltonian preserves S_a and/or S_b symmetry, no asymptotic current is possible.

Experimentally, directed current generation was first studied in solid state devices, quantum dots and Josephson junctions [8]. More recently, the precise control achievable in cold atom experiments opened up the possibility of realizing directed atomic currents for Hamiltonian systems with controllable or no dissipation in the time scale of the measurements [9–11]. Recently, a very clean realization of a coherent quantum ratchet was experimentally demonstrated in a Bose-Einstein condensate exposed to a sawtooth potential realized with an optical lattice which was periodically modulated in time [12]. Directed transport of atoms was observed when the driving lattice potential broke the spatio-temporal symmetries. The current oscillations and the dependence of the current on the initial time and the resonant frequencies [13] were measured, demonstrating the quantum character of the ratchet.

Although the generation of an asymptotic directed current needs the breaking of the spatio-temporal symmetries S_a and S_b simultaneously, there has been a recent discussion on the possibility of obtaining long-lasting directed currents without it [14–17]. The analyzed schemes involve either strong interparticle interactions such that the directed atom transport endures over a time scale related to the validity of the mean-field description applied [14] or strong driving fields in the presence of an accidental degeneracy in the quasienergy spectrum [15–17]. We show here that one can exploit a structural degeneracy in the quasienergy spectrum in order to generate long-lasting currents without breaking the symmetries of the system. In contrast to previous works [14–17] we obtain directed current for a weakly driven system. In addition, the tunability of our model allows for the control of not only the amplitude and oscillation period of the generated current, but also of the amount of kinetic energy in the system.

One useful way of treating time-periodic quantum Hamiltonians, $H(t) = H(t + T)$, is the Floquet formalism [18]. The Floquet states or cyclic states $|\phi_j(t)\rangle = |\phi_j(t + T)\rangle$ are the eigenstates of the evolution operator for one period while the quasienergies ε_j are the eigenvalues. It is precisely the symmetry breaking needed for the creation of an asymptotic current that implies the desymmetrization of Floquet states of the system [7]. Even if the symmetries are not broken, we show here that it is possible to populate a superposition of cyclic eigenstates with non-zero average momentum for experimentally relevant times by taking advantage of a resonance condition of the Floquet states. The solution to the time-dependent Schrödinger equation $H(t)\,|\psi(t)\rangle = i\hbar\partial\,|\psi(t)\rangle/\partial t$ can be spanned in the cyclic eigenbasis ($\hbar = 1$)

$$|\psi(t)\rangle = \sum_j e^{-i\varepsilon_j t} c_j \,|\phi_j(t)\rangle \qquad (1)$$

where $c_j = \langle\phi_j(0)|\psi(0)\rangle$ [18]. The average current generated during n cycles is given by $\mathcal{I}(t = nT) = \frac{1}{n}\sum_{m=1}^{n}\mathcal{I}_m$ with

$$\mathcal{I}_m = \frac{1}{T}\int_{(m-1)T}^{mT}\langle p(t)\rangle dt, \qquad (2)$$

where the expectation value of the momentum operator at time t reads $\langle p(t)\rangle = \sum_{j,j'} c_j c_{j'}^* e^{-it(\varepsilon_j - \varepsilon_j')}\langle\phi_{j'}(t)|\, p\, |\phi_j(t)\rangle$. Note that due to the periodicity of the cyclic states the average current during n cycles can be simplified in terms of integrals of the cyclic states during the first period

$$\mathcal{I}(nT) \;=\; \frac{1}{n}\sum_{j,j'} c_j c_{j'}^* \langle p\rangle_{jj'} \frac{1 - e^{-inT(\varepsilon_j - \varepsilon_j')}}{1 - e^{iT(\varepsilon_j - \varepsilon_j')}}, \qquad (3)$$

$$\langle p\rangle_{jj'} \;\equiv\; \frac{1}{T}\int_0^T \langle\phi_j(t)\,|\,p\,|\,\phi_{j'}(t)\rangle\, e^{-it(\varepsilon_j - \varepsilon_j')}dt. \qquad (4)$$

In general, the oscillatory off-diagonal terms decay rapidly [19] and for long times only the diagonal terms in Eq. (3) remain. If both S_a and S_b are broken, the cyclic eigenstates desymmetrise and carry net momentum, i.e. $\langle p\rangle_{jj} \neq 0$ [7]. In such case, the asymptotic average current, $\mathcal{I}(\infty) = \sum_j |c_j|^2\langle p\rangle_{jj}$, is nonzero. Correspondingly, if either of the relevant symmetries is not broken $\langle p\rangle_{jj} = 0$ and thus $\mathcal{I}(\infty) = 0$. Note, however, that the off-diagonal terms in Eq. (3) become relevant if the initial state projects into degenerate or quasidegenerate cyclic states with $\langle p\rangle_{jj'} \neq 0$. This fact can give rise to measurable long-lasting currents if one tunes the parameters of the model in an appropriate way.

To illustrate this we consider a system of non-interacting bosons in a lattice of L sites with periodic boundary conditions driven by a time-periodic potential such that $H(t) = H_0 + V(t)$ and

$$H_0 \;=\; -J\sum_{i=1}^{L} |i\rangle\langle i+1| + |\,i+1\rangle\langle i\,|, \qquad (5)$$

$$V(i,t) \;=\; V\sin(\omega t)\left[\sin(Mx_i) + \alpha\sin(2Mx_i + \phi)\right], \qquad (6)$$

where J is the tunneling probability, $x_i = 2\pi i/L$, M is an integer and $\omega = 2\pi/T$. This driving potential does not break the time reversal symmetry S_b and thus no asymptotic current is expected [7]. Consequently $\langle p\rangle_{jj} = 0$ and the current generated after a few cycles arises only from the off-diagonal terms in Eq. (3). In order to enhance the off-diagonal contribution to the current we need to tune the system to resonance. We use the resonant condition $2\omega = E_{2M} - E_0$ introduced for $M = 1$ in [15, 20], where $E_k = -2J\cos(2\pi k/L)$ with $k = [-k_{\max}, k_{\max}]$ are the eigenenergies of H_0 with corresponding momentum eigenvectors $\langle i\,|\,k\rangle = e^{-ikx_i}/\sqrt{L}$ and $k_{\max} = (L-1)/2$ for L odd. We consider an initial state of zero momentum $|\,\psi(0)\rangle = |\,0\rangle$. Note that the introduction of $M \leq k_{\max}$ in the driving potential allows for the coupling of the initial state to very high momentum states $|\,M\rangle$ and $|\,2M\rangle$. We show in figure 1 the average current per cycle $\mathcal{I}_m$, and the integrated average current $\mathcal{I}(t)$ as a function of the number of cycles. We note that the average current per cycle Eq. (2) oscillates and achieves values on the order of $2M$ recoil units. On the other hand, the integrated current vanishes for long times, as the potential does not break the time-reversal S_b symmetry [7]. We observe that the current changes direction with a sign change in ϕ and that for the weak driving strength used here the current is zero if $\phi = 0$ for any initial time phase shift.

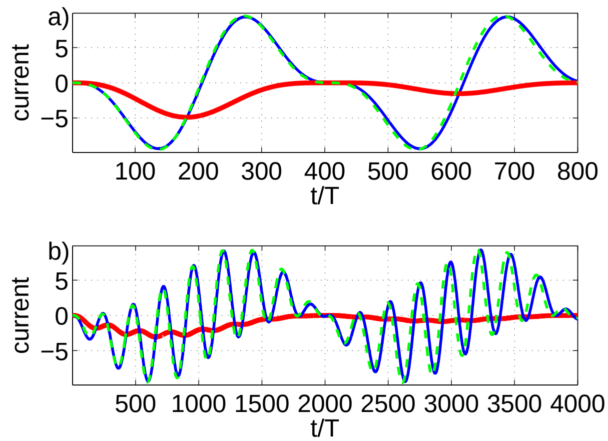

FIG. 1: Average current in recoil units (thick line) in Eq. (3) as a function of time for an initial zero momentum state. Thin solid line is the current average per cycle in Eq. (2) obtained with a numerical integration of the Schrödinger equation using Floquet theory for $M = 5$, $L = 41$, $V/\omega = 0.1$ and $\alpha = 1.2$. Dashed line corresponds to the current of the effective 3-level system in Eq. (7) and Eq. (8). The average current per cycle depends only on M/L, its amplitude scales with M and the oscillation time scales with $(\omega/V)^2 T$ where $T \simeq \pi J^{-1}$. a) $\phi/\pi = 0.2579$ and b) $\phi/\pi = 1/20$.

Close to the resonance and for weak driving potential $V/J < 1$ [20], the dynamics of the system involve only three cyclic states $\{|\,s_{2M}, 2\rangle, |\,0,0\rangle, |\,a_{2M}, 2\rangle\}$, where $\langle i,t\,|\,a[s]_k, n\rangle = \sqrt{2/L}\sin(kx_i)[\cos(kx_i)]e^{-in\omega t}$. Following [20] we do time-independent perturbation theory in Floquet space, using the $\mathcal{T}$-matrix approach $\mathcal{T}(\epsilon) = V + VG_0(\epsilon)\mathcal{T}$ [21], where $G_0(\epsilon) = \sum_j \frac{|\,j\rangle\langle j\,|}{\epsilon - \varepsilon_j^0}$ and $|\,j\rangle = |\,k,n\rangle$ with corresponding $\varepsilon_j^0 \equiv E_k - n\omega$. The system dynamics are governed by an effective Hamiltonian given by the $\mathcal{T}$-matrix obtained after expanding to second order in V around the ground state quasienergy $\epsilon = \varepsilon_0^0$,

$$\frac{4\mathcal{T}}{V^2} = \begin{pmatrix} a + d\alpha^2 + e\alpha^2\sin^2\phi & b & i\frac{e}{2}\alpha^2\sin 2\phi \\ b & c + f\alpha^2 & 0 \\ -i\frac{e}{2}\alpha^2\sin 2\phi & 0 & a + d\alpha^2 + e\alpha^2\cos^2\phi \end{pmatrix}. \qquad (7)$$

Here the constants a to c are due to coupling through $\sin(Mx_i)$ whereas d to e involve coupling through

$\sin(2Mx_i)$. All of them are real numbers, sums of inverse quasienergy differences which depend only on M/L. If $M \leq k_{\max}/4$, they are of the same order $\mathcal{O} \sim 1/\omega$. From Eq.(7) one extracts that the quasienergies $\varepsilon_j \simeq \frac{V^2}{\omega}$. Hence, the time scale of the dynamics depends only on the ratio M/L and scales as $\sim \omega/V^2$. Note that we need $\phi \neq 0$ in order to mix the asymmetric and symmetric momentum states in Eq. (7). As we show below, this is a key element in the generation of particle motion when the initial state has a defined momentum symmetry as in the ratchet problem when the initial state is the zero momentum eigenstate.

For an initial state $|\,\psi(0)\rangle = |\,0\rangle$, the average current at cycle m after evolution with the effective Hamiltonian Eq.(7) reduces to a sum of 3 oscillations with different frequencies and the same weight

$$\mathcal{I}_m \;=\; \mathcal{C}(\alpha,\phi)F_m(\alpha,\phi), \tag{8}$$

where $\mathcal{C}(\alpha,\phi) = 4ic_1^*c_2\langle p\rangle_{12}$, $F_m = \sum_{j<j'}\sin(mT\Delta\varepsilon_{jj'})$ and $\Delta\varepsilon_{jj'} \equiv (-1)^{j+j'}(\varepsilon_j - \varepsilon'_j)$. As shown in figure 1, the three-mode approximation Eq. (8) fits perfectly the exact numerical results.

Let us now analyze the two parts of Eq. (8). The amplitude $\mathcal{C}$ is shown in the left panel of figure 2 for different parameters α and ϕ . For $M = k_{\max}/4$, it attains its maximum at $\alpha \simeq 1$ when both terms of the driving potential have the same weight. We observe that the current is periodic in ϕ with π periodicity and has vanishing values for $\phi = l\pi/2$ with l integer. This is due to the fact that for weak driving, when only these three cyclic eigenstates are involved the dynamics, the only way to obtain nonvanishing values of the momentum is when the eigenvectors of $\mathcal{T}$ mix both symmetric and asymmetric basis states. One thus needs that $\langle s_{2M}, 2\,|\,\mathcal{T}\,|\,a_{2M}, 2\rangle \neq 0$ and hence $\phi \neq l\pi/2$ in order to obtain particle motion. Note that for strong driving potential this mixing can be achieved through coupling to other states. However, the current obtained in this way is noticeable smaller and highly dependent on accidental degeneracies [17].

The current $\mathcal{I}_m$ is a sum of three sinusoidals F_m whose arguments sum up to 0 as shown in the right panel of figure 2. By tuning ϕ we can find different situations for the oscillation frequencies $\Delta\varepsilon_{jj}$: i) There is a particular ϕ when two slow frequencies $\Delta\varepsilon_s$ degenerate and become half the fast frequency. In such case $\sum_{m=1}^{n} F_m = 4\sin^2(nT\Delta\varepsilon_f/2)\sin^2((n+1)T\Delta\varepsilon_f/2)/\sin(T\Delta\varepsilon_f)$ and the average current $\mathcal{I}$ achieves its maximum after $nT = 2\pi/(3\Delta\varepsilon_f)$ cycles. This case is shown in figure 1 a). ii) Around $\phi \simeq 0$, there is a slow frequency $\Delta\varepsilon_s \to 0$ and 2 fast frequencies of opposite sign $\Delta\varepsilon_f$. In this case shown in 1 b) the current $\mathcal{I}_m$ oscillates at the fast frequency $\Delta\varepsilon_f$ modulated with a slow frequency envelope. The average current $\mathcal{I}$ is modulated by

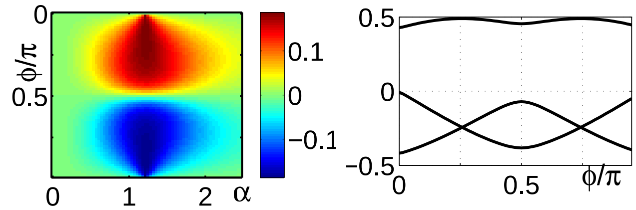

FIG. 2: Left panel: Amplitude $\mathcal{C}/4M$ of the current in Eq.(8) for different values of the potential parameters ϕ and α for an initial zero momentum state and $L = 41$, $M = 5$. Right panel: $\Delta\varepsilon_{13}$, $\Delta\varepsilon_{12}$, $\Delta\varepsilon_{23}$ in units of ω/V^2 as a function of ϕ for $\alpha = 1.2$, $M/k_{\max} = 4$. For fixed values ϕ and α the average current per cycle in Eq. (8) is a sum of sinusoidals with these frequencies with an amplitude shown in the left panel.

the slow frequency $\Delta\varepsilon_s$ and attains its maximum at $nT = 2\pi/(3|\Delta\varepsilon_s|)$. iii) any other value of ϕ results in an intermediate situation. The maxima of $\mathcal{I}_m$ is similar for all the above situations, but in case i) the maximum, with tunable amplitude $\sim 2M$, is reached earlier at a time which can be independently adjusted with V/ω. This leads to a higher and smoother average current $\mathcal{I}$ (fig. 1).

In the context of cold atoms it may be of interest not only the generation of a current from an initial zero momentum state, but also the control of the quantum state of the system. We have explored the capabilities of our model to control the momentum and the kinetic energy in the system. We plot in figure 3 the projection of the particle state in the momentum basis at different times for different initial states. We observe in figure 3 a) that the zero momentum state can be indeed converted into an almost pure momentum state $|\pm 2M\rangle$, eigenstate of H_0, at times $t = 66T$ and $t = 132T$. One could then switch off the driving and use this scheme to generate an asymptotic current. It is shown in figure 3 b) that one could reduce an initial state with finite momentum into a zero momentum eigenstate. Thus the application of the driving potential slows down a particle beam and reduces the kinetic energy of the system. It can be seen from figure 3 c) that the average tunneling energy per cycle reduces almost to zero for an initial state $|\,s_{2M}\rangle$ after 57 cycles. All these are examples of the high controllability of our system and open up the possibility of using this scheme for slowing down particle beams or cooling atomic or even molecular systems.

Finally, let us analyze the feasibility of the model. We have shown that starting from an initial zero momentum state one can obtain long-lasting tunable currents when two requirements are met: the system is tuned to resonance and the driving field is weakly coupled. The resonance condition yields for $M = k_{\max}/4$ values of $T \simeq \pi J^{-1}$. We obtain currents with amplitudes which can be tuned up to approximately half the maximum

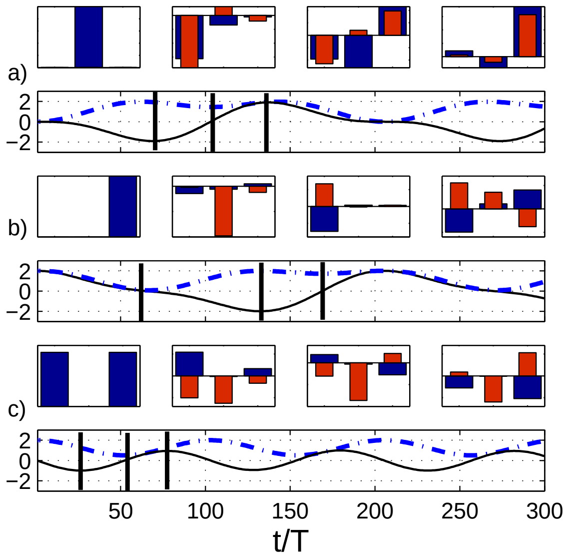

FIG. 3: Upper panel: Real (thick bar) and imaginary part (thin bar) of the $\{\langle -2M\,|\,, \langle 0\,|\,, \langle 2M\,|\}\,|\,\psi(t)\rangle$ at $t = 0$ and at the times showed by vertical lines in the lower panel. Lower panel: Average current per cycle $\mathcal{I}_m/M$ in recoil units and average tunneling energy per cycle (dashed line) in units of $1/\omega$ as a function of time. a) $|\,\psi(0)\rangle = |\,0\rangle$ b) $|\,\psi(0)\rangle = |\,2M\rangle$ c) $|\,\psi(0)\rangle = |\,s_{2M}\rangle$.The parameters used are $M = 5$, $L = 41$, $\phi/\pi = 0.257$, $V/\omega = 0.1$ and $\alpha = 1.2$.

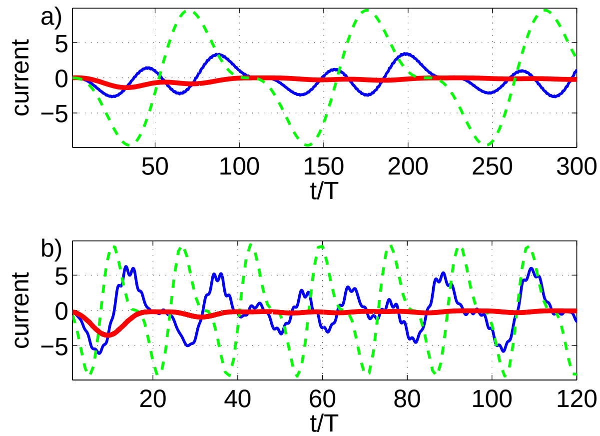

FIG. 4: Average current in recoil units (thick solid line) in Eq. (3) as a function of time for an initial zero momentum state. The thin solid line is the current average at each cycle Eq. (2) obtained with a numerical integration of the Schrödinger equation using Floquet theory for $M = 5$, $L = 41$ and $\alpha = 1.2$. Dashed line corresponds to the current of the effective 3-level system in Eq. (7) and Eq.(8). a) $V/\omega = 0.2$, $\phi/\pi = 0.2562$ and $\omega = 1.01(E_{2M} - E_0)/2$ b) $V/\omega = 0.5$, $\phi/\pi = 0.2579$.

achievable momentum in the lattice $k_{\max}$. The current oscillation period scales with the coupling strength as $\simeq (\omega/V)^2 T$. Note that this time scale can be tuned to fit the experimental observation time. It can be enlarged at will for smaller couplings while the weak coupling condition $V/\omega < 1$ sets a lower limit. To show the robustness of the method we plot in figure 4 the current when the system is not perfectly tuned to resonance and for stronger driving fields. We find that couplings up to $V/\omega = 0.5$ and errors of 1% in the resonant frequency still give rise to strong particle currents.

We have presented a model system where one can obtain currents with amplitudes orders of magnitude larger than those observed in recent experiments with coherent ratchet currents [12]. The oscillation period of the current can be controlled by the amplitude of the driving potential and in particular, by decreasing the driving strength one obtains currents which do not decay during the lifetime of the experiment. This effect is obtained with a potential which does not break the time-reversal symmetry and is due to crossed terms between the cyclic eigenstates of the system. The proposed scheme requires that the system is tuned to resonance and that the driving potential is weakly coupled such that only a few cyclic eigenstates are involved in the dynamics. We have checked the robustness of the method by tuning the system out of resonance and increasing the coupling strength. Furthermore, we have shown that it is possible to control the quantum state and the amount of kinetic energy in the system, using the proposed scheme to convert a zero momentum state into a state with high finite momentum, and viceversa. Our results show that the use of unbiased oscillatory potentials for slowing down or cooling particle beams deserves further study.

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