CHARACTERIZATIONS OF Γ-AG**-GROUPOIDS BY THEIR Γ-IDEALS

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Abstract. In this paper we have discusses Γ-left, Γ-right, Γ-bi-, Γ-quasi-, Γ-interior and Γ-ideals in Γ-AG**-groupoids and regular Γ-AG**-groupoids. Moreover we have proved that the set of Γ-ideals in a regular Γ-AG**-groupoid form a semilattice structure. Also we have characterized a regular Γ-AG**-groupoid in terms of left ideals.

1. Introduction

Kazim and Naseeruddin [4] have introduced the concept of an LA-semigroup. This structure is the generalization of a commutative semigroup. It is closely related with a commutative semigroup and commutative groups because if an LA-semigroup contains right identity then it becomes a commutative semigroup and if a new binary operation is defined on a commutative group which gives an LA-semigroup [9]. The connection of the class of LA-semigroups with the class of vector spaces over finite fields and fields has been given as: Let W be a sub-space of a vector space V over a field F of cardinal $2r$ such that $r > 1$. Many authors have generalized some useful results of semigroup theory.

In 1981, the notion of Γ-semigroups was introduced by M. K. Sen [6] and [7].

T. Shah and I. Rehman [14] defined Γ-AG-groupoids analogous to Γ-semigroups and then they introduce the notion of Γ-ideals and Γ-bi-ideals in Γ-AG-groupoids. It is easy to see that Γ-ideals and Γ-bi-ideals in Γ-AG-groupoids are infect a generalization of ideals and bi-ideals in AG-groupoids (for a suitable choice of Γ).

In this paper we define Γ-quasi-ideals and Γ-interior ideals in Γ-AG**-groupoids and generalize some results. Also we have proved that Γ-AG-groupoids with left identity and AG-groupoids with left identity coincide.

Let G and Γ be two non-empty sets. G is said to be a Γ-AG-groupoid if there exist a mapping $G \times \Gamma \times G \to G$, written $(a,\, \gamma,\, b)$ as $a\gamma b$, such that G satisfies the identity $(a\gamma b)\,\delta c = (c\gamma b)\,\delta a$, for all $a,\, b,\, c \in G$ and $\gamma,\, \delta \in \Gamma$ [14].

Definition 1. *An element $e \in S$ is called a left identity of Γ-AG-groupoid if $e\gamma a = a$ for all $a \in S$ and $\gamma \in \Gamma$.*

Lemma 1. *If a Γ-AG-groupoid contains left identity, then it becomes an AG-groupoid with left identity.*

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Proof. Let G be a Γ-AG-groupoid and e be the left identity of G and let $a, b \in G$ and $\alpha, \beta \in \Gamma$ therefore we have

$$a\alpha b = a\alpha(e\beta b) = e\alpha(a\beta b) = a\beta b.$$

Hence Γ-AG-groupoid with left identity becomes and an AG-groupoid with left identity. □

Remark 1. *From Lemma 1, it is easy to see that all the results given in* [14] *and* [15] *for a* Γ*-AG-groupoid with left identity is identical to the results given in* [10] *and* [11].

Definition 2. *A* Γ*-AG-groupoid is called a* Γ*-AG****-groupoid if it satisfies the following law*

$$a\alpha(b\beta c) = b\alpha(a\beta c), \text{ for all } a, b, c \in S \text{ and } \alpha, \beta \in \Gamma.$$

The following results and definition from definition 3 to lemma 3 have been taken from [14].

Definition 3. *Let G be a* Γ*-AG-groupoid, a non-empty subset S of G is called sub* Γ*-AG-groupoid if $a\gamma b \in S$ for all a, $b \in S$ and $\gamma \in \Gamma$ or S is called sub* Γ*-AG-groupoid if $S\Gamma S \subseteq S$.*

Definition 4. *A subset I of a* Γ*-AG-groupoid G is called left(right)* Γ*-ideal of G if $G\Gamma I \subseteq I$ $(I\Gamma G \subseteq I)$ and I is called* Γ*-ideal of G if it is both left and right* Γ*-ideal.*

Definition 5. *An element a of a* Γ*-AG-groupoid G is called regular if there exist $x \in G$ and β, $\gamma \in \Gamma$ such that $a = (a\beta x)\gamma a$. G is called regular* Γ*-AG-groupoid if all elements of G are regular.*

Definition 6. *A sub* Γ*-AG-groupoid B of a* Γ*-AG-groupoid G is called* Γ*-bi-ideal of G if $(B\Gamma G)\Gamma B \subseteq B$.*

Definition 7. *Let G and* Γ *be any non-empty sets. If there exists a mapping $G \times \Gamma \times G \to G$, written (x, γ, y) as $x\gamma y$, G is called a* Γ*-medial if it satisfies $(x\alpha y)\beta(l\gamma m) = (x\alpha l)\beta(y\gamma m)$, and called* Γ*-paramedial if it satisfies $(x\alpha y)\beta(l\gamma m) = (m\alpha l)\beta(y\gamma x)$ for all x, y, l, $m \in G$ and α, β, $\gamma \in \Gamma$.*

Lemma 2. *If A and B are any* Γ*-ideals of a regular* Γ*-AG-groupoid G then $A\Gamma B = B\Gamma A$.*

Definition 8. *A* Γ*-ideal P of a* Γ*-AG-groupoid G is called* Γ*-prime(*Γ*-semiprime) if for any* Γ*-ideals A and B, $A\Gamma B \subseteq P$ $(A\Gamma A \subseteq P)$ implies either $A \subseteq P$ or $B \subseteq P$ $(A \subseteq P)$.*

Lemma 3. *Any* Γ*-ideal A of a regular* Γ*-AG-groupoid is a* Γ*-idempotent that is $A\Gamma A = A$.*

It is important to note that every Γ-AG-groupoid G is Γ-medial and every Γ-AG**-groupoid G is Γ-paramedial because for any x, y, l, $m \in G$ and α, β, $\gamma \in \Gamma$, we have

$$(x\alpha y)\beta(l\gamma m) = ((l\gamma m)\alpha y)\beta x = ((y\gamma m)\alpha l)\beta x = (x\alpha l)\beta(y\gamma m).$$

We call it as Γ-medial law.

Theorem 1. *If L and R are left and right* Γ*-ideals of a* Γ*-AG****-groupoid G then $L \cup L\Gamma G$ and $R \cup G\Gamma R$ are* Γ*-ideals of G.*

Proof. Let L be a left Γ-ideal of G then we have

$$\begin{aligned}(L \cup L\Gamma G)\,\Gamma G &= (L\Gamma G) \cup (L\Gamma G)\,\Gamma G = (L\Gamma G) \cup (G\Gamma G)\,\Gamma L \\ &\subseteq L\Gamma G \cup (G\Gamma L) \subseteq L\Gamma G \cup L = L \cup L\Gamma G \text{ and} \\ G\Gamma\,(L \cup L\Gamma G) &= G\Gamma L \cup G\Gamma\,(L\Gamma G) \subseteq L \cup L\Gamma\,(G\Gamma G) = L \cup L\Gamma G.\end{aligned}$$

Again let R be a right Γ-ideal of G then we have

$$\begin{aligned}(R \cup G\Gamma R)\,\Gamma G &= R\Gamma G \cup (G\Gamma R)\,\Gamma G \subseteq R \cup (G\Gamma R)\,\Gamma\,(G\Gamma G) \\ &= R \cup (G\Gamma G)\,\Gamma\,(R\Gamma G) \subseteq R \cup G\Gamma R, \text{ and} \\ G\Gamma\,(R \cup G\Gamma R) &= G\Gamma R \cup G\Gamma\,(G\Gamma R) = G\Gamma R \cup (G\Gamma G)\,\Gamma\,(G\Gamma R) \\ &= G\Gamma R \cup (R\Gamma G)\,\Gamma\,(G\Gamma G) \subseteq G\Gamma R \cup R\Gamma G \\ &\subseteq G\Gamma R \cup R = R \cup G\Gamma R.\end{aligned}$$

□

Lemma 4. *Right identity in a Γ-AG-groupoid G becomes identity of G and hence G becomes commutative Γ-semigroup.*

Proof. Let e be the right identity of G , $g \in G$, α and $\beta \in \Gamma$, then

$$e\alpha g = (e\beta e)\,\alpha g = (g\beta e)\,\alpha e = g\alpha e = g.$$

Again for a, b, $c \in G$ and α, $\beta \in \Gamma$ we have

$$a\gamma b = (e\alpha a)\,\gamma b = (e\alpha a)\,\gamma(e\alpha b) = (b\alpha e)\gamma\,(a\alpha e) = b\gamma a.$$

Now

$$\begin{aligned}(a\alpha b)\,\beta c &= (a\alpha b)\,\beta\,(e\alpha c) = (a\alpha e)\,\beta\,(b\alpha c) = e\alpha\,((a\alpha e)\,\beta\,(b\alpha c)) \\ &= (a\alpha e)\,\alpha\,(e\beta\,(b\alpha c)) = a\alpha\,(e\beta\,(b\alpha c)) = a\alpha\,(b\beta\,(e\alpha c)) \\ &= a\alpha\,(b\beta c)\,.\end{aligned}$$

□

Definition 9. *A sub Γ-AG-groupoid Q of a Γ-AG-groupoid G is called a quasi-ideal of G if $G\Gamma Q \cap Q\Gamma G \subseteq Q$.*

Definition 10. *A sub Γ-AG-groupoid I of a Γ-AG-groupoid G is called a Γ-interior ideal of G if $(G\Gamma I)\,\Gamma G \subseteq I$.*

Lemma 5. *Every one sided (left or right) Γ-ideal of a Γ-AG-groupoid G is a Γ-quasi ideal of G.*

Proof. Let L be a left Γ-ideal of G then we have

$$L\Gamma G \cap G\Gamma L \subseteq G\Gamma L \subseteq L.$$

Which implies L is a Γ-quasi ideal of G. Similarly if R is a right Γ-ideal of G then it is a Γ-quasi ideal of G. □

Lemma 6. *Every right Γ-ideal and left Γ-ideal of a Γ-AG-groupoid G is a Γ-bi-ideal of G.*

Proof. Let R be a right Γ-ideal of G then we have

$$(R\Gamma G)\,\Gamma R \subseteq R\Gamma R \subseteq R\Gamma G \subseteq R.$$

Again let L be a left Γ-ideal of G then we have

$$(L\Gamma G)\,\Gamma L \subseteq (G\Gamma G)\,\Gamma L \subseteq G\Gamma L \subseteq L.$$

□

Corollary 1. *Every* Γ*-ideal of a* Γ*-AG-groupoid* G *is a* Γ*-bi-ideal of* G*.*

Proof. It follows from lemma 6. □

Lemma 7. *If* B_1 *and* B_2 *are* Γ*-bi-ideals of a* Γ*-AG**-groupoid* G *then* $B_1\Gamma B_2$ *is also a* Γ*-bi-ideals of* G*.*

Proof. Let B_1 and B_2 be Γ-bi-ideals of G then we have

$$\begin{aligned} ((B_1\Gamma B_2)\,\Gamma G)\,\Gamma\,(B_1\Gamma B_2) &= ((B_1\Gamma B_2)\,\Gamma\,(G\Gamma G))\,\Gamma\,(B_1\Gamma B_2) \\ &= ((B_1\Gamma G)\,\Gamma\,(B_2\Gamma G))\,\Gamma\,(B_1\Gamma B_2) \\ &= ((B_1\Gamma G)\,\Gamma B_1)\,\Gamma\,((B_2\Gamma G)\,\Gamma B_2) \\ &\subseteq B_1\Gamma B_2. \end{aligned}$$

□

Lemma 8. *Every* Γ*-idempotent quasi-ideal of a* Γ*-AG-groupoid* G *is a* Γ*-bi-ideal of* G*.*

Proof. Let Q be an Γ-idempotent quasi-ideal of G. Now

$$\begin{aligned} (Q\Gamma G)\,\Gamma Q &\subseteq (G\Gamma G)\,\Gamma Q \subseteq G\Gamma Q, \text{ and} \\ (Q\Gamma G)\,\Gamma Q &= (Q\Gamma G)\,\Gamma\,(Q\Gamma Q) = (Q\Gamma Q)\,\Gamma\,(G\Gamma Q) = Q\Gamma\,(G\Gamma Q) \\ &\subseteq Q\Gamma\,(G\Gamma G) \subseteq Q\Gamma G, \text{ which implies that} \\ (Q\Gamma G)\,\Gamma Q &\subseteq G\Gamma Q \cap Q\Gamma G \subseteq Q. \end{aligned}$$

□

Lemma 9. *Every* Γ*-ideal of a* Γ*-AG-groupoid* G *is a* Γ*-interior ideal of* G*.*

Proof. Let I be a Γ-ideal of G then we have

$$(G\Gamma I)\,\Gamma G \subseteq I\Gamma G = I.$$

□

Lemma 10. *A subset* I *of a* Γ*-AG**-groupoid* G *is a* Γ*-interior ideal if and only if it is right* Γ*-ideal.*

Proof. Let I be a right Γ-ideal G then it becomes a left Γ-ideal so is Γ-ideal and by lemma 9 it is Γ-interior ideal.

Conversely assume that I is a Γ-interior ideal of G. Using Γ-paramedial law, we have

$$\begin{aligned} I\Gamma G &= I\Gamma\,(G\Gamma G) = G\Gamma\,(I\Gamma G) = (G\Gamma G)\,\Gamma\,(I\Gamma G) \\ &= (G\Gamma I)\,\Gamma\,(G\Gamma G) \subseteq (G\Gamma I)\,\Gamma G \subseteq G. \end{aligned}$$

Which shows that I is a right Γ-ideal of G. □

Example 1. *Let $G = \{1, 2, 3, 4, 5\}$ with binary operation "$\cdot$" given in the following Cayley's table, an AG-groupoid with left identity* 4.

$\cdot$	1	2	3	4	5
1	4	5	1	2	3
2	3	4	5	1	2
3	2	3	4	5	1
4	1	2	3	4	5
5	5	1	2	3	4

It is easy to observe that G is a simple AG-groupoid that is there is no left or right ideal of G. Now let $\Gamma = \{\alpha, \beta, \gamma\}$ defined as

α	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1

β	1	2	3	4	5
1	2	2	2	2	2
2	2	2	2	2	2
3	2	2	2	2	2
4	2	2	2	2	2
5	2	2	2	2	2

γ	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	3	3

It is easy to prove that G is a Γ-AG-groupoid because $(a\pi b)\,\psi c = (c\pi b)\,\psi a$ for all $a, b, c \in G$ and $\pi, \psi \in \Gamma$ also G is non-associative because $(1\alpha 2)\,\beta 3 \neq 1\alpha\,(2\beta 3)$. This Γ-AG-groupoid does not contain left identity because $4\alpha 5 \neq 5$, $4\beta 5 \neq 5$ and $4\gamma 5 \neq 5$. It is easy to see that every AG-groupoid with left identity not necessarily implies Γ-AG-groupoid with left identity. Clearly $A = \{1, 2, 3\}$ is a Γ-ideal of G. $B = \{1, 2, 4\}$ is a right Γ-ideal but is not a left Γ-ideal. A and B both are Γ-bi-ideals of G. $C = \{1, 2, 3, 4\}$ is a Γ-interior ideal of G.

Lemma 11. *For a regular Γ-AG-groupoid G $A\Gamma G = A$ and $G\Gamma B = B$ for every right Γ-ideal A and for every left Γ-ideal B.*

Proof. Let A be a right Γ- ideal of G then $A\Gamma G \subseteq A$. Let $a \in A$, since G is regular so there exist $x \in G$ and $\alpha, \gamma \in \Gamma$ such that

$$a = (a\alpha x)\,\gamma a \in (A\Gamma G)\,\Gamma A \subseteq (A\Gamma G)\,\Gamma G \subseteq A\Gamma G.$$

Now again let B be a left Γ-ideal of G then $G\Gamma B \subseteq B$. Let $b \in B$, also G is regular so there exist $t \in G$ and $\pi, \sigma \in \Gamma$ such that

$$b = (b\pi t)\,\sigma b \in (B\Gamma G)\,\Gamma B \subseteq (G\Gamma G)\,\Gamma B \subseteq G\Gamma B.$$

□

Lemma 12. *If G is a Γ-AG**-groupoid then $g\Gamma G$ and $G\Gamma g$ are Γ-bi-ideals for all $g \in G$.*

Proof. Using the definition of Γ-AG**-groupoid we have

$$\begin{aligned} ((g\Gamma G)\,\Gamma G)\,\Gamma\,(g\Gamma G) &= ((G\Gamma G)\,\Gamma g)\,\Gamma\,(g\Gamma G) \subseteq (G\Gamma g)\,\Gamma\,(g\Gamma G) \\ &= g\Gamma\,((G\Gamma g)\,\Gamma G) \subseteq g\Gamma\,((G\Gamma G)\,\Gamma G) \subseteq g\Gamma\,(G\Gamma G) \\ &\subseteq g\Gamma G. \end{aligned}$$

Again using Γ-paramedial law we have

$$\begin{aligned}((G\Gamma g)\Gamma G)\Gamma(G\Gamma g) &= (((G\Gamma g)\Gamma g)\Gamma G)\Gamma G=(((g\Gamma g)\Gamma G)\Gamma G)\Gamma G\\ &= ((G\Gamma G)\Gamma G)\Gamma(g\Gamma g)\subseteq(G\Gamma G)\Gamma(g\Gamma g)\\ &= (g\Gamma g)\Gamma(G\Gamma G)\subseteq(g\Gamma g)\Gamma G=(G\Gamma g)\Gamma g\\ &\subseteq (G\Gamma G)\Gamma g\subseteq G\Gamma g.\end{aligned}$$

□

Corollary 2. *If G is a regular Γ-AG^{**}-groupoid then $a\Gamma G$ is a Γ-bi-ideal in G, for all $a\in G$.*

Proof. Let G be a regular Γ-AG-groupoid then for every $a\in G$ there exist $x\in G$ and $\alpha,\ \beta\in\Gamma$ such that $a=((a\alpha x)\beta a)$ therefore we have

$$\begin{aligned}((a\Gamma G)\Gamma G)\Gamma(a\Gamma G) &= ((((a\alpha x)\beta a)\Gamma G)\Gamma G)\Gamma(a\Gamma G)\\ &= ((G\Gamma G)\Gamma((a\alpha x)\beta a))\Gamma(a\Gamma G)\\ &\subseteq (G\Gamma((a\alpha x)\beta a))\Gamma(a\Gamma G)=((a\alpha x)\Gamma(G\beta a))\Gamma(a\Gamma G)\\ &\subseteq ((a\alpha x)\Gamma(G\beta G))\Gamma(G\Gamma G)\subseteq((a\alpha x)\Gamma G)\Gamma G\\ &= (G\Gamma G)\Gamma(a\alpha x)\subseteq G\Gamma(a\alpha x)=a\Gamma(G\alpha x)\subseteq a\Gamma(G\Gamma G)\\ &\subseteq a\Gamma G.\end{aligned}$$

□

Lemma 13. *For a Γ-bi-ideal B in a regular Γ-AG-groupoid G, $(B\Gamma G)\Gamma B=B$.*

Proof. Let B be a Γ-bi-ideal in G then $(B\Gamma G)\Gamma B\subseteq B$. Let $x\in B$, since G is a regular Γ-AG-groupoid therefore there exist $a\in G$ and $\alpha,\ \beta\in\Gamma$ such that

$$x=(x\alpha a)\beta x\in(B\Gamma G)\Gamma B.$$

Which implies that $B\subseteq(B\Gamma G)\Gamma B$. □

Lemma 14. *If G is a regular Γ-AG-groupoid then, $G\Gamma G=G$.*

Proof. Since $G\Gamma G\subseteq G$. Let $x\in G$, since G is a regular Γ-AG-groupoid therefore there exist $a\in G$ and $\alpha,\ \beta\in\Gamma$ such that

$$x=(x\alpha a)\beta x\in(G\Gamma G)\Gamma G\subseteq G\Gamma G.$$

Which implies that $G\subseteq G\Gamma G$. □

Lemma 15. *A subset I of a regular Γ-AG^{**}-groupoid G is a left Γ-ideal if and only if it is a right Γ-ideal of G.*

Proof. Let I be a left Γ-ideal of G then $G\Gamma I\subseteq I$. Let $i\gamma g\in I\Gamma G$ for $g\in G$, $i\in I$ and $\gamma\in\Gamma$, also G is a regular Γ-AG-groupoid therefore there exist $x,\ y\in G$ and α, $\beta,\ \gamma,\ \delta,\ \pi\in\Gamma$ such that

$$\begin{aligned}i\gamma g &= ((i\alpha x)\beta i)\gamma((g\delta y)\pi g)=((i\alpha x)\beta(g\delta y))\gamma(i\pi g)\\ &= ((((i\alpha x)\beta i)\alpha x)\beta(g\delta y))\gamma(i\pi g)=((y\alpha g)\beta((i\beta(i\alpha x))\delta x))\gamma(i\pi g)\\ &= (i\beta(((y\alpha g)\beta(i\alpha x))\delta x))\gamma(i\pi g)=((i\pi g)\beta(((y\alpha g)\beta(i\alpha x))\delta x))\gamma i\\ &\in (G\Gamma I)\subseteq I.\end{aligned}$$

Conversely let I be a right Γ-ideal then there exist $x\in G$ and $\alpha,\ \beta\in\Gamma$ such that

$$g\gamma i=((g\alpha x)\beta g)\gamma i=(i\beta g)\gamma(g\alpha x)\in(I\Gamma G)\Gamma G\subseteq I\Gamma G\subseteq I.$$

□

Theorem 2. *for a* Γ*-*AG^{**}*-groupoid* G*, following statements are equivalent.*
(i) *G is regular* Γ*-*AG*-groupoid.*
(ii) *Every left* Γ*-ideal of* G *is* Γ*-idempotent.*

Proof. $(i) \Rightarrow (ii)$
Let G be a regular Γ-AG-groupoid then by lemma 3 every Γ-ideal of G is Γ-idempotent.
$(ii) \Rightarrow (i)$
Let every left Γ-ideal of a Γ-AG**-groupoid G is Γ-idempotent, since $G\Gamma a$ is a left Γ-ideal of G for all $a \in G$ [14], so is Γ-idempotent and by Γ-paramedial law, lemma **??** and Γ-medial law, we have, $a \in G\Gamma a$ implies

$$\begin{aligned}
a &\in (G\Gamma a)\,\Gamma\,(G\Gamma a) = ((G\Gamma a)\,\Gamma a)\,\Gamma G = ((a\Gamma a)\,\Gamma G)\,\Gamma G \\
&= ((a\Gamma a)\,\Gamma\,(G\Gamma G))\,\Gamma G = ((G\Gamma G)\,\Gamma\,(a\Gamma a))\,\Gamma G \\
&= (a\Gamma\,((G\Gamma G)\,\Gamma a))\,\Gamma G = (G\Gamma\,((G\Gamma G)\,\Gamma a))\,\Gamma a \\
&= (G\Gamma\,(G\Gamma a))\,\Gamma a = (G\Gamma\,((G\Gamma a)\,\Gamma\,(G\Gamma a)))\,\Gamma a \\
&= (G\Gamma\,((a\Gamma G)\,\Gamma\,(a\Gamma G)))\,\Gamma a = ((G\Gamma G)\,\Gamma\,((a\Gamma G)\,\Gamma\,(a\Gamma G)))\,\Gamma a \\
&= ((G\Gamma\,(a\Gamma G))\,\Gamma\,(G\Gamma\,(a\Gamma G)))\,\Gamma a = (((a\Gamma G)\,\Gamma G)\,\Gamma\,((a\Gamma G)\,\Gamma G))\,\Gamma a \\
&= ((((a\Gamma G)\,\Gamma G)\,\Gamma G)\,\Gamma\,(a\Gamma G))\,\Gamma a = (a\Gamma\,((((a\Gamma G)\,\Gamma G)\,\Gamma G)\,\Gamma G))\,\Gamma a \\
&\subseteq (a\Gamma G)\,\Gamma a.
\end{aligned}$$

Which shows that G is a regular Γ-AG**-groupoid. □

Lemma 16. *Any* Γ*-ideal of a regular* Γ*-*AG*-groupoid* G *is* Γ*-semiprime.*

Proof. It is an easy consequence of lemma 3. □

Theorem 3. *Set of all* Γ*-ideals in a regular* Γ*-*AG*-groupoid* G *with forms a semi-lattice* $(G, \circ)$ *where* $A \circ B = A\Gamma B$*, for all* Γ*-ideals* A *and* B *of* G.

Proof. Let A and B be any Γ-ideals in G, then by Γ-medial law we have

$$\begin{aligned}
(A\Gamma B)\,\Gamma G &= (A\Gamma B)\,\Gamma\,(G\Gamma G) = (A\Gamma G)\,\Gamma\,(B\Gamma G) \subseteq A\Gamma B. \text{ And} \\
G\Gamma\,(A\Gamma B) &= (G\Gamma G)\,\Gamma\,(A\Gamma B) = (G\Gamma A)\,\Gamma\,(G\Gamma B) \subseteq A\Gamma B.
\end{aligned}$$

Also by lemma 2, we have $A\Gamma B = B\Gamma A$ which implies that

$$(A\Gamma B)\,\Gamma C = C\Gamma\,(A\Gamma B) = A\Gamma\,(C\Gamma B) = A\Gamma\,(B\Gamma C)\,.$$

And by lemma 3, $A\Gamma A = A$.

□

References

[1] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, John Wiley & Sons,($vol.1$)1961.

[2] P. Holgate, Groupoids satisfying a simple invertive law, The Math. Stud., $1-4, 61(1992), 101-106$.

[3] J. Ježek and T. Kepka, A note on medial division groupoids, Proc. Amer. Math. Soc., $2, 119(1993), 423-426$.

[4] M. A. Kazim and M. Naseeruddin, On almost semigroups, The Alig. Bull. Math., $2(1972), 1-7$.

[5] M. Khan and Naveed Ahmad, Characterizations of left almost semigroups by their ideals, Journal of Advanced Research in Pure Mathematics, $2, 3.$ $(2010), 61-73$.

[6] M. K. Sen, On Γ-semigroups, Proceeding of International Symposium on Algebra and Its Applications, Decker Publication, New York, $(1981), 301-308$.

[7] M. K. Sen and N. K. Saha, On Γ-semigroups I, Bull. Cal. Math. Soc., 78(1986), 180-186.

[8] Q. Mushtaq and S. M. Yousuf, On LA-semigroups, The Alig. Bull. Math., $8(1978), 65-70$.

[9] Q. Mushtaq and S. M. Yousuf, On LA-semigroup defined by a commutative inverse semigroup, Math. Bech., 40(1988), $59-62$.

[10] Q., Mushtaq and Madad Khan, M-Systems in LA-Semigroups, Southeast Asian Bulletin of Mathematics 33(2009), $321-327$.

[11] Q., Mushtaq and Madad Khan, Ideals in Left Almost Semigroups, http://arxiv.org/abs/0904.1635$v1$.

[12] Naseeruddin, N., Some studies in almost semigroups and flocks, Ph.D., thesis, Aligarh Muslim University, Aligarh, India, 1970.

[13] P. V. Protić and N. Stevanović, AG-test and some general properties of Abel-Grassmann's groupoids, PU. M. A., 4., 6 (1995), $371-383$.

[14] T. Shah and I. Rehman, On Γ-Ideals and Γ-Bi-Ideals in Γ-AG-groupoids, International Journal of Algebra, $4, 6$ (2010), $267-276$.

[15] T. Shah and I. Rehman, On M-systems in Γ-AG-groupoids, Proc. Pakistan Acad. Sci., $47(1): 33-39.2010$.

[16] O. Steinfeld, *Quasi-ideals in ring and semigroups*, Akademiaikiado, Budapest, 1978.

[17] N. Stevanović and P. V. Protić, Composition of Abel-Grassmann's 3-bands, Novi Sad, J. Math., 2, 34(2004).