

# New Potential in Chameleon Mechanism

Kh. Saaidi<sup>1</sup>, H. Sheikahmadi<sup>2</sup>

*Department of Physics, Faculty of Science, University of Kurdistan,  
Sanandaj, Iran*

## Abstract

Chameleon mechanism is a new model, which introduced to provide a mechanism for exhibiting accelerating universe. It has several interesting aspects, such as field dependence on the local matter density. For this model we introduce a new potential which has run away form and satisfies chameleon constraints. The results are comparable with the other potentials which are studied up to now.

PACS: 04.20.-q, 04.90.+e, 98.80.-k

Keywords: Chameleon Mechanism; Dark Energy; chameleon constraints.

## 1 Introduction

The observation that universe appears to be accelerating at present times has caused one of the greatest problem to modern cosmology. The recent cosmological observations suggests that the universe consist of about 24% cold dark matter and 76% dark energy (DE) [1], while DE has a negative pressure, is used to explain the present cosmic acceleration. It is explicit that the nature of DE is unknown for researchers up to now, but they can describe it by some candidates. One of that candidates is cosmological constant,  $\Lambda$ , but it has two well-known difficulties, the "fine tuning" problem and the "cosmic coincidence" problem [2]. Whereas,  $\Omega_\Lambda = 0.763$  and  $\Omega_m = 0.237$ , the large value of  $\Omega_\Lambda$ , obviously predicts that the universe is accelerating today, rather than decelerating as had long been believed[3]. The observation evidence tells us that the rate of expansion in the high- $z$  region is slower than that in our neighborhood. In this condition, whereas variation of the  $\rho_\Lambda$  with respect to the time is equal to zero, this provide a problem in cosmology,

---

<sup>1</sup>ksaaidi@uok.ac.ir

<sup>2</sup>h.shahmadi@uok.ac.ir

called fine tuning, the quintessence (cosmon-field) solve this problem, by using coupling between scalar field and dark matter [4]. There are several different theories, which have been proposed by people, to interpret the accelerating universe, such as, holographic DE model [5], agegraphic DE model [6] and scalar field models of DE, which including phantom field[7], tachyon field [8], quintum [9] and quintessence [10]. While the quantity of cosmological constant is not zero, the dark energy component is more generally modeled as quintessence mechanism. It is a scalar field rolling down a flat potential[11]. Whereas the quintessence mechanism is a massless scalar field which couples directly to matter with gravitational strength, leads to undesirable large violations of the equivalence principle(EP). The authors of [12] have suggested a scalar field where having coupling to matter of order unity. They called this mechanism as chameleon mechanism. In this mechanism the scalar field acquires a mass whose magnitude depends on the local matter density. Indeed the chameleon mechanism is a way to give an effective mass to a light scalar field via field self interaction and interaction between matter. We exhibit chameleon behavior by new potential where has a run away form, but has not  $\phi^4$  form at quintessence model [13, 14, 15]. Where the consequences of a run away potential for chameleon mechanism can play the role of dark energy [16], we select our potential in this class of potential. The scheme of the present paper is as follows: In sec. 2 we study the preliminary of chameleon mechanism, by using two potentials, power law and exponential. In sec. 3 we introduce another potential, where has run away form. For this model we obtain several parameters which are useful in cosmology. One of that parameters is the matter density in earliest time. The last section is devoted to conclusion.

## 2 Preliminary

We consider the general action as

$$S = \int d^4x \sqrt{-g} \left( \frac{M_{pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{1}{\sqrt{-g}} \mathcal{L}_m(\psi_m, g_{\mu\nu}) \right) \quad (1)$$

where  $\phi$  is the chameleon scalar field and the potential  $V(\phi)$  has run away form and  $M_{pl} = (8\pi G)^{-\frac{1}{2}} = 2.44 \times 10^{18} Gev$ , is the reduce planck mass. Each matter field,  $\psi$ , coupled to a metric in Jordan fame, is related to the Einstein frame metric by a conformal transformation,  $\tilde{g}_{\mu\nu} = e^{\frac{2\beta\phi}{M_{pl}}} g_{\mu\nu}$ . Here  $\beta$  is the coupling constant without dimension. For the matter that described

by pressureless perfect fluid we have  $\tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu} = -\tilde{\rho}$ , where

$$\tilde{T}^{\mu\nu} = -\frac{2}{\sqrt{-\tilde{g}}}\frac{\partial\mathcal{L}_m}{\partial\tilde{g}_{\mu\nu}}, \quad (2)$$

where  $\tilde{\rho}$  is the energy density. We need an effective potential to govern the dynamic of the chameleon field. It be known that [17] :

$$\nabla^2\phi = V_{eff,\phi}(\phi), \quad (3)$$

where

$$V_{eff}(\phi) := V(\phi) + \rho e^{\frac{\beta\phi}{M^{p_l}}}. \quad (4)$$

It is necessary to be reminded that Khoury and Weltman had a good discussion on chameleon model at two different potentials, Ratra-Peebles and exponential as

$$V(\phi) = \frac{M^{4+n}}{\phi^n},$$

and

$$V(\phi) = M^4 \exp\left(\frac{M^n}{\phi^n}\right).$$

Note that, for large value of the field, only power law can tend to zero, see [18, 19, 20, 21, 22] For further reviews. Also some researchers such as Waterhouse, Brax and Bruck have considered chameleon model at different aspects, such as chameleon cosmology, radion and dark energy [13, 15, 16], respectively. We are going to introduce another kind of potential which its results are acceptable and also satisfies chameleon condition.

### 3 The Model

We introduce a new potential such as

$$V(\phi) = \frac{a + b(q\phi)^n}{1 + (q\phi)^n}, \quad (5)$$

where  $a$  and  $b$  are constant by dimension  $GeV^4$ ,  $q$  is a constant by  $GeV^{-1}$  and  $n$  is dimensionless real number.

This potential satisfies the constraints which emphasis in [13], and the asymptotic behavior of  $V(\phi)$  is as

1.  $\lim_{\phi \rightarrow \infty} V(\phi) = b$ ,

2.  $\lim_{\phi \rightarrow 0} V(\phi) = a$ ,
3.  $V_{,\phi}(\phi)$  is increasing and negative,
4.  $V_{,\phi\phi}(\phi)$  is decreasing and positive.

The most advantage of this potential is capability of bring experiencing. Because one can obtain the constraints (1)...(4) by different class of  $a, b, q$  and  $n$ . This potential is similar to exponential potential which is studied in [22]. It has no conflict whit local experiments because is very flexible function. The theoretical results got by this potential can be very closed to observation evidence. We note that this potential has run away behavior, therefore effective potential has a minimum, so that from Eq. (3), we have

$$V_{eff,\phi}(\phi_{min}) = 0, \quad (6)$$

consequently using (4) and (6), we get

$$\frac{nq^n \phi_{min}^{n-1}(b-a)}{(1+(q\phi_{min})^n)^2} + \rho \frac{\beta}{M_{pl}} e^{\frac{\beta\phi_{min}}{M_{pl}}} = 0, \quad (7)$$

we can obtain the mass of the small fluctuations,  $m_{min}$ , as

$$m_{min}^2 = V_{,\phi\phi}(\phi_{min}) + \rho \frac{\beta^2}{M_{pl}^2} e^{\frac{\beta\phi_{min}}{M_{pl}}}, \quad (8)$$

so that by substituting (6) into (8) we have

$$m_{min}^2 = \frac{V_{,\phi}(\phi_{min})}{\phi_{min}} \left( n - 1 - \frac{2n(q\phi_{min})^n}{1+(q\phi_{min})^n} \right) + \rho \frac{\beta^2}{M_{pl}^2} e^{\frac{\beta\phi_{min}}{M_{pl}}}. \quad (9)$$

Assuming that universe is just composed of matter and dark energy, so that by using the following data [1, 13]

$$\Omega_{matter} = 0.237, \quad \Omega_{DE} = 1 - \Omega_{matter} = 0.763,$$

$$\rho_{matter} = 1.04 \times 10^{-47} Gev^4, \quad \rho_{DE} = 3.34 \times 10^{-47} Gev^4,$$

and  $\rho = e^{\frac{3\beta\phi}{M_{pl}}} \tilde{\rho}$ , in conformal transformation, we can rewrite Eq. (7) as

$$\frac{nq^n \phi_{min}^{n-1}(b-a)}{(1+(q\phi_{min})^n)^2} + \rho_m \frac{\beta}{M_{pl}} e^{\frac{4\beta\phi_{min}}{M_{pl}}} = 0. \quad (10)$$

By choosing the parameters  $a, b, q, n$  and  $\beta$  as

$$a = 1.1 \times 10^{-12} \text{Gev}^4, \quad b = 2.0 \times 10^{-48} \text{Gev}^4, \quad (11)$$

$$q = 2.05 \times 10^{20} \text{Gev}^{-1}, \quad n = 0.9, \quad \beta = 9, \quad (12)$$

we obtain  $\phi_{min} = 1.1775 \times 10^{18} \text{Gev}$ . We have drawn  $V(\phi), V_{,\phi}(\phi)$  and  $V_{,\phi\phi}(\phi)$ , by these constants for more introduction. From figure (1) it is seen that this potential satisfies the constraint of [13, 22]. We assume that in the large scale universe(today)  $\phi = \phi_{min}$ , therefore according to Eq.(4) which shows that  $V(\Phi)$  and  $\rho$  have the same dimension, one can define the density of dark energy as  $V(\phi_{min})$ . So we have

$$\rho_{DE} = V(\phi_{min}) = \frac{a + b(q\phi_{min})^n}{1 + (q\phi_{min})^n},$$

by making use of  $\phi_{min}$  and other constant  $a, b, q$  and  $n$ , we can obtain  $\rho_{DE} = 3.34 \times 10^{-47} \text{Gev}^4$ . This result exactly is equaled with main quantity which is brought in [13]. Now we want obtain  $m_{min}$  for this model. For instance we take the atmosphere of the Earth as matter with  $\tilde{\rho} = 4 \times 10^{-21} \text{Gev}^4$ , using relation (9) we arrive  $m_{min}$  as

$$m_{min} = 4.03 \times 10^{-24} \text{Gev}.$$

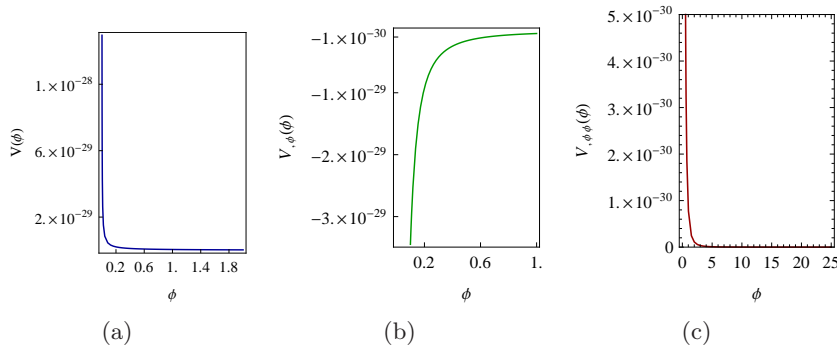


Figure 1: (a)  $V(\phi)$ , which is a positive and decreasing function of  $\phi$ , (b)  $V_{,\phi}(\phi)$ , which is a negative and increasing function of  $\phi$ , (c)  $V_{,\phi\phi}(\phi)$ , which is positive and decreasing. In these figures we use the constants of Eqs. (11) and (12).

We should note that  $m_{min}$  is small fluctuation around minimum. However, we obtain the relation between  $V_{,\phi}(\phi)$  and momentum-energy tensor

as

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}_m}{\partial g_{\mu\nu}}. \quad (13)$$

In Jordan frame we have  $\tilde{T}^{\mu\nu} \tilde{g}_{\mu\nu} = 3\tilde{p} - \tilde{\rho}$ , by substituting  $\tilde{p} = \omega\tilde{\rho}$  we have  $\tilde{T}^{\mu\nu} \tilde{g}_{\mu\nu} = (3\omega - 1)\tilde{\rho}$ . But in Einstein frame we obtain

$$T^{\mu\nu} = \tilde{T}^{\mu\nu} e^{\frac{6\beta\phi}{M_{pl}}}, \quad (14)$$

from relation (12) we have  $\tilde{T}^{00} \tilde{g}_{00} = -\tilde{\rho}$ , this means that  $\omega = 0$ . Eventually we can obtain

$$T^{00} = \tilde{\rho} e^{\frac{4\beta\phi}{M_{pl}}}, \quad (15)$$

by substituting (13) in Eq. (10), we have

$$T^{00} = \frac{M_{pl}}{\beta} \left( \frac{nq\phi_{min}^{n-1}(a-b)}{(1+(q\phi_{min})^n)^2} \right), \quad (16)$$

by making use of the value of  $\phi_{min}$  and other relevant constant we obtain  $T^{00} = -6.5 \times 10^{-48} Gev^4$  for this model. This is another result which is agree with other works. Now we want focus on the chameleon behavior in the earlier universe by  $\omega = -1$ , note that in earlier universe, we use

$$V_{eff}(\phi) = \frac{a + b(q\phi)^n}{1 + (q\phi)^n} + \rho e^{\frac{(1-3\omega)\beta\phi}{M_{pl}}}, \quad (17)$$

for simplicity we define  $\Omega_m$  as

$$\Omega_m = \frac{\rho_m}{\rho_c} e^{\frac{\beta\phi_{min}}{M_{pl}}},$$

where  $\rho_c = 3H^2 M_{pl}^2$ . Therefore from Eq. (9) we can obtain

$$\frac{m_{min}^2}{H^2} = \frac{3\beta\Omega_m M_{pl}}{\phi_{min}} \left( (1-n) + \frac{2n(q\phi_{min})^n}{1+(q\phi_{min})^n} \right) + 3\beta^2\Omega_m, \quad (18)$$

for investigating the cosmology behavior we consider two regimes, i)  $\phi_{min} \geq b^{\frac{1}{4}}$ , and ii)  $\phi_{min} \gg b^{\frac{1}{4}}$ . For  $\phi_{min} \geq b^{\frac{1}{4}}$  regime, where  $(q\phi_{min})^n \simeq (10^{26})^n$  is very larger than unity in denominator, and  $\frac{M_{pl}}{\phi_{min}} \simeq 10^{30}$ , so that we can rewrite Eq. (18) as

$$\frac{m_{min}^2}{H^2} \simeq 3\beta\Omega_m(n+1) \times 10^{30}, \quad (19)$$

it is well known that for  $\Omega_m > 10^{-28}$  we have  $\frac{m_{min}^2}{H^2} \gg 1$ . We can obtain the similar result for the case which we have coupling constant cosmology only. In this case, equation of state is  $P = -\rho$  this means that  $\omega = -1$ , therefore according to Eq. (9) one can obtain the similar result by replacing  $4\beta$  instead of  $\beta$ . So that we have

$$\frac{m_{min}^2}{H^2} = \frac{12\beta\Omega_{vac}M_{pl}}{\phi_{min}} \left( (1-n) + \frac{2n(q\phi_{min})^n}{1+(q\phi_{min})^n} \right) + 48\beta^2\Omega_{vac}, \quad (20)$$

therefore we can obtain  $\frac{m_{min}^2}{H^2} \simeq 48\beta^2\Omega_{vac} \gg 1$ , as

$$\Omega_{vac} = \frac{\rho_{vac} e^{\frac{4\beta\phi_{min}}{M_{pl}}}}{\rho_c}.$$

For  $\phi_{min} \gg b^{\frac{1}{4}}$  regime, from Eq.(18), we have

$$\frac{m_{min}^2}{H^2} \simeq \frac{3\beta\Omega_m M_{pl}}{\phi_{min}} (n+1), \quad (21)$$

By using the definitions of  $\Omega_m$ , we obtain

$$\frac{m_{min}^2}{H^2} \simeq \frac{3\beta\rho_m e^{\frac{\beta\phi_{min}}{M_{pl}}}}{\rho_c\phi_{min}} (n+1), \quad (22)$$

using  $\rho_c = 3H_0^2 M_{pl}^2$ , and Eq. (22), we get

$$\rho_m = \frac{m_{min}^2 \phi_{min} M_{pl}}{(n+1) e^{\frac{\beta\phi_{min}}{M_{pl}}}}, \quad (23)$$

By making use of the value of  $m_{min}$  and other introduced constant, we can arrive at

$$\rho_m = 0.4 \times 10^{-16} Gev^4.$$

This result is an estimate for matter density in earliest universe. Note that in present time  $\rho_m \simeq 10^{-47} Gev^4$  therefore our estimate says that the density falls off in proportion to the volume of the universe. Also this condition is for earliest time, and  $\phi_{min}$  increase with time, so that the matter density is diluted.

## 4 Conclusion

In this paper, we have discussed chameleon behavior and chameleon condition. Khoury and Weltman have introduced chameleon model and obtain several important results by two potentials as power law and exponential type.

We have introduced a new potential which is useful for chameleon model and has acceptable results. For this potential we obtain  $\phi_{min}$ ,  $m_{min}$ , mass of small fluctuations (can be interpreted radion mass as [16]), and other useful quantity such as  $\rho_m$ . Then we have investigated two regimes for  $\rho_m$ , on the present time and the earliest time, our results are comparable with other articles [12, 13, 18].



## References

- [1] Y. Fujii, and K. Maeda, The scalar -tensor theory of gravitation, Cambridge University Press(2003).
- [2] P. J. Steinhardt, in critical problems in physics, edited by V. L. Fitch and D. R. Marlow(Printed University Press, Princeton, NJ, 1997).
- [3] S. prlmutter *et al.*, Nature 391,(1998); S. prlmutter *et al.*, Astrophysics. J. 509,(1999).
- [4] Steven Weinberg, Reviess of Modern Physics. 61, 1 (1989); Y. Fujii, and T. Nishioka. Phys. Lett . B 254, (1991); Y. Fujii. Astropart. Phys. 5, (1996); Y. Fujii, Phys.Rev. D 62, 064004, (2000).
- [5] M. R. Setare, Phys. Lett. B 644, 99, (2007)[hep-th/0610190]; M. R. Setare, Phys. Lett. B 648, 329, (2007); M. Li, Phys. Lett. B 603, 1 (2004); K. Enqvist and M. S. Sloth, Phys. Rev. Lett. 93,221302 (2004) [hep-th/0406019]; A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. 82, 4971 (1999); K. Karami, J. Fehri, [arXiv:0911.4932]; Kourosch Nozari, Narges Rashidi, Int. J. Mod. Phys. D 19, (2010), [gr-qc:0912.4802]
- [6] M. R. Setare, [gr-qc: 0908.0196], (2009); R.G. Cai, Phys. Lett. B 657, 228 (2007); H. Wei, R.G. Cai, Phys. Lett. B 660, 113 (2008); K.Y. Kim, H. W. Lee, Y. S. Myung, Phys. Lett. B 660, 118 (2008); H. Wei, R.G. Cai, Phys. Lett. B 663, 1 (2008); J. P. Wu, D. Z. Ma, Y. Ling, Phys. Lett. B 663, 152 (2008); Y.W. Kim et al., Mod. Phys. Lett. A 23, 3049 (2008); K. Karami, M. S. Khaledian, F. Felegary, Z. Azarmi, [arXiv: 0912.1536]; Kourosch Nozari, Tahereh Azizi, Phys. Lett .B 680: 205-211, (2009).
- [7] M. R. Setare, Eur. Phys. J. C50, 991, (2007); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301(2003)[astro-ph/0302506]; S. Nojiri and S. D. Odintsov, Phys. Lett., B 565, 1 (2003) [hep-th/0304131].
- [8] M. R. Setare, Phys. Lett. B 653, 116,(2007), arXiv: 0705.3517 [hep-th]; T. Padmanabhan, Phys. Rev. D 66, 021301 (2002) [hep-th/0204150].
- [9] M. R. Setare, J. Sadeghi, [gr-qc: 0805.1117]; B. Feng, X. L. Wang, X. Zhang, Phys. Lett. B 607, 35, (2005); B. Feng, astro-ph/0602156; E. Elizalde, S. Nojiri, S. D. Odintsov, Phys. Rev. D 70, 043539,(2004).

- [10] P. J. E. Peebles and B. Ratra, *Astrophys. J.* 325, L17 (1988); B. Ratra and P. J. E. Peebles, *Phys. Rev. D* 37, 3406 (1988); C. Wetterich, *Nucl. Phys. B* 302, 668 (1988).
- [11] J. P. Ostriker, and P. J. Steinhardt, *Sci. Am.*, Jan.issue(2001); R. R. Caldwell, R. Dave, P. J. Steinhardt, *Phys. Rev. Lett.* 80, 1582(1998), [astro-ph/9708069]; Steven, Weinberg, *Cosmology*, Oxford University Press, (2008).
- [12] David F. Mota, John D. Barrow, *Phys. Lett. B* 581, 141, (2004); Justin Khoury, and Amanda Weltman, [astro-ph/0309300]; Justin Khoury, and Amanda Weltman, [astro-ph/0410103].
- [13] T. P. Waterhouse, [astro-ph/061186], (2006); Ph. Brax, C. Vande bruke, A. C. Davis, J. Khoury and A. Weltman, [astro-ph/0410103].
- [14] A. Upadhye, S. S. Gusber and J. Khoury, [hep-ph/0608186]; Justin Khoury, and Amanda Weltman. [astro-ph/0309411], (2003).
- [15] Ph. Brax, C. Vande bruke, A. C. Davis, and C. S. Rhodes, *Phys. Rev. D* 67, 023512(2003).
- [16] Ph. Brax, C. Vande bruke, A. C. Davis, [astro-ph/0408464], (2004).
- [17] I. Zlatev, L. M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* 82, 896, (1999) [astro-ph/9807002];
- [18] S. S. Gusber and J. Khoury, [hep-ph/0405231]; A. R. Liddle and R. J. Scherrer, *Phys. Rev. D* 59, 023509 (1999) [astro-ph/9809272]; V. Faraoni, *Class. Quant. Grav.* 22, 3235, (2005).
- [19] Z. G. Huang, H. Q. Lu, and W. Fang, *Class. Quant. Grav.* 23, 6215, (2006), [hep-th/0604160 ].
- [20] P. Brax, and J. Martin, *Phys. Rev. D* 61, 103502, (2000); I. Affleck, M. Dien, and N. Seiberg, *Nucl. Phys. B* 256, (1985).
- [21] J. Yokoyama, and K. Maeda, *Phys. Lett. B* 207, (1988); Y. Fujii, *Prog. Theor. Phys.* 99, (1998).
- [22] P. Binétruy, *International Journal of theoretical physics* 39, 1859-75 (2000).