The maximal regularity operator on tent spaces

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En l'honneur des 60 ans de Michel Pierre

Abstract

Recently, Auscher and Axelsson gave a new approach to non-smooth boundary value problems with L^2 data, that relies on some appropriate weighted maximal regularity estimates. As part of the development of the corresponding L^p theory, we prove here the relevant weighted maximal estimates in tent spaces $T^{p,2}$ for p in a certain open range. We also study the case $p = \infty$.

1 Introduction

Let $-L$ be a densely defined closed linear operator acting on $L^2(\mathbb{R}^n)$ and generating a bounded analytic semigroup $(e^{-tL})_{t\geq 0}$. We consider the maximal regularity operator defined by

$$\mathcal{M}_L f(t,x) = \int_0^t L e^{-(t-s)L} f(s,.)(x) ds,$$

for functions $f \in C_c(\mathbb{R}_+ \times \mathbb{R}^n)$. The boundedness of this operator on $L^2(\mathbb{R}_+ \times \mathbb{R}^n)$ was established by de Simon in [15]. The $L^p(\mathbb{R}_+ \times \mathbb{R}^n)$ case, for $1 < p < \infty$, turned out, however, to be much more difficult. In [10], Kalton and Lancien proved that $\mathcal{M}_L$ could fail to be bounded on L^p as soon as $p \neq 2$. The necessary and sufficient assumption for L^p boundedness was then found by Weis [16] to be a vector-valued strengthening of analyticity, called R-analyticity. As many differential operators L turn out to generate R-analytic semigroups, the L^p boundedness of $\mathcal{M}_L$ has subsequently been successfully used in a variety of PDE situations (see [13] for a survey).

Recently, maximal regularity was used as an important tool in [2], where a new approach to boundary value problems with L^2 data for divergence form elliptic systems on Lipschitz domains, is developed. More precisely, in [2], the authors establish and use the boundedness of $\mathcal{M}_L$ on weighted spaces $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dtdx)$, for certain values of $\beta \in \mathbb{R}$. The use of these spaces is common in the study of boundary value problems, where they are seen as variants of the tent space $T^{2,2}$, introduced by Coifman, Meyer and Stein in [6]. For $p \neq 2$, the corresponding spaces are weighted versions of the tent spaces $T^{p,2}$, which are defined, for parameters $\beta \in \mathbb{R}$ and $m \in \mathbb{N}$, as the completion of $C_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$\|g\|_{T^{p,2,m}(t^\beta dtdy)} = \left(\int_{\mathbb{R}^n} \Big(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x,t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}} \left|g(t,y)\right|^2 t^\beta dydt \Big)^{\frac{p}{2}} dx \right)^{\frac{1}{p}},$$

the classical case corresponding to $\beta = -1$, $m = 1$, and being denoted simply by $T^{p,2}$. The parameter m is used to allow various homogeneities, and thus to make these spaces relevant in the study of differential operators L of order m. To develop an analogue of [2] for L^p data, we need boundedness results for the maximal operator $\mathcal{M}_L$ on these tent spaces. This is the purpose of this note. It will be used in the study of such boundary value problems, as well as in the context of well-posedness of non-autonomous Cauchy problems, in subsequent papers. The proof is based on recent results and methods developed in [9], building on ideas from [5] and [8]. In Section 2 we recall the relevant definitions and result from [9]. In Section 3 we then state and prove the adequate boundedness results.

2 Tools

When dealing with tent spaces, the key estimate needed is a change of aperture formula, i.e., a comparison between the $T^{p,2}$ norm and the norm

$$\|g\|_{T^{p,2}_{\alpha}} := \left(\int\limits_{\mathbb{R}^n}\Big(\int\limits_0^{\infty}\int\limits_{\mathbb{R}^n}\frac{1_{B(x,\alpha t)}(y)}{t^n}\,\big|g(t,y)\big|^2\frac{dydt}{t}\Big)^{\frac{p}{2}}dx\right)^{\frac{1}{p}},$$

for some parameter $\alpha > 0$. Such a result was first established in [6], building on similar estimates in [7], and analogues have since been developed in various contexts. Here we use the following version given in [9, Theorem 4.3].

Theorem 2.1. *Let $1 < p < \infty$ and $\alpha \geq 1$. There exists a constant $C > 0$ such that, for all $f \in T^{p,2}$,*

$$\|f\|_{T^{p,2}} \leq \|f\|_{T^{p,2}_{\alpha}} \leq C(1+\log\alpha)\alpha^{n/\tau}\|f\|_{T^{p,2}},$$

where $\tau = \min(p, 2)$.

Theorem 2.1 is actually a special case of the Banach space valued result obtained in [9] corresponding to the scalar valued situation. Note, however, that it improves the power of α appearing in the inequality from the n given in [6] to $\frac{n}{\tau}$. This is crucial in what follows, and has been shown to be optimal in [9].

Applying this to $(t,y) \mapsto t^{\frac{m(\beta+1)}{2}} f(t^m, y)$ instead of f, we also have the weighted result, where

$$\|g\|_{T^{p,2,m}_{\alpha}(t^{\beta}dtdy)} = \left(\int\limits_{\mathbb{R}^n}\Big(\int\limits_0^{\infty}\int\limits_{\mathbb{R}^n}\frac{1_{B(x,\alpha t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}}\,\big|g(t,y)\big|^2 t^{\beta}dydt\Big)^{\frac{p}{2}}dx\right)^{\frac{1}{p}}.$$

Corollary 2.2. *Let $1 < p < \infty$, $m \in \mathbb{N}$, $\alpha \geq 1$, and $\beta \in \mathbb{R}$. There exists a constant $C > 0$ such that, for all $f \in T^{p,2,m}(t^{\beta}dtdy)$,*

$$\|f\|_{T^{p,2,m}(t^{\beta}dtdy)} \leq \|f\|_{T^{p,2,m}_{\alpha}(t^{\beta}dtdy)} \leq C(1+\log\alpha)\alpha^{n/\tau}\|f\|_{T^{p,2,m}(t^{\beta}dtdy)},$$

where $\tau = \min(p, 2)$.

To take advantage of this result, one needs to deal with families of operators, that behave nicely with respect to tent norms. As pointed out in [9], this does not mean considering R-bounded families (which means R-analytic semigroups when one considers $(tLe^{-tL})_{t\geq 0}$) as in the $L^p(\mathbb{R}_+ \times \mathbb{R}^n)$ case, but tent bounded ones, i.e. families of operators with the following L^2 off-diagonal decay, also known as Gaffney-Davies estimates.

Definition 2.3. A family of bounded linear operators $(T_t)_{t\geq 0} \subset B(L^2(\mathbb{R}^n))$ is said to satisfy off-diagonal estimates of order M, with homogeneity m, if, for all Borel sets $E, F \subset \mathbb{R}^n$, all $t > 0$, and all $f \in L^2(\mathbb{R}^n)$:

$$\|1_E T_t 1_F f\|_2 \lesssim \Big(1 + \frac{dist(E,F)^m}{t}\Big)^{-M}\|1_F f\|_2.$$

In what follows $\|\cdot\|_2$ denotes the norm in $L^2(\mathbb{R}^n)$.

As proven, for instance, in [4], many differential operators of order m, such as (for $m = 2$) divergence form elliptic operators with bounded measurable complex coefficients, are such that $(tLe^{-tL})_{t\geq 0}$ satisfies off-diagonal estimates of any order, with homogeneity m. This condition can, in fact, be seen as a replacement for the classical gaussian kernel estimates satisfied in the case of more regular coefficients.

3 Results

Theorem 3.1. *Let $m \in \mathbb{N}$, $\beta \in (-\infty, 1)$, $p \in \left(\frac{2n}{n+m(1-\beta)}, \infty\right) \cap (1,\infty)$, and $\tau = \min(p,2)$. If $(tLe^{-tL})_{t\geq 0}$ satisfies off-diagonal estimates of order $M > \frac{n}{m\tau}$, with homogeneity m, then $\mathcal{M}_L$ extends to a bounded operator on $T^{p,2,m}(t^\beta dtdy)$.*

Proof. The proof is very much inspired by similar estimates in [5] and [9]. Let $f \in \mathscr{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$. Given $(t,x) \in \mathbb{R}_+ \times \mathbb{R}^n$, and $j \in \mathbb{Z}_+$, we consider

$$C_j(x,t) = \begin{cases} B(x,t) \text{ if } j = 0, \\ B(x,2^j t)\backslash B(x, 2^{j-1}t) \text{ otherwise.} \end{cases}$$

We write $\|\mathcal{M}_L f\|_{T^{p,2}} \leq \sum\limits_{k=1}^{\infty}\sum\limits_{j=0}^{\infty} I_{k,j} + \sum\limits_{j=0}^{\infty} J_j$ where

$$I_{k,j} = \left(\int_{\mathbb{R}^n}\Big(\int_0^\infty\int_{\mathbb{R}^n} \frac{1_{B(x,t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}}\Big|\int_{2^{-k-1}t}^{2^{-k}t} Le^{-(t-s)L}(1_{C_j(x,4t^{\frac{1}{m}})}f(s,.))(y)ds\Big|^2 t^\beta dy\,dt\Big)^{\frac{p}{2}}dx\right)^{\frac{1}{p}},$$

$$J_j = \left(\int_{\mathbb{R}^n}\Big(\int_0^\infty\int_{\mathbb{R}^n} \frac{1_{B(x,t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}}\Big|\int_{\frac{t}{2}}^{t} Le^{-(t-s)L}(1_{C_j(x,4s^{\frac{1}{m}})}f(s,.))(y)ds\Big|^2 t^\beta dy\,dt\Big)^{\frac{p}{2}}dx\right)^{\frac{1}{p}}.$$

Fixing $j \geq 0$, $k \geq 1$ we first estimate $I_{k,j}$ as follows. For fixed $x \in \mathbb{R}^n$,

$$\begin{aligned}
&\int_0^\infty\int_{B(x,t^{\frac{1}{m}})}\Big|\int_{2^{-k-1}t}^{2^{-k}t} Le^{-(t-s)L}(1_{C_j(x,4t^{\frac{1}{m}})}f(s,\cdot))(y)\,ds\Big|^2 t^{\beta-\frac{n}{m}}dy\,dt \\
\leq\ & \int_0^\infty\int_{B(x,t^{\frac{1}{m}})}\Big(\int_{2^{-k-1}t}^{2^{-k}t}\Big|(t-s)Le^{-(t-s)L}(1_{C_j(x,4t^{\frac{1}{m}})}f(s,\cdot))(y)\Big|\frac{ds}{t-s}\Big)^2 t^{\beta-\frac{n}{m}}dy\,dt \\
\lesssim\ & \int_0^\infty\int_{2^{-k-1}t}^{2^{-k}t} 2^{-k}t\Big(\int_{B(x,t^{\frac{1}{m}})}\Big|(t-s)Le^{-(t-s)L}(1_{C_j(x,4t^{\frac{1}{m}})}f(s,\cdot))(y)\Big|^2dy\Big)\,t^{\beta-\frac{n}{m}-2}ds\,dt \\
\lesssim\ & \int_0^\infty\int_{2^{-k-1}t}^{2^{-k}t} 2^{-k}\Big(1+\frac{2^{jm}t}{t-s}\Big)^{-2M}\|1_{B(x,2^{j+2}t^{\frac{1}{m}})}f(s,\cdot)\|_2^2\,t^{\beta-\frac{n}{m}-1}ds\,dt \\
\lesssim\ & 2^{-k}2^{-2jmM}\int_0^\infty\Big(\int_{2^k s}^{2^{k+1}s} t^{\beta-\frac{n}{m}-1}dt\Big)\|1_{B(x,2^{j+\frac{k}{m}+3}s^{\frac{1}{m}})}f(s,\cdot)\|_2^2\,ds \\
\lesssim\ & 2^{-k(\frac{n}{m}+1-\beta)}2^{-2jmM}\int_0^\infty\|1_{B(x,2^{j+\frac{k}{m}+3}s^{\frac{1}{m}})}f(s,\cdot)\|_2^2\,s^{\beta-\frac{n}{m}}ds.
\end{aligned}$$

In the second inequality, we use Cauchy-Schwarz inequality for the integral with respect to t, the fact that $t-s\sim t$ for $s \in \cup_{k\geq 1}[2^{-k-1}t, 2^{-k}t] \subset [0,\frac{t}{2}]$ and Fubini's theorem to exchange the integral in t and the integral in y. The next inequality follows from the off-diagonal estimate verified by $(t-s)Le^{-(t-s)L}$ and again the fact that $t-s\sim t$. By Corollary 2.2 this gives

$$I_{k,j} \lesssim (j+k)2^{-k(\frac{1}{2}(\frac{n}{m}+1-\beta)-\frac{n}{m\tau})}2^{-j(mM-\frac{n}{\tau})}\|f\|_{T^{p,2,m}(t^\beta dtdy)},$$

where $\tau = \min(p,2)$. It follows that $\sum\limits_{k=1}^{\infty}\sum\limits_{j=0}^{\infty} I_{k,j} \lesssim \|f\|_{T^{p,2,m}(t^\beta dtdy)}$ since $M > \frac{n}{m\tau}$ and $\frac{n}{m}+1-\beta > \frac{2n}{m\tau}$ (Note that for $p \geq 2$, this requires $\beta < 1$).

We now turn to J_0 and remark that $J_0 \leq \left(\int_{\mathbb{R}^n} J_0(x)^{\frac{p}{2}} dx\right)^{\frac{1}{p}}$, where

$$J_0(x) = \int_0^\infty \int_{\mathbb{R}^n} \Big| \int_{\frac{t}{2}}^{t} Le^{-(t-s)L}(g(s,\cdot)(y) ds \Big|^2 t^{\beta - \frac{n}{m}} dy\, dt$$

with $g(s,y) = 1_{B(x,4s^{\frac{1}{m}})}(y) f(s,y)$. The inside integral can be rewritten as

$$\mathcal{M}_L g(t,\cdot) - e^{-\frac{t}{2}L} \mathcal{M}_L g(\frac{t}{2},\cdot).$$

As $\mathcal{M}_L$ is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^{\beta-\frac{n}{m}} dydt)$ by [3, Theorem 3.1] and $(e^{-tL})_{t\geq 0}$ is uniformly bounded on $L^2(\mathbb{R}^n)$, we get

$$J_0(x) \lesssim \int_0^\infty \|1_{B(x,4s^{\frac{1}{m}})} f(s,\cdot)\|_2^2 \, s^{\beta-\frac{n}{m}} ds.$$

We finally turn to J_j, for $j \geq 1$. For fixed $x \in \mathbb{R}^n$,

$$\begin{aligned}
&\int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{\frac{1}{m}})}(y) \Big| \int_{\frac{t}{2}}^{t} Le^{-(t-s)L}(1_{C_j(x,4s^{\frac{1}{m}})} f(s,.))(y) ds \Big|^2 t^{\beta-\frac{n}{m}} dy\, dt \\
\leq\; & \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{\frac{1}{m}})}(y) \Big(\int_{\frac{t}{2}}^{t} |(t-s)Le^{-(t-s)L}(1_{C_j(x,4s^{\frac{1}{m}})} f(s,.))(y)| \frac{ds}{t-s}\Big)^2 t^{\beta-\frac{n}{m}} dy\, dt \\
\lesssim\; & \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{\frac{1}{m}})}(y) \int_{\frac{t}{2}}^{t} |(t-s)Le^{-(t-s)L}(1_{C_j(x,4s^{\frac{1}{m}})} f(s,.))(y)|^2 \frac{ds}{(t-s)^2} t^{\beta-\frac{n}{m}+1} dy\, dt \\
\lesssim\; & \int_0^\infty \int_{\frac{t}{2}}^{t} (t-s)^{-2} \Big(1 + \frac{2^{jm} t}{t-s}\Big)^{-2M} \|1_{B(x,2^{j+2}s^{\frac{1}{m}})} f(s,.)\|_2^2 \, s^{\beta-\frac{n}{m}+1} ds\, dt \\
\lesssim\; & 2^{-jm(2M-2)} \int_0^\infty \left(\int_s^{2s} s(t-s)^{-2} \Big(1 + \frac{2^{jm} t}{t-s}\Big)^{-2} dt \right) \|1_{B(x,2^{j+2}s^{\frac{1}{m}})} f(s,.)\|_2^2 \, s^{\beta-\frac{n}{m}} ds \\
\lesssim\; & 2^{-2jmM} \int_0^\infty \|1_{B(x,2^{j+2}s^{\frac{1}{m}})} f(s,.)\|_2^2 \, s^{\beta-\frac{n}{m}} ds,
\end{aligned}$$

where we have used Cauchy-Schwarz inequality in the second inequality, the off-diagonal estimates and the fact that $s \leq t$ in the third, Fubini's theorem and the fact that $s \geq \frac{t}{2}$ in the fourth, and the change of variable $\sigma = \frac{t}{t-s}$ in the last. An application of Corollary 2.2, then gives

$$J_j \lesssim 2^{-jmM} j 2^{j\frac{n}{\tau}} \|f\|_{T^{p,2,m}(t^\beta dtdy)} = j 2^{-j(mM-\frac{n}{\tau})} \|f\|_{T^{p,2,m}(t^\beta dtdy)},$$

and the proof is concluded by summing the estimates. □

An end-point result holds for $p = \infty$. In this context the appropriate tent space consists of functions such that $|g(t,x)|^2 \frac{dxdt}{t}$ is a Carleson measure, and is defined as the completion of the space $\mathscr{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$\|g\|_{T^{\infty,2}}^2 = \sup_{(x,r)\in\mathbb{R}^n\times\mathbb{R}_+} r^{-n} \int_{B(x,r)} \int_0^r |g(t,x)|^2 \frac{dxdt}{t}.$$

We also consider the weighted version defined by

$$\|g\|^2_{T^{\infty,2,m}(t^\beta dtdy)} := \sup_{(x,r)\in\mathbb{R}^n\times\mathbb{R}_+} r^{-\frac{n}{m}} \int_{B(x,r^{\frac{1}{m}})} \int_0^r |g(t,x)|^2 t^\beta dxdt.$$

Theorem 3.2. *Let $m \in \mathbb{N}$, and $\beta \in (-\infty, 1)$. If $(tLe^{-tL})_{t\geq 0}$ satisfies off-diagonal estimates of order $M > \frac{n}{2m}$, with homogeneity m, then $\mathcal{M}_L$ extends to a bounded operator on $T^{\infty,2,m}(t^\beta dtdy)$.*

Proof. Pick a ball $B(z, r^{\frac{1}{m}})$. Let

$$I^2 = \int_{B(z,r^{\frac{1}{m}})} \int_0^r |(\mathcal{M}_L f)(t,x)|^2 t^\beta dxdt.$$

We want to show that $I^2 \lesssim r^{\frac{n}{m}} \|f\|^2_{T^{\infty,2}(t^\beta dtdy)}$. We set

$$I_j^2 = \int_{B(x,r^{\frac{1}{m}})} \int_0^r |(\mathcal{M}_L f_j)(t,x)|^2 t^\beta dxdt$$

where $f_j(s,x) = f(s,x) 1_{C_j(z,4r^{\frac{1}{m}})}(x) 1_{(0,r)}(s)$ for $j \geq 0$. Thus by Minkowsky inequality, $I \leq \sum I_j$. For I_0 we use again [3, Theorem 1.3] which implies that $\mathcal{M}_L$ is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n, t^\beta dxdt)$. Thus

$$I_0^2 \lesssim \int_{B(z,4r^{\frac{1}{m}})} \int_0^r |f(t,x)|^2 t^\beta dxdt \lesssim r^{\frac{n}{m}} \|f\|^2_{T^{\infty,2,m}(t^\beta dtdy)}.$$

Next, for $j \neq 0$, we proceed as in the proof of Theorem 3.1 to obtain

$$\begin{aligned} I_j^2 \quad &\lesssim \quad \sum_{k=1}^\infty \int_0^r \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} t \Big(1 + \frac{2^{jm} r}{t-s}\Big)^{-2M} \|f_j(s,.)\|^2_{L^2} t^{\beta-2} ds\, dt \\ &\quad + \int_0^r \int_{\frac{t}{2}}^t t(t-s)^{-2} \Big(1 + \frac{2^{jm} r}{t-s}\Big)^{-2M} \|f_j(s,.)\|^2_{L^2} t^\beta ds\, dt. \end{aligned}$$

Exchanging the order of integration, and using the fact that $t \sim t-s$ in the first part and that $t \sim s$ in the second, we have the following.

$$\begin{aligned} I_j^2 \quad &\lesssim \quad \sum_{k=1}^\infty 2^{-k} 2^{-2jmM} r^{-2M} \int_0^{2^{-k}r} \int_{2^k s}^{2^{k+1}s} t^{\beta+2M-1} \|f_j(s,.)\|^2_{L^2} dtds \\ &\quad + \int_0^r \int_s^{2s} r(t-s)^{-2} \Big(1 + \frac{2^{jm} r}{t-s}\Big)^{-2M} \|f_j(s,.)\|^2_{L^2} s^\beta dtds \\ &\lesssim \quad \sum_{k=1}^\infty 2^{-k} 2^{-2jmM} \int_0^{2^{-k}r} (2^k s)^\beta \|f_j(s,.)\|^2_{L^2} ds + \int_0^r \int_1^\infty \big(1 + 2^{jm}\sigma\big)^{-2M} \|f_j(s,.)\|^2_{L^2} s^\beta d\sigma ds \\ &\lesssim \quad 2^{-2jmM} \int_0^r \|f_j(s,.)\|^2_{L^2} s^\beta ds, \end{aligned}$$

where we used $\beta < 1$. We thus have

$$I_j^2 \lesssim 2^{-2jmM}(2^j r^{\frac{1}{m}})^n \|f\|^2_{T^{\infty,2,m}(t^\beta dtdy)},$$

and the condition $M > \frac{n}{2m}$ allows us to sum these estimates. □

Remark 3.3. Assuming off-diagonal estimates, instead of kernel estimates, allows to deal with differential operators L with rough coefficients. The harmonic analytic objects associated with L then fall outside the Calderón-Zygmund class, and it is common (see for instance [1]) for their boundedness range to be a proper subset of $(1,\infty)$. Here, our range $(\frac{2n}{n+m(1-\beta)},\infty]$ includes $[2,\infty]$ as $\beta < 1$, which is consistent with [2]. In the case of classical tent spaces, i.e., $m = 1$ and $\beta = -1$, it is the range $(2_*,\infty]$, where 2_* denotes the Sobolev exponent $\frac{2n}{n+2}$. We do not know, however, if this range is optimal.

Remark 3.4. Theorem 3.2 is a maximal regularity result for parabolic Carleson measures norms. This is quite natural from the point of view of non-linear parabolic PDE (where maximal regularity is often used), and such norm have, actually, already been used in the context of Navier-Stokes equations in [11], and, subsequently, for some geometric non-linear PDE in [12]. Theorem 3.1 is also reminiscent of Krylov's Littlewood-Paley estimates, and of their recent far-reaching generalization in [14]. In fact, the methods and results from [9], on which this paper relies, use the same circle of ideas (R-boundedness, Kalton-Weis γ multiplier theorem...) as [14]. The combination of these ideas into a "conical square function" approach to stochastic maximal regularity will be the subject of another forthcoming paper.

References

[1] P. Auscher, On necessary and sufficient conditions for L^p estimates of Riesz transforms associated to elliptic operators on $\mathbb{R}^n$ and related estimates. *Mem. Amer. Math. Soc. 871* (2007).

[2] P. Auscher, A. Axelsson, Weighted maximal regularity estimates and solvability of elliptic systems I. To appear in *Inventiones Math.*, arXiv:0911.4344.

[3] P. Auscher, A. Axelsson, Remarks on maximal regularity estimates To appear in *Parabolic Problems : Herbert Amann Festschrift, Birkhäuser.* arXiv:0912.4482.

[4] P. Auscher, S. Hofmann, M. Lacey, A. McIntosh, P. Tchamitchian, The solution of the Kato square root problem for second order elliptic operators on $\mathbb{R}^n$. *Ann. of Math. 156(2)* (2002) 633–654.

[5] P. Auscher, A. McIntosh, E. Russ, Hardy spaces of differential forms and Riesz transforms on Riemannian manifolds. *J. Geom. Anal. 18* (2008) 192–248.

[6] R. Coifman, Y. Meyer, E.M. Stein, Some new function spaces and their applications to harmonic analysis. *J. Funct. Anal. 62* (1985) 304–335.

[7] C. Fefferman; E.M. Stein, H^p spaces of several variables. *Acta Math. 129* (1972) 137–193.

[8] T. Hytönen, A. McIntosh, P. Portal, Kato's square root problem in Banach spaces. *J. Funct. Anal. 254* (2008) no. 3, 675–726.

[9] T. Hytönen, J. van Neerven, P. Portal, Conical square function estimates in UMD Banach spaces and applications to H^∞-functional calculi. *J. Analyse Math. 106* (2008) 317–351.

[10] N. Kalton, G. Lancien, A solution to the problem of L_p maximal-regularity. *Math. Z. 235* (2000), 559-568.

[11] H. Koch, D. Tataru, Well-posedness for the Navier-Stokes equations. *Adv. Math. 157* (2001), 22–35.

[12] H. Koch, T. Lamm, Geometric flows with rough initial data. preprint, arXiv:0902.1488v1.

[13] P.C. Kunstmann, L. Weis, Maximal L^p regularity for parabolic problems, Fourier multiplier theorems and H^∞-functional calculus, in *Functional Analytic Methods for Evolution Equations (Editors: M. Iannelli, R. Nagel, S. Piazzera). Lect. Notes in Math. 1855*, Springer-Verlag (2004).

[14] J. van Neerven, M. Veraar, L. Weis, Stochastic maximal L^p regularity. submitted, arXiv:1004.1309v2.

[15] L. de Simon. Un'applicazione della theoria degli integrali singolari allo studio delle equazioni differenziali lineare astratte del primo ordine. *Rend. Sem. Mat., Univ. Padova* (1964) 205-223.

[16] L.Weis, Operator-valued Fourier multiplier theorems and maximal L_p-regularity. *Math.Ann. 319* (2001), 735–758.

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