Retrograde Hot Jupiters from Secular Planet–Planet Interactions

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The search for extra-solar planets has led to the surprising discovery of many Jupiter-like planets in very close proximity to their host star[1], the so-called "hot Jupiters." Even more surprisingly, many of these hot Jupiters have orbits that are eccentric or highly inclined with respect to the equator of the star, and some (about 25%) appear to be in retrograde orbits[2]. How they get so close to the star in such orbits remains an open question. Slow migration though a protoplanetary disk[3,4] would produce orbits with low eccentricities and inclinations. Some models[7,8] invoke a companion star in the system, which perturbs the inner orbit and can produce increases in eccentricity and inclination but not retrograde orbits. Here we show that the presence of an additional, moderately inclined and eccentric massive planet in the system can naturally explain close, inclined, eccentric, and even retrograde orbits. We provide a complete and accurate treatment of the secular dynamics including both the key octupole-order effects and tidal friction. The flow of angular momentum from the inner orbit to the orbit of the perturber can lead to both high eccentricities and inclinations, and even flip the inner orbit. Previous treatments of the secular dynamics focusing on stellar-mass perturbers would not allow for such an outcome. In our treatment the component of the inner orbit's angular momentum perpendicular to the stellar equatorial plane can change sign; a brief excursion to very high

eccentricity during the chaotic evolution of the inner orbit can then lead to rapid "tidal capture," forming a retrograde hot Jupiter.

Despite many attempts[7−12], there is no model that can account for all the properties of the known hot Jupiter (HJ) systems. One model suggests that HJs formed far away from the star and slowly spiraled in, losing angular momentum and orbital energy to the protoplanetary disk[3,4]. This "migration" process should produce planets with low orbital inclinations and eccentricities. However, many HJs are observed to be on orbits with high eccentricities, and misaligned with respect to the rotation axis of the star (as measured through the Rossiter–McLaughlin effect[5]) and some of these (8 out of 32) appear to be in *retrograde* orbits. Secular perturbations from a distant binary star companion can produce increases in the eccentricity and inclination of a planetary orbit[6], but they cannot produce a retrograde orbit. During the evolution to high eccentricity, tidal dissipation near pericenter can force the planet's orbit to decay, potentially forming a HJ[7,8]. Another mechanism to produce a tilted orbit is via planet–planet scattering[9], possibly combined with other perturbers and tidal friction[13].

In our new treatment we allow for the magnitude and orientation of both orbital angular momenta to change (see Figure 1). The additional body (either an outer planet or a brown-dwarf companion) gravitationally perturbs the inner planet on time scales long compared to the orbital period (i.e., we consider the secular evolution of the system). We define the orientation of the inner orbit so that a prograde (retrograde) orbit has $i_1 < 90°$ ($i_1 > 90°$), where i_1 is the inclination of the inner orbit with respect to the *total* angular momentum, assumed parallel to the stellar rotation axis[1]. We assume a hierarchical configuration, with the outer

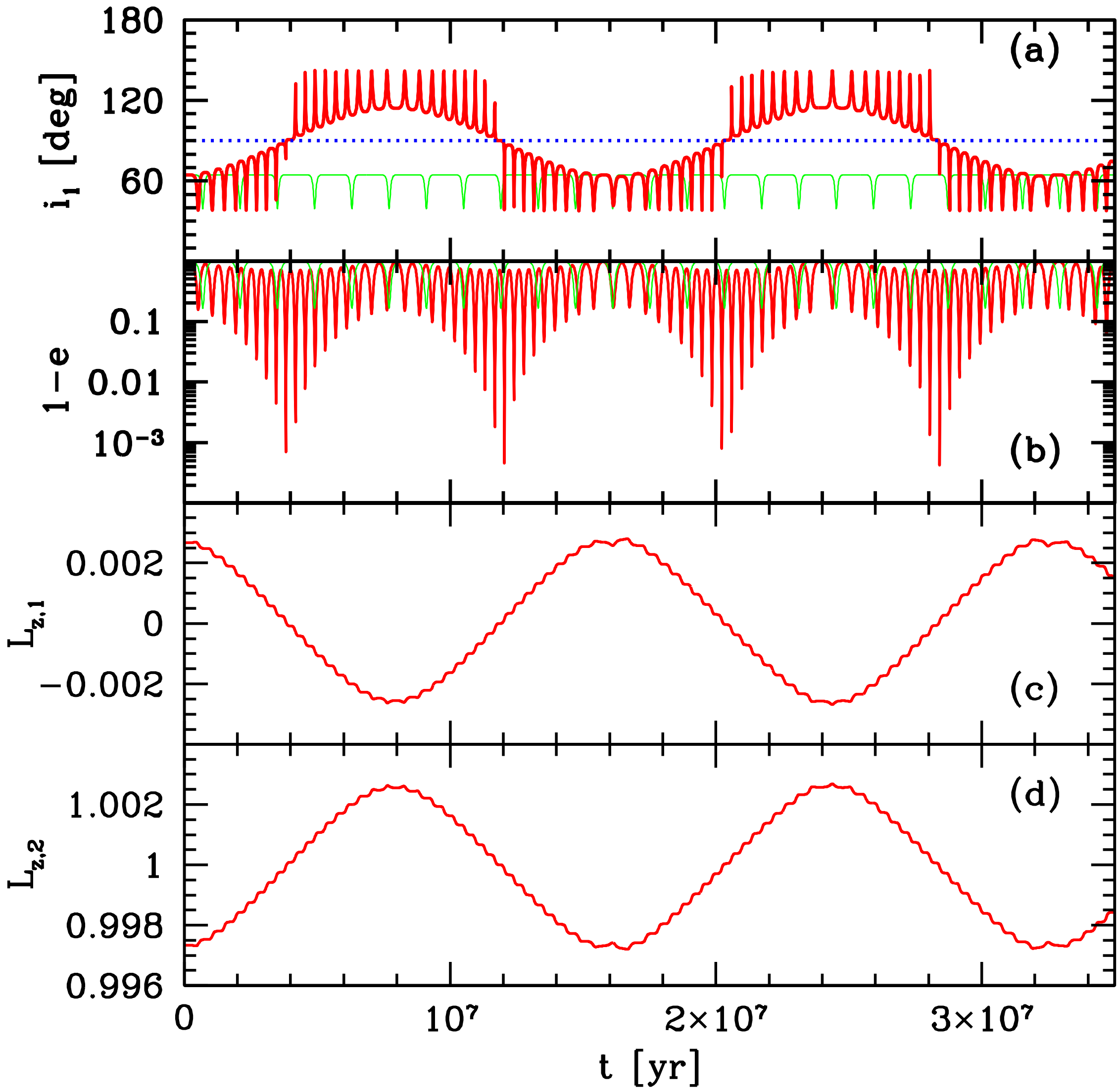
[1]The directly observed parameter is actually the *projected* angle between the spin axis of the star and the planet's angular momentum. However, for simplicity we focus here on the physical angle i_1.

perturber in a much wider orbit than the inner one. In the secular approximation the orbits may change shape and orientation but the semi-major axes (SMA) are strictly conserved in the absence of tidal dissipation[14,15]. In particular, the Kozai-Lidov mechanism[16,17,18] produces large-amplitude oscillations of the eccentricity and inclination when the initial relative inclination between the inner and outer orbits is sufficiently large ($40^\circ \lesssim i \lesssim 140^\circ$).

We have derived the secular evolution equations to octupole order using Hamiltonian perturbation theory[19,20,14]. In contrast to previous derivations of "Kozai-type" evolution, our treatment allows for changes in the z-components of the orbital angular momenta (i.e., the components along the total angular momentum) $L_{\rm z,1}$ and $L_{\rm z,2}$ (see supplementary material). The octupole-order equations allow us to calculate more accurately the evolution of systems with more closely coupled orbits and with planetary-mass perturbers. The octupole-level terms can give rise to fluctuations in the eccentricity maxima to arbitrarily high values[14,20], in contrast to the regular evolution in the quadrupole potential[7,8,18], where the amplitude of eccentricity oscillations is constant. Many previous studies of secular perturbations in hierarchical triples considered a stellar-mass perturber, for which $L_{\rm z,1}$ is very nearly constant[7,8,18]. Moreover, the assumption that $L_{\rm z,1}$ is constant has been built into previous derivations[21−24]. However, this assumption is only valid as long as $L_2 \gg L_1$, which is not the case in comparable-mass systems (with two planets). Unfortunately, an immediate consequence of this assumption is that a prograde orbit can never be turned into a retrograde orbit. Figure 1 shows the evolution of a representative system (here without tidal effects for simplicity): the inner planet oscillates between prograde and retrograde orbits as angular momentum flows back and forth between the two orbits.

Previous calculations of planet migration through "Kozai cycles with tidal friction" (KCTF)[7,8,15,18] produced a slow, gradual spiral-in of the inner planet. Instead, our more accurate treatment shows that the eccentricity can occasionally reach a much higher value than in the regular "Kozai cycles" calculated to quadrupole order. Thus, the pericenter distance will occasionally shrink on a short time scale (compared to the Kozai period), and the planet can then suddenly be tidally captured by the star. We propose to call this "Kozai capture". Kozai capture provides a new way to form HJs. If the capture happens after the inner orbit has flipped the HJ will appear in a retrograde orbit. This is illustrated in Figure 2. During the evolution of the system the inner orbit shrinks in steps (Fig. 2c) whenever the dissipation becomes significant, i.e., near unusually high eccentricity maxima. The inner orbit can then eventually become tidally circularized. This happens near the end of the evolution, on a very short time scale (see Fig. 2, right panels). In this final step, the inner orbit completely and quickly decouples from the outer perturber, and the orbital angular momenta then become constant. Therefore, the final SMA for the HJ is $\approx 2r_p$, where r_p is the pericenter distance at the beginning of the capture phase[25].

The same type of evolution shown in Figure 2 is seen with a broad range of initial conditions. Our mechanism requires that the outer perturber's orbit start with high inclination ($i_2 \gtrsim 50°$). The particular configuration in Figure 2 has a very wide outer orbit similar to those of directly imaged planets such as Fomalhaut b[26] and HR 8799b[27]. In this case the inner Jupiter could have formed in its original location in accordance with the standard core accretion model[28]. An alternative path to such a configuration involves strong planet–planet scattering in a closely packed initial system of several giant planets[13]. Independent of any particular planet formation mechanism, we predict that systems with misaligned HJs should also contain a much more distant massive planet on an inclined orbit.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(c)
1.002
1
0.998
0.996
$L_{z,2}$
(d)
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10^7
2×10^7
3×10^7
t [yr]

Fig. 1.— Dynamical evolution of a representative planet and brown dwarf system in the *point-mass limit* (i.e., with no tidal dissipation). Here the star has mass $1\,M_\odot$, the inner planet $1\,M_\mathrm{J}$ and the outer brown dwarf $40\,M_\mathrm{J}$. The inner orbit has SMA $a_1 = 6\,\mathrm{AU}$ and the outer orbit has SMA $a_2 = 100\,\mathrm{AU}$. The initial eccentricities are $e_1 = 0.001$ and $e_2 = 0.6$ and the initial relative inclination $i = 65^\circ$. We show from top to bottom: (a) the inner orbit's inclination (i_1); (b) the eccentricity of the inner orbit (as $1 - e_1$); (c) and (d) the z-component of the inner- and outer-orbit's angular momentum, normalized to the total angular momentum (where the z-axis is defined to be along the total angular momentum). The thin horizontal line in (a) marks the 90° boundary, separating prograde and retrograde orbits. The initial mutual inclination of 65° corresponds to an inner and outer inclination with respect to the invariable plane (perpendicular to z) of 64.7° and 0.3°, respectively. During the evolution, eccentricity and inclination of the inner orbit oscillate, but, in contrast to what would be predicted from evolution equations truncated to quadrupole order [shown by the thin curves in panels (a) and (b)], the eccentricity of the inner orbit can occasionally reach extremely high values and its inclination can become higher then 90°. The outer orbit's inclination always remains near its initial value. We note that more compact systems usually do not exhibit the same kind of regular oscillations between retrograde and prograde orbits illustrated here, as nonlinear effects become more important and are revealed at octupole order (see Fig. 2).

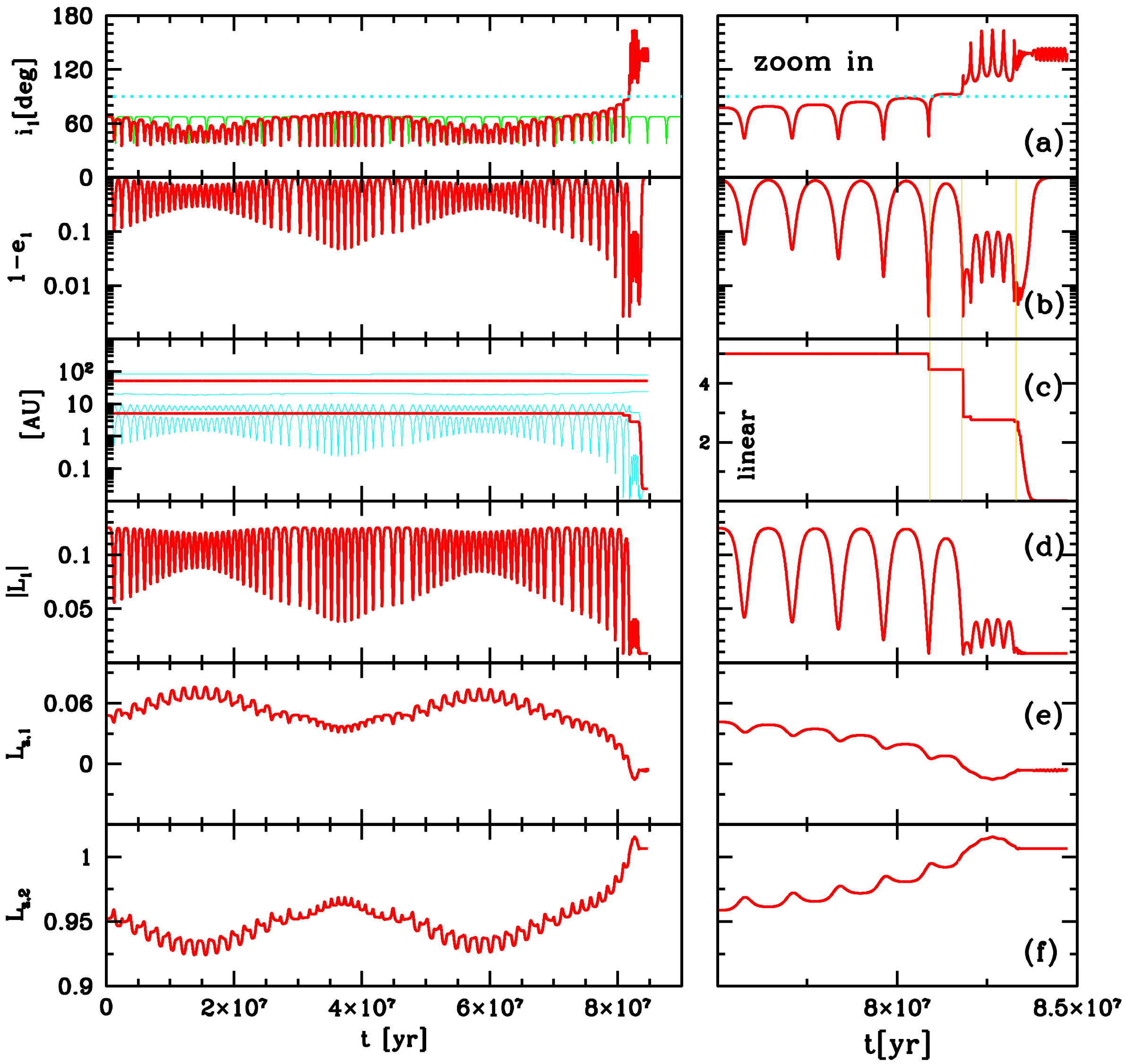
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Fig. 2.— Dynamical evolution of a representative two-planet system with tidal dissipation included. The inner planet becomes retrograde at 82 Myr, and remains retrograde after circularizing into a Hot Jupiter. Here the star has mass $1\,M_\odot$, the inner planet $1\,M_{\rm J}$ and the outer planet $3\,M_{\rm J}$. The inner orbit has SMA $a_1 = 5\,$AU and the outer orbit has SMA $a_2 = 51\,$AU. The initial eccentricities are $e_1 = 0.001$ and $e_2 = 0.6$ and the initial relative inclination $i = 74.5^\circ$. We show: (a) the inner orbit's inclination (i_1); (b) the eccentricity of the inner orbit (as $1 - e_1$); (c) the SMA, peri-, and apo-center distances for the inner orbit and the peri- and apo-center distance for the outer orbit; (d) the magnitude of the angular momentum of the inner orbit; and, in (e) and (f) the z-components of the inner and outer orbit's angular momentum, normalized to the total angular momentum. The initial mutual inclination of 74.5° corresponds to inner- and outer-orbit inclinations of 67.6° and 6.9°, respectively. During each excursion to very high eccentricity for the inner orbit [marked with vertical lines in panels (b) and (c)], tidal dissipation becomes significant. Eventually the inner planet is tidally captured by the star and its orbit becomes decoupled from the outer body. After this point the orbital angular momenta remain nearly constant. The final SMA for the inner planet is at 0.024 AU, typical of a hot Jupiter. The thin curve in panel (a) shows the evolution in the quadrupole approximation (but including tidal friction), demonstrating that the octupole-order effects lead to a qualitatively different behavior. For the tidal evolution in this example we assume tidal quality factors $Q_\star = 5.5 \times 10^6$ for the star and $Q_{\rm J} = 5.8 \times 10^6$ for the hot Jupiter (see supplementary material). We monitor the pericenter distance of the inner planet to ensure that it always remains outside the Roche limit[29].

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Supplementary material

1. Octupole-order Evolution Equations and Angular Momentum Conservation

Our new derivation corrects an error in previous Hamiltonian derivations of the secular evolution equations.

We consider a hierarchical triple system consisting of an inner binary (m_1 and m_2) and a third body (m_3) in a wider exterior orbit. We describe the system using canonical variables, known as Delaunay's elements, which provide a particularly convenient dynamical description of our three-body system. The coordinates are chosen to be the mean anomalies, l_1 and l_2, the longitudes of ascending nodes, h_1 and h_2, and the arguments of periastron, g_1 and g_2, where subscripts $1, 2$ denote the inner and outer orbits, respectively. Their conjugate momenta are:

$$\begin{aligned} L_1 &= \frac{m_1 m_2}{m_1+m_2}\sqrt{k^2(m_1+m_2)a_1}\ , \qquad (1)\\ L_2 &= \frac{m_3(m_1+m_2)}{m_1+m_2+m_3}\sqrt{k^2(m_1+m_2+m_3)a_2}\ , \end{aligned}$$

$$G_1 = L_1\sqrt{1-e_1^2}\ , \quad G_2 = L_2\sqrt{1-e_2^2}\ , \qquad (2)$$

and

$$H_1 = G_1\cos i_1\ , \quad H_2 = G_2\cos i_2\ , \qquad (3)$$

where G_1 and G_2 are the absolute values of the angular momentum vectors ($\mathbf{G}_1$ and $\mathbf{G}_2$), and H_1 and H_2 are the z-components of these vectors.

We choose to work in a coordinate system where the total initial angular momentum of the system lies along the z axis. The transformation to this coordinate system is known as the elimination of the nodes[30,16]; the x-y plane in this coordinate system is known as the invariable plane. Figure 3 shows the resulting configuration of the orbits. We obtain simple relations between H_1, H_2, G_1 and G_2, using $\mathbf{G}_{\rm tot} = \mathbf{G}_1 + \mathbf{G}_2$:

$$\cos i = \frac{G_{\rm tot}^2 - G_1^2 - G_2^2}{2G_1G_2}\ , \qquad (4)$$

$$H_1 = \frac{G_{\rm tot}^2 + G_1^2 - G_2^2}{2G_{\rm tot}}\ , \qquad (5)$$

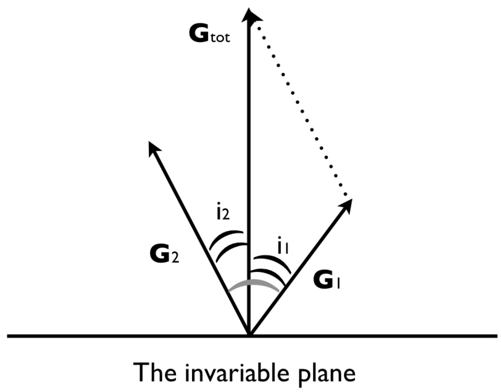

Fig. 3.— The angular momenta of the bodies after the elimination of the nodes (see also Ref. [14]). Note that all three vectors are in the same plane. The mutual inclination $i = i_1 + i_2$ is the angle between $\mathbf{G}_1$ and $\mathbf{G}_2$.

$$H_2 = \frac{G_{\rm tot}^2 + G_2^2 - G_1^2}{2G_{\rm tot}}\,, \tag{6}$$

where the relation for H_1 comes from setting $\mathbf{G}_2 = \mathbf{G}_{\rm tot} - \mathbf{G}_1$ (and similarly for H_2). Because total angular momentum is conserved by the evolution of the system, we must have $\mathbf{G}_1(t) + \mathbf{G}_2(t) = \mathbf{G}_{\rm tot} = G_{\rm tot}\hat{z}$, implying that

$$h_1(t) = h_2(t) - \pi. \tag{7}$$

The Hamiltonian for the three-body system can be transformed into the form

$$\mathcal{H} = \mathcal{H}_1^K(L_1) + \mathcal{H}_2^K(L_2) + \mathcal{H}_{12}, \tag{8}$$

where $\mathcal{H}_1^K$ and $\mathcal{H}_2^K$ represent the Keplerian interaction between bodies 1 and 2 and the central body, and $\mathcal{H}_{12}$ represents the interaction between body 1 and body 2. The Kepler Hamiltonians depend only on the momenta L_1 and L_2, while the interaction Hamiltonian, $\mathcal{H}_{12}$, depends on all the coordinates and momenta. Due to the rotational symmetry of the problem, $\mathcal{H}_{12}$ depends on h_1 and h_2 only through the combination $h_1 - h_2$. Because we are interested in secular effects, we average the Hamiltonian over the coordinates (angles) l_1 and l_2, obtaining the secular Hamiltonian

$$\bar{\mathcal{H}} = \mathcal{H}_1^K(L_1) + \mathcal{H}_2^K(L_2) + \bar{\mathcal{H}}_{12}, \tag{9}$$

where

$$\bar{\mathcal{H}}_{12} = \frac{1}{4\pi^2} \int_0^{2\pi} dl_1 \int_0^{2\pi} dl_2 \, \mathcal{H}_{12}. \tag{10}$$

For simplicity we first focus on the quadrupole approximation, where the error is more easily shown; it is then straightforward to see its effects at all orders in the hierarchical triple system's secular dynamics expansion. The quadrupole Hamiltonian results from expanding $\bar{\mathcal{H}}_{12}$ to second order[2] in a_1/a_2:

$$\bar{\mathcal{H}}_{12} = \bar{\mathcal{H}}_{12}^{(2)} + \mathcal{O}\left(\frac{a_1}{a_2}\right)^3. \tag{11}$$

The resulting quadrupole-order Hamiltonian, $\bar{\mathcal{H}}_{12}^{(2)}$, depends only on the coordinates g_1, h_1, and h_2, with the latter two appearing only in the combination $h_1 - h_2$:

$$\bar{\mathcal{H}}_{12}^{(2)} = \bar{\mathcal{H}}_{12}^{(2)}(g_1, h_1 - h_2). \tag{12}$$

Previous calculations[16,19] eliminated h_1 and h_2 from the Hamiltonian using eq. (7), obtaining a quadrupole Hamiltonian that depends only on g_1. But, this is incorrect! Such a Hamiltonian would imply that all quantities in eq. (5) are constant *except* G_1, i.e. that eq. (5) is incorrect. Thus the previously used formalism did not conserve angular momentum. The initial Hamiltonian is spherically symmetric, and therefore *does* conserve angular momentum; the correct quadrupole Hamiltonian does as well. Because the correct quadrupole Hamiltonian depends on h_1 and h_2 through the combination $h_1 - h_2$, we have

$$\dot{H}_1 = -\dot{H}_2, \tag{13}$$

or

$$H_1 + H_2 = G_{\rm tot} = {\rm const.} \tag{14}$$

The mathematical error affects all orders in secular perturbations. The independence of the secular quadrupole Hamiltonian on $h_{1,2}$ was the source[16] of the famous relation $\cos i_{1,2}\sqrt{1 - e_{1,2}^2} =$

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[2]The first order term in a_1/a_2 averages to zero, so the quadrupole term is the first term to contribute to $\bar{\mathcal{H}}_{12}$

const. In the correct derivation, this relation does not always hold. However, in a certain limit, it does. From eq. (5), we see that

$$\dot{H}_1 = \frac{G_1}{G_{\rm tot}}\dot{G}_1 - \frac{G_2}{G_{\rm tot}}\dot{G}_2. \tag{15}$$

When $G_2 \sim G_{\rm tot} \gg G_1$, we have

$$\dot{H}_1 \approx -\frac{G_2}{G_{\rm tot}}\dot{G}_2. \tag{16}$$

At the quadrupole level $\bar{H}_{12}^{(2)}$ is independent of g_2, so $\dot{G}_2 = 0$, implying

$$\dot{H}_1 \approx 0, \tag{17}$$

when $G_2 \sim G_{\rm tot} \gg G_1$. This is precisely the limit considered in previous works[16,19], so their conclusion that $H_{1,2} = \cos i_{1,2}\sqrt{1-e_{1,2}^2} = $ const is correct (though not for the reason they claim), but the limit where $G_2 \gg G_1$ is not sufficient for our work.

In some later studies, the assumption that $H_1 =$ const was built into the calculations of secular evolution for various astrophysical systems[21,22,23,24], even when the condition $G_2 \gg G_1$ was not satisfied. Moreover many previous studies simply set $i_2 = 0$, which is repeating the same error. In fact, given the mutual inclination i, the inner and outer inclinations i_1 and i_2 are set by the conservation of total angular momentum:

$$\cos i_1 = \frac{G_{\rm tot}^2 + G_1^2 - G_2^2}{2G_{\rm tot}G_1}\ , \tag{18}$$

$$\cos i_2 = \frac{G_{\rm tot}^2 + G_2^2 - G_1^2}{2G_{\rm tot}G_2}\ . \tag{19}$$

2. Tidal Friction

We adopt the tidal evolution equations of Eggleton, Kiseleva & Hut (1998), which are based on the equilibrium tide model of Hut (1981). The complete equations can be found in Fabrycky & Tremaine (2007, eqs A1–A5). Following their approach (see their eq. A10) we set the tidal quality factors $Q_{1,2} \propto P_{\rm in}$ [see also Hansen 2010, eq. (11)]. This means that the viscous times of the star and planet remain constant; the representative values we adopt here are 5 yr for the star and 1.5 yr for the planet, which correspond to $Q_\star = 5.5 \times 10^6$ and $Q_J = 5.8 \times 10^6$, respectively, for a 1-day period.

3. Comparison to Observations

The observable parameter from the Rossiter–McLaughlin effect is the *projected* angle between the star's spin and the orbital angular momentum (the projected obliquity)[5]. Here instead we focus on the true angle between the orbital angular momentum of the inner planet and the invariable plane. Projection effects can cause these two quantities to differ in magnitude, or even sign.

Moreover, several mechanisms have been proposed in the literature that could, under certain assumptions, directly affect the spin axis of the star. These mechanisms can re-align the stellar spin axis through tidal interactions with either a slowly spinning star[29] or with the outer convective layer of a sufficiently cold star[10]. Additionally, a magnetic interaction between the star and a significantly charged protoplanetary disk with negligible accretion could also lead to misalignment between the stellar spin and the disk[12].

These effects can potentially complicate the interpretation of any specific observation. Nevertheless, if hot Jupiters are produced by the simple mechanism described here, many of their orbits should indeed be observed with large projected obliquities.

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