Relativistic Constraints for a Naturalistic Metaphysics of Time

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Abstract

The traditional metaphysical debate between static and dynamic views in the philosophy of time is examined in light of considerations concerning the nature of time in physical theory. Adapting the formalism of Rovelli (1995, 2004), I set out a precise framework in which to characterise the formal structure of time that we find in physical theory. This framework is used to provide a new perspective on the relationship between the metaphysics of time and the special theory of relativity. An integral feature of this relationship is the dual representations of time we find in special relativity. I extend this analysis to the general theory of relativity with a view to prescribing the constraints that must be heeded for a metaphysical theory of time to remain within the bounds of a naturalistic metaphysics.

Key words: static time, dynamic time, special relativity, Minkowski spacetime, general relativity.

1 Introduction

The A-theory of time proclaims that temporal passage is an objective feature of reality. Implicit in this view is that the temporal instant that embodies this passage, the present, maintains a privileged status over and above the temporal instants that have already 'passed' (the past) and that are yet to 'pass' (the future). In contrast, the B-theory of time is characterised by its rejection of temporal passage as a real and objective feature of the world. As such, there is no privileged instant and all times, past, present and future, are considered to be equally real according to this view. The division between these opposing temporal theories defines what we will call the *traditional metaphysical debate* on the nature of time. It has been suggested that Einstein's special theory of relativity seriously compromises the viability of various formulations of the A-theory of time[1];

[1]See, for instance, Rietdijk (1966), Putnam (1967), Maxwell (1985) and Saunders (2002).

Minkowski's formulation of the special theory of relativity as a four dimensional spacetime has been instrumental in creating the perception that it provides strong evidence for a B-theory of time. On the other hand, much work has been carried out attempting to show the compatibility of special relativity and A-theories of time[2] with a general sentiment emerging that Minkowski spacetime is the wrong sort of entity to definitively adjudicate either way on the traditional debate in the philosophy of time.

This project is not an attempt to enter this debate and argue for or against either the A- or B-theory of time. Nor it is a concern of this project to attempt to argue the consistency of either of these temporal models with classical relativity theory. The purpose of this paper is to investigate and outline the constraints, imposed by the temporal structure of classical physical theory[3], that the traditional debate must heed in order to remain within the bounds of a naturalistic metaphysics.[4] As one can infer from the introductory remarks above, the special theory of relativity has been conspicuously present in the traditional debate and, therefore, this might make one wonder whether such a project is already *fait accompli*. There are two reasons to be cautious of this presumption. To begin with, existing attempts to answer the question as to why the formal temporal structure of Minkowski spacetime does not preclude the possibility of objective temporal passage (some of which we will meet in §3) appear to lack a precise characterisation of exactly how time is represented in special relativity. The initial goal of this paper is to adopt a *formal characterisation* of time in special relativity (§4) with the resulting picture providing a new perspective on why the constraints imposed by special relativity on the traditional debate are not so restrictive as to quash the debate. The second reason is that the temporal structure of general relativity must be considered also if one is to remain within the bounds of a naturalistic metaphysics. The ultimate goal of this paper, then, is to extend the precise characterisation of time in special relativity to general relativity which, as we will see, imposes much more restrictive constraints.

1.1 Background

The traditional metaphysical debate has its origin in an analysis due to McTaggart (1908), who is credited with clearly distinguishing two ways in which we differentiate positions in time (the contents of these positions being events): each position is either past, present or future; or each position is either earlier than or later than some other position. The series of positions running from the past to the present, and then from the present to the future McTaggart labels the A-series. The A-series is characterised by the position of the present; those positions in time earlier than the present are in the past and those later

[2] See, for instance, McCall (1976), Hinchliff (1996), Tooley (2000), Zimmerman (2008) and Savitt (forthcoming).

[3] Other physical theories, especially quantum theory, may impose further constraints on our temporal models but these will not be considered here.

[4] A naturalistic metaphysics is meant here *à la* Ladyman and Ross (2007, Ch. 1): a "metaphysics that is motivated exclusively by attempts to unify hypotheses and theories that are taken seriously by contemporary science".

than the present are in the future. The present is in some sense in motion through the A-series as positions in time change from future to present, then from present to past. The series of positions which run from earlier to later independently of the present McTaggart labels the B-series. The B-series is not defined by any position in time labelled as the present. Any position in time is related to any other position in time by either the 'earlier than', 'later than' or 'simultaneous with' relation irrespective of which position might be referred to as the present. These relations are defined by the temporal ordering of the positions along the series and are thus unchanging and objective.

By distinguishing between the ways that we differentiate positions in time, we can construct two types of temporal models. The A-theory of time is the view that an adequate description of temporal reality requires either the A-series alone, or both the A-series and the B-series together. The A-theory is often referred to as a *dynamic* view of time. We will characterise dynamic time here as the claim that we exist in a privileged present that is in some sense 'flowing' through successive instants of time.[5] The present is thus conceived as a privileged element of our reality which demarcates the past from the future in some objective respect. There are two ways that we can understand this privileged present. We can understand the present as ontologically privileged, whereby the notion of flow is envisaged as the existential displacement of the privileged time instant by its successor. Accordingly, each time instant then 'comes into' existence as the present instant and then 'goes out of' existence as a new time instant becomes the present. We can alternatively understand the present as metaphysically privileged, whereby flow is interpreted as the evolution of some property of 'presentness' across consecutive time instants that are ontologically undifferentiated.[6]

The second of the temporal models that we can construct, the B-theory of time, is the view that a complete description of temporal reality can be given by the B-series. The B-theory is often referred to as a *static* or *block universe* model of time. We will take the block universe model to be characterised by two claims: every position in the B-series exists (the past, present and future are equally real), and the A-series is unreal (there is no privileged instant nor any objective flow of time). One can imagine constructing the block by arranging all the positions of the B-series as an unchanging temporal map of the universe and then augmenting this with the spatial dimensions of the universe to create a four dimensional block which contains all the spatial and temporal relations between events. The block universe view forges a strong analogy between the static conception of time and our ordinary conception of space; there is nothing objective about labelling a particular position in space 'here' nor claiming the contents of 'here' to be more real than the contents of 'there', just as there is nothing objective about labelling a particular time

[5] For a nice illustration of some of the various ways this conception of dynamic time can be expressed, see Williams (1951).

[6] It is well known that the metaphysical notion of flow falls afoul of McTaggart's original analysis of the A- and B-series of time since a changing property such as presentness requires a separate temporal dimension in which to change, which is thought to be undesirable. We will leave this issue to one side for the purposes of this project.

'now' whose contents can be thought of as any more real than the contents of any other position within the block. As we will see in §2, there exist persuasive arguments that this is more than just a mere analogy.[7]

1.2 Outline of the paper

This investigation will proceed as follows. I begin in §2 by introducing the special theory of relativity, paying particular attention to the geometric formalism and temporal structure of Minkowski spacetime. The dual formulation of time in special relativity as both proper time and coordinate time is also introduced here. In §3 I sketch an argument from the literature, which I call the proper time argument, to the effect that neither static nor dynamic views of time are precluded solely by the formal temporal structure of Minkowski spacetime and briefly examine some possible explanations as to why this might be the case. I suggest that the proper time argument crucially turns on the dual formulation of time in special relativity. I set out in §4 a precise framework for characterising time and use this framework to formalise the ambiguity with respect to the dual formulation of time in special relativity. I employ this formalism to argue for a more general explanation as to why the formal temporal structure of Minkowski spacetime alone precludes neither metaphysical position in the traditional debate. In §5 I extend the analysis to the picture of time that arises in the general theory of relativity and set out the classical constraints that must be respected by a metaphysical theory of time to remain within the scope of a naturalistic metaphysics.

2 Minkowski spacetime

Einstein (1952) developed the special theory of relativity in a 1905 paper, on the shoulders of the pioneering work of FitzGerald, Lorentz and Poincaré. He was motivated in part by the fact that any model of space and time must tell some story about how information concerning distant spatial and temporal regions is made available to a spatiotemporally bound observer. He realised that it is *light signals* that connect us with distant parts of space and time. Einstein used this insight to construct a principled theory of space and time that forms an axiomatic basis for deriving the Lorentz transformations. In 1908, Minkowski (1952) built upon Einstein's special theory of relativity by formally uniting the structure of space and time into a four dimensional object, which he called spacetime: three dimensions representing space and one dimension representing time.

[7] The distinction here between A- and B- theories of time follows that of Dainton (2001, p. 11). The A- and B-theories of time can also be characterised as the 'tensed' and 'tenseless' theories of time, respectively (as in Le Poidevin (1998), for instance). Under such a construal, the A-theory takes the properties picked out by terms such as past, present and future (known as 'tenses') to be real, i.e. to be objective properties of reality. On the other hand, the B-theory denies the reality of tenses. Despite this alternative construal, the core difference between the A- and B-theory remains whether temporal passage is objective or not, as in the above characterisation, and thus 'tense' is not taken to be a significant notion here.

The significance of Minkowski's extension of Einstein's theory is that he formulated the theory in a geometric framework. In this section I introduce the geometric structure of Minkowski spacetime, paying particular attention to the picture of time that arises therein.[8]

Minkowski spacetime can be represented by a geometry $(\mathcal{M}^4, g_{\mu\nu})$, which consists of a differentiable, four dimensional manifold, $\mathcal{M}^4$, and a Lorentzian metric, $g_{\mu\nu}$, with signature $(1,3)$. The manifold is interpreted as representing the set of spacetime points and the metric can be interpreted as the geometric instructions by which these spacetime points are connected. Given a particular point p in $\mathcal{M}^4$, and a four-vector $\mathrm{d}x^\mu$ in the tangent space $T_p\mathcal{M}^4$ at p, we can use the metric to construct the line element, $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$, for each spacetime point in $\mathcal{M}^4$. The invariance of $\mathrm{d}s^2$ according to the special theory of relativity endows Minkowski spacetime with a conformal structure: we say that $\mathrm{d}x^\mu$ is a *timelike* vector if $\mathrm{d}s^2 > 0$, a *lightlike* vector if $\mathrm{d}s^2 = 0$ and a *spacelike* vector if $\mathrm{d}s^2 < 0$. The metric thus determines a *lightcone structure* in the tangent space at every point of $\mathcal{M}^4$, where the lightlike vectors define the boundary of the cone, the timelike vectors the inside of the cone and the spacelike vectors the outside of the cone.

Since we want to be able to use this formalism to model the behaviour of objects in spacetime, we need to extend these classifications to *curves* in $\mathcal{M}^4$. We say that a curve is timelike, lightlike or spacelike if its tangent vector field is characterised as such at every point. We can now interpret timelike curves as the possible spacetime paths of massive particles and lightlike curves as the possible spacetime paths of massless particles (i.e. photons); the actual paths of such objects in spacetime are called *worldlines*. This then gives us a causal structure to Minkowski spacetime: an observer situated at any position in spacetime can divide their surrounding spacetime into a causally contiguous region (the timelike region plus the lightlike boundaries) and causally separated (spacelike) region (see Figure 1). The division of spacetime in this way is unique to each spacetime point.

We say that a Minkowski spacetime is *temporally orientable* if there exists a continuous timelike vector field on $\mathcal{M}^4$. We can then stipulate a temporal orientation to this vector field, simply by picking a future direction, and thus define any timelike or lightlike vector at a point of $\mathcal{M}^4$ as *future directed* or *past directed* with respect to this orientation. As above, a curve is future directed or past directed with respect to this orientation if its tangent vector field is characterised as such at every point. Time is then associated with the parameter employed to parametrise a future directed timelike curve in $\mathcal{M}^4$; such a parametrised curve describes the dynamical behaviour of an object in spacetime (it is only through such a parametrisation that we can begin to speak of 'time instants' in special relativity). There are two ways that a curve can be parametrised according to an arbitrary observer in spacetime: the curve can be parametrised by the time as measured by a clock moving along the curve in question; or the curve can be parametrised by the time as measured by a clock at rest in the observer's frame of reference. Let us consider

[8]The exposition here mostly follows Malament (2007).

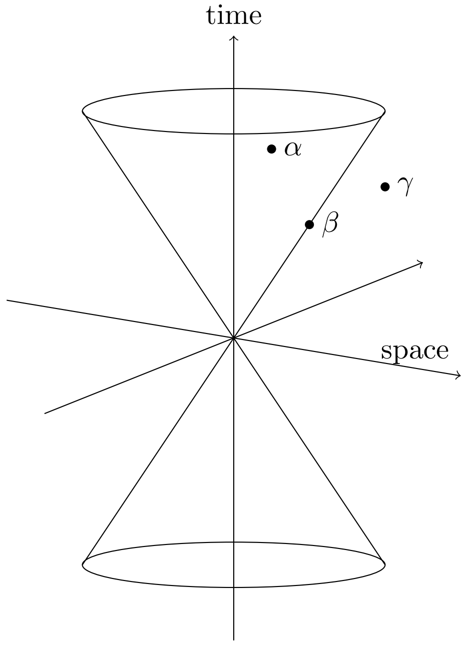

Figure 1: The causal structure of Minkowski spacetime for an observer at the origin: α is timelike separated from the observer, β lightlike separated and γ spacelike separated.

these options more formally.

Given a future directed timelike curve, γ, between spacetime points s_1 and s_2 in $\mathcal{M}^4$ with tangent field $\mathrm{d}x^\mu$, we can define the elapsed time between s_1 and s_2, τ, with which to parametrise γ, as the arc length of the curve:

$$\tau = |\gamma| = \int_{s_1}^{s_2} (g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu)^{\frac{1}{2}}\ \mathrm{d}s. \tag{1}$$

The parametrisation of γ by τ is a 'natural' parametrisation since the arc length, as a function of the invariant line element, is an observer independent quantity. We thus call τ *proper time* and associate it with the time that a clock will measure along its own (not necessarily inertial) worldline. One can also generate an observer dependent parametrisation of γ: an observer in some arbitrary reference frame can employ a clock at rest with respect to the observer (proper time along the observer's worldline) to define the elapsed time, t, with which to parametrise γ. Such an observer has, in effect, stipulated an arbitrary coordinatisation of the manifold, with a time coordinate coinciding with the observer's trajectory through spacetime, with which to describe spacetime dynamics. We thus call t *coordinate time* and associated it with a global time measure corresponding to the fourth coordinate of the spacetime manifold (so long as the motion of the observer in question is inertial). Since coordinate time is observer dependent, while proper time is observer independent, the latter is taken to have direct physical significance, while the former is not. This dual formulation of time, as proper time and as coordinate time, in

Minkowski spacetime will play a crucial role in the discussion below.[9]

The formal relationship between proper time and coordinate time is given by the Lorentz transformations (which are embodied in $g_{\mu\nu}$) and is dependent upon the relative velocity of the reference frames from which each time measure is procured. Time intervals (and the relations between them) as measured for a set of events in one reference frame vary from those measured in a second frame moving relative to the first. In addition, the temporal *order* of events at spacelike separation from an observer is also frame dependent; observers in motion relative to one another may record a different temporal order for the very same observed events. It follows that whether two events are *simultaneous* or not is again observer dependent. Thus there is no absolute fact of the matter as to whether two spacelike separated events stand in either the 'earlier than', 'later than' or 'simultaneous with' relation to each other; this relation is dependent upon the observer's state of motion. Due to this Lorentzian temporal structure, Minkowski spacetime cannot in general be decomposed into distinctly spatial and temporal elements.[10] However, provided that one has stipulated a particular time coordinate coinciding with an inertial timelike trajectory, one is able to *foliate* the Minkowski manifold into spacelike slices each consisting of a set of simultaneous events.

We now turn our attention to the relationship between the temporal structure of Minkowski spacetime and the traditional metaphysical debate on the nature of time.

3 Finding time in special relativity

It is clear from the above characterisation that Minkowski spacetime conspicuously constitutes a four dimensional object; indeed, I believe that it is this fact that is at the heart of the conception that Minkowski spacetime *actually is* the block universe. Upon closer analysis, however, the argument from the formal temporal structure of Minkowski spacetime against the dynamic view of time is not quite so straightforward. As mentioned in the introduction, particular A-theories of time have been defended against claims that Minkowski spacetime precludes their possibility and, rather than revisit some well trodden ground, I will instead present here an argument (following Dieks (2006) and Ellis (2007)) that will not only illustrate the sort of analysis that leads one to this conclusion but will also start us in the right direction to providing in the next section a more general argument as to why the formal temporal structure of Minkowski spacetime alone precludes neither static nor dynamic views of time. This is all, of course, with a view to describing in §5 the constraints prescribed by the general theory of relativity that must be recognised by a metaphysical theory of time to fall within the scope of a naturalistic

[9] The distinction between proper time and coordinate time as formulations of time in the special theory of relativity has also been emphasised by Kroes (1985) and Rovelli (1995) and, more recently, by Savitt (forthcoming).

[10] In contrast, recall that the ordinary Euclidean metric imposed on a four dimensional manifold results in a Newtonian spacetime in which space and time can be globally separated as distinct elements of the manifold.

metaphysics.

The most pressing concern for an A-theorist when presented with Minkowski spacetime is the question of how to endow the manifold with an objective temporal passage. Since temporal passage invariably involves change, for Minkowski spacetime to include temporal passage as an objective element something within the manifold would have to undergo some sort of 'change'. The most attractive candidate for this "something" is an objective 'now': a hyperplane of simultaneity within spacetime which privileges a particular time instant and which embodies the passage of time. The problem at this point for the A-theorist is that no such hyperplane of simultaneity is privileged as such; due to the relativity of simultaneity, many hyperplanes of simultaneity can be specified depending on the relative motion of the observer and none of these can claim any special status as being a privileged time instant. Thus, it seems as if there is no scope for an objective 'now' and thus no scope for objective temporal passage.

Not all is lost for the dynamic view of time though. While this argument does provide some important restrictions on the form that an objective temporal passage can take, it does not show that objective temporal passage is incompatible with the formal temporal structure of Minkowski spacetime. There is no objective hyperplane of simultaneity in Minkowski spacetime and thus no objective *global* 'now'. However, a global 'now' is not the only candidate for the basis of temporal passage. While an integral element of the special theory of relativity is that there is no absolute fact of the matter about global temporal orderings, there are some facets of Minkowski spacetime that are absolute. Recall (Figure 1) that the conformal structure of Minkowski spacetime separates the manifold into timelike separated events (inside the light cone) and spacelike separated events (outside the light cone). Observers at the same position in spacetime but in motion relative to one another will define their hyperplane of simultaneity and their local direction of time skewed with respect to one another, but the conformal structure of Minkowski spacetime is inherent in the geometry; they will agree on which regions of spacetime are timelike separated and which regions of spacetime are spacelike separated.

We encountered the claim above that according to relativity theory there is no objective fact of the matter as to whether two spacelike separated events stand in either the 'earlier than', 'later than' or 'simultaneous with' relation to each other. In contrast, the causal structure of Minkowski spacetime permits that for future directed timelike curves there *is* an objective fact of the matter as to which events are past and which events are future. This temporal ordering of events is only local (i.e. applicable to a single point on a worldline) since observers at different spacetime locations with varied relative motions will disagree on the ordering of spacelike separated, or nonlocal, events. One can then imagine any single spacetime point on a future directed worldline as a candidate for an objective *local* 'now'. Minkowski spacetime would then contain many such local objective 'nows', each associated with a single worldline. The formal geometric structure of Minkowski spacetime then does *not* preclude the possibility of an objective local 'now' (though it certainly does limit the scope of such a ‘now’) and therefore does not preclude out of hand this particular form of objective temporal passage. Let us call this argument the *proper time argument*.

It is far from obvious that the metaphysical notion of dynamic time that arises from the proper time argument is indeed a viable metaphysical position. The A-theorist who wishes to develop such a view faces a tough challenge explaining exactly how consistency can be maintained between the dynamic local nows to produce the sort of phenomenology that we (as spatiotemporal beings) experience. I contend, however, that if the resulting picture of time is apt to be rejected as unfavourable, this would evidently not be as a result of the formal geometric structure of Minkowski spacetime. These issues aside, it is interesting to ask then why, as a result of the proper time argument, we find ourselves unable to undermine either static or dynamic views of time with solely the temporal structure of Minkowski spacetime. Let us briefly consider two possible explanations.

Dieks (1991) makes the suggestion that it is the universality of physical theories that prevents them from including specific metaphysical commitments concerning the flow of time. According to Dieks, physical theories are concerned with the task of giving descriptions of universal laws, valid at all times and places and this can only be achieved if all times and places are treated on an equal footing; there are no times or places to which the laws of physics must be anchored. This is the source of the purely relational nature of physical laws. An absolute and global difference between past and future, for instance, simply does not and cannot exist in a physical theory. The specific properties of events are disregarded in physical theories and only what is common to all processes of a particular kind is retained. As Dieks puts it, “the laws of physics by themselves cannot reveal what time it is” (1991, p. 258). Thus according to this view, the ‘now’ of experience may indeed reflect something objectively real but we should not expect it to play a role in our physical theories, including Minkowski spacetime, and thus our physical theories should not be able to provide evidence for any particular metaphysical view of time.[11]

While there is at least something compelling about this argument, I think it does not get to the heart of the reason as to why Minkowski spacetime cannot provide definitive adjudication on the metaphysical nature of time. Depending upon how one wishes to characterise universality, to lean on the universality of physical theories to argue against a metaphysical position in some sense begs the question: if objective temporal passage were an element of our reality, a truly universal physical theory would pick out this element of reality and account for it in its structure. For instance, if it were the case that the past and future were unreal and only the present were real, a universal theory valid at all times and places would only need to be valid at the present time to be universal. In a certain way, then, the laws of physics *would* be anchored to a particular time, namely the present, opening the door for a possible metaphysical thesis concerning the flow of

[11]This sentiment is echoed by McCall (2001): “Although not endorsing time flow, modern physics does not rule it out as a logical impossibility. Its attitude is empirical rather than logical; time flow may exist, but (i) there is no hard experimental evidence for it, and (ii) it plays no role in physical theory.”

time.[12]

Another possible explanation as to why the formal temporal structure of Minkowski spacetime might not preclude either static or dynamic views concerns the underdetermination of metaphysics by physics. One could argue that we should never expect conclusive determination of the metaphysics of our best physical theories, in the same way that we do not expect complete determination of our physical theories from observation. Again, though, this does not provide the explanation we are looking for. The claim against which the proper time argument is levelled is that formal geometric structure of Minkowski spacetime precludes the possibility of a dynamic view of time. Therefore, just in the same way that it is possible for an observation to falsify a physical theory, so it is possible that a physical theory falsify one or more of our metaphysical beliefs. So although we should not expect any particular physical theory to provide definitive evidence for any particular metaphysical view, it seems reasonable to expect that a physical theory might eliminate the possibility of a certain metaphysical thesis.[13]

The more compelling explanation, which has already been alluded to in the previous section, is that the proper time argument turns on the ambiguity we find in the picture of time that arises in the special theory of relativity. We saw two ways in which time is formulated in special relativity: the first is as a time measure along an individual worldline, proper time; and the second is as a time measure associated with a coordinatisation of the manifold, coordinate time. Due to the conformal structure of Minkowski spacetime, there are restrictions on how a particular manifold can be foliated. However, given these restrictions there still remains an infinite number of ways to coordinatise the manifold. Thus we are not left any option for stipulating a global objective 'now'. Although objective temporal passage cannot correspond consistently with some objective time coordinate of the manifold, we *are* able to imagine that objective temporal passage corresponds with the incremental evolution along an object's worldline, or the proper time in some reference frame (namely, the reference frame that contains the object in question). This variable characterisation of time in the special theory of relativity thus gives us a good clue as to why an argument can be made *against* the possibility of dynamic time in Minkowski spacetime, in the first place, and why Minkowski spacetime does not formally *preclude* either metaphysical position in the traditional debate, thereafter.

Having said this, however, it is not obvious that the opposing views of the traditional debate emerge from these considerations unscathed, either. For instance, it might seem that relying on the local proper time along a worldline to locate objective temporal passage indeed results in quite a significant modification of the metaphysical position that the A-theorist originally intended. Thus depending upon which features of dynamic time the A-theorist thinks essential, the possibility arises that the metaphysical theory resulting

[12] Zimmerman (2008), for instance, suggests this possibility.

[13] A lengthy discussion of this issue has played out in the literature with respect to the metaphysical underdetermination of quantum field theory; see van Fraassen (1991), French (1998, 1989), Ladyman (1998) and French and Ladyman (2003).

from the above considerations does not do justice to dynamic time. We must keep in mind at this point, however, that rejecting the logical space circumscribed by the proper time argument is tantamount to rejecting a naturalistic metaphysics.

The discussion of this section has conspicuously lacked the formal machinery with which we introduced the geometry of special relativity in §2. Let us turn then to the promised formal characterisation of the temporal structure of Minkowski spacetime; doing so will serve to illustrate precisely why the proper time argument functions as it does.

4 Characterising time

In both his (1995) and his (2004), Rovelli sets about characterising the various roles that the concept of time plays in different scientific theories.[14] The terminological project associated with this analysis is complicated by the multitude of features that are attributed to time in natural language. Not often are the entirety of these features found bundled together in some formulation of time in a physical theory. I will adapt Rovelli's formalism here to provide a more precise grounding to the explanation above with respect to the temporal structure of Minkowski spacetime. Let us begin by considering time as it is often characterised, as a variable t which parametrises the real line $\mathbb{R}$.

The real line can be described by the following structure: a manifold, $\mathcal{M}^1$, consisting of a set of objects (which in this case is simply all the real numbers) with a one dimensional topology and a differential structure; a metric, g, which ensures that the distance between any two members of the set is meaningfully measurable; an ordering, $<$, which sequences the members of the set within the topological structure; and an origin, φ, which fixes a preferred member of the set. Let us represent this as $\mathbb{R} : \{\mathcal{M}^1, g, <, \varphi\}$ (Rovelli, 1995, p. 83). It is clear to see that this structure maps into the features we ordinarily associate with the notion of time; the set of objects represent the instants of time, the metric represents a measure of temporal duration, the ordering represents the sequential structure of the instants and the preferred fixed time instant is the present. We should note, however, that this short list of attributes represented by the real line is not a consequence of any particular physical theory. If we consider the picture of time in Newtonian mechanics, for instance, there is no preferred fixed point in the theory that is necessarily labelled as the present. This is not to say that the characterisation of time as the real line is incompatible with Newtonian time; on the contrary, time characterised by the real line is quite consistent with the temporal structure of Newtonian theory. Let us represent the structure of Newtonian time as $N : \{\mathcal{M}^1, g, <\}$ and represent that it is consistent with a richer structure by $N : \{\mathcal{M}^1, g, < \mid \varphi\}$.

As was introduced in §2, the characterisation of time in special relativity is not so straightforward. We saw that the dynamical behaviour of objects in spacetime is described

[14] As well as Rovelli, the different features of time have also been discussed with respect to the special theory of relativity by Kroes (1985) and with respect to both special and general relativity by Callender (2006).

by future directed timelike curves in a four dimensional geometry, $(\mathcal{M}^4, g_{\mu\nu})$, and that the notion of time is associated with the parametrisation of such curves. The significant feature of time in special relativity that sets it apart from Newtonian time is that, for all p in $\mathcal{M}^4$, a whole family of future directed timelike curves through p provide a multitude of candidate structures with which time might be identified; the conformal structure of the Minkowski geometry simply does not permit a unique global one dimensional time to be abstracted from $\mathcal{M}^4$. In other words, a global ordering of all the spacetime points in $\mathcal{M}^4$ is not possible; we can only define a partial ordering on the set of spacetime points, $<'$.[15] There are, however, two avenues open to us for reinstating a total ordering to a set of spacetime points in $\mathcal{M}^4$ which correspond to characterising time as coordinate time and proper time, respectively. Let us consider coordinate time first.

While the structure of Minkowski spacetime may not permit a unique global one dimensional time to be *abstracted* from $\mathcal{M}^4$, we are at liberty to *impose* such a structure on the set of all spacetime points. We can simply choose the reference frame of an arbitrary observer and take the time as measured by a clock at rest in that frame to provide a unique foliation of the manifold. Of course, a global time measure of this sort is just coordinate time and the unique foliation of $\mathcal{M}^4$ into hyperplanes of simultaneity does indeed yield a one dimensional set of time instants (the global hyperplanes), $\mathcal{M}^1$, with a total ordering, $<$. A caveat arises at this point, however, when one considers that there are an uncountably infinite number of ways that one can choose such a coordinatisation of the manifold. For every inertial future directed timelike curve through some $p \in \mathcal{M}^4$ there is a corresponding foliation of the manifold. Thus there is an infinite number of ways that one might measure the temporal duration between any particular pair of events and thus any such measurement in an arbitrary coordinate system is physically meaningless. The characterisation of time in the special theory of relativity as coordinate time thus lacks *metricity*. Let us represent the structure of coordinate time in special relativity as $C_S : \{\mathcal{M}^1, <\}$.

A second methodology that we can adopt to find a total ordering of a set of spacetime points in $\mathcal{M}^4$ is to restrict ourselves to a subset of points in the manifold. Rather than search for a unique *global* one dimensional time, we can instead make use of the linear structure of a single future directed timelike curve to provide a *local* measure of time. Of course, a time measure of this sort is just proper time and the local parametrisation of such a curve yields a one dimensional set of time instants, $\mathcal{M}^1$, with a total ordering, $<$. Since proper time is an invariant time measure, the associated parametrisation of

[15] A *total order* on a set S is defined by a binary relation $(\leq)$ with the following properties:

1. $\forall x \in S$, $x \leq x$,
2. $\forall x, y \in S$, $x \leq y$ & $y \leq x \;\Rightarrow\; x = y$,
3. $\forall x, y, z \in S$, $x \leq y$ & $y \leq z \;\Rightarrow\; x \leq z$, and
4. $\forall x, y \in S$, $x \leq y$ or $y \leq x$.

A *partial order* on a set is a binary relation that satisfies (i)-(iii) but not (iv).

a particular worldline is observer independent and thus is a physically meaningful time measure (of temporal durations along the curve only), i.e. proper time is locally metrical. The structure of proper time can thus be represented $P_S : \{\mathcal{M}^1, g, <\}$. In addition, since proper time is only defined locally, fixing a preferred time instant amounts to privileging merely a single spacetime point rather than some global hyperplane. Thus a preferred fixed time instant is consistent with the structure of proper time, $P_S : \{\mathcal{M}^1, g, < \mid \varphi\}$.

As a final element to analysing the proper time argument, let us attempt an equally precise construal of dynamic time. We are taking the dynamic view of time here as the claim that we exist in a privileged present that is in some sense 'flowing' through successive instants of time. Let us consider which of the above attributes might best fit with this notion of time. Dynamic time is certainly linear, has a well defined order (directed towards the future) and fixes a preferred time instant (the present). Inherent in the idea of 'flow' is a notion of continuity that is meaningful only when there exists a measure across the flowing time instants, i.e. dynamic time requires a definite metric. Thus it seems as though dynamic time can be construed as having the structure of the real line as above, $D = \mathbb{R} : \{\mathcal{M}^1, g, <, \varphi\}$ (which is hardly a surprise). Let us now reconsider the proper time argument in light of these considerations.

The charge was made against the A-theorist that there can be no objective temporal passage in Minkowski spacetime because there is no scope for an objective hyperplane of simultaneity. This amounts to a claim that not only is there no preferred time instant in special relativity, but a preferred time instant is incompatible with the temporal structure of special relativity. It is clear that this argument aims to characterise time in special relativity as coordinate time and, in light of the above analysis, $C_S \neq D$; not only is C_S incompatible with a preferred time instant, C_S is incompatible with any general and natural definition of a metric. If the structure of coordinate time were the only formulation of time in special relativity then, to stay within the bounds of a naturalistic metaphysics, dynamic time as we have presented it here would need to be reconsidered as a metaphysical position.

However, we know that time can be construed in special relativity in terms of the structure $P_S : \{\mathcal{M}^1, g, < \mid \varphi\}$. By formalising the temporal structure of both Minkowski spacetime and the dynamic view of time in this way, we can see immediately that P_S is completely consistent with $D : \{\mathcal{M}^1, g, <, \varphi\}$. Thus the dynamic view of time is *not* precluded by the formal temporal structure of Minkowski spacetime. This is then the more compelling explanation as to why the proper time argument functions as it does: the picture of proper time that arises in special relativity ensures that the dynamic view of time is compatible with the temporal structure of Minkowski spacetime due to the correspondence between the characterisations of time that each of them yield. The constraints imposed by the temporal structure of Minkowski spacetime on a naturalistic metaphysics are thus not so restrictive as to force an A-theorist into a major rethink of her position (or a B-theorist, either, for that matter).

This result is the first of the goals I set out at the beginning of the paper: the precise characterisation of the features of time in special relativity illustrates more clearly why the formal constraints imposed by special relativity on the traditional debate are not so restrictive as to quash the debate. In the next section I wish to address the prime goal of this paper: to show that the general theory of relativity provides much sterner restrictions on a naturalistic metaphysics of time. The argument which leads us to these restrictions hinges on an additional feature of time that we find within general relativity but not within special relativity.

5 The traditional debate constrained

Our description in §2 of the dynamical behaviour of objects in spacetime according to the special theory of relativity is in terms of curves through $\mathcal{M}^4$; insofar as this is the case, we are treating spatiotemporal objects as point particles. To provide a more general description of dynamical behaviour in spacetime, we can extend our formalism with the addition of *matter fields*. A matter field is represented by a smooth tensor field, $T_{\mu\nu}$, on $\mathcal{M}^4$ and is assumed to satisfy field equations relating $T_{\mu\nu}$ and $g_{\mu\nu}$. A crucial element to recovering the correspondence between future directed timelike curves on $\mathcal{M}^4$ and worldlines of massive particles in spacetime in the special theory of relativity is the latent assumption that the background spacetime structure, $(\mathcal{M}^4, g_{\mu\nu})$, remains fixed *independently* of the $T_{\mu\nu}$ that live on $\mathcal{M}^4$.[16] It is the fact that $T_{\mu\nu}$ is independent of the background spacetime in special relativity that allows us to describe the dynamical behaviour of matter in spacetime in terms of evolution in a time parameter with metric properties (proper time). In contrast, as we will address below, when a dependency exists between $T_{\mu\nu}$ and $g_{\mu\nu}$ the evolution of the system *defines* proper time and not *vice versa*. We will call time 'independent' when the metric defining time is independent of the matter and energy distribution in the manifold and represent this as an independent metric, g_I.

Time is not independent in general relativity. The geometric structure of general relativity is much the same as the structure we introduced in §2 for special relativity: we have a geometry $(\mathcal{M}^4, g_{\mu\nu})$ and we define proper time (1) and coordinate time as before. In addition, however, the dynamical behaviour of general relativity is given by the Einstein field equations,

$$G_{\mu\nu}(g_{\mu\nu}) = 8\pi T_{\mu\nu}, \tag{2}$$

that define an explicit relation between the matter/energy content of spacetime, represented by the stress-energy tensor $T_{\mu\nu}$, and the curvature of the spacetime manifold, represented by the Einstein tensor $G_{\mu\nu}$ (which is a function of the metric, $g_{\mu\nu}$). Due to this relation, the metric, of which proper time is a function, is a dynamical entity that is directly dependent upon the matter/energy across the entire four dimensional manifold.[17]

[16] Malament (2007, p. 242). We can think of an independent $T_{\mu\nu}$ in the special theory of relativity as representing "test particles" in spacetime.

[17] More accurately, there is a *mutual dependency* between the stress-energy tensor and the metric.

Let us represent a dependent metric such as this as g_D.

The picture of time that arises in general relativity is again much like the picture that arises in special relativity. Coordinate time can be thought of as an arbitrary foliation of the manifold which gives a unique slicing of four dimensional spacetime into a sequence of three dimensional configurations. The linear substructure determined by the foliation yields a total ordering of the slices. However, the parametrisation of the time slices defined by coordinate time in general relativity can be arbitrarily rescaled, which forbids any notion of meaningfully measuring time intervals between pairs of events and so destroys any notion of metricity. We thus represent coordinate time in general relativity as $C_G : \{\mathcal{M}^1, <\}$. Again, coordinate time is merely an imposition of an arbitrary variable determining time evolution, and because general relativity is fundamentally foliation invariant, coordinate time has no physical significance. Similarly, proper time in general relativity is defined exactly as in special relativity (1), except that in general relativity it is determined by a dependent metric, g_D, as above. The local parametrisation of a general relativistic worldline in terms of proper time again yields a one dimensional set of time instants, $\mathcal{M}^1$, with a total ordering, $<$. Thus we represent proper time in general relativity with the structure $P_G : \{\mathcal{M}^1, g_D, <\}$.

If we now consider the structure of dynamic time, D, we can see immediately that similar arguments to those above could be constructed claiming dynamic time to be inconsistent with the structure of general relativity *if* time were characterised simply by C_G: coordinate time in general relativity has no physically meaningful metric properties. We know, however, not to be persuaded by such argumentation. The case for the consistency of dynamic time with proper time in general relativity, on the other hand, is not so clear cut. Using our updated notation for dependent and independent metrics, we can compare proper time in special relativity, $P_S : \{\mathcal{M}^1, g_I, <\}$, to proper time in general relativity, $P_G : \{\mathcal{M}^1, g_D, <\}$, and see that the only difference between the two is the dependency of the metric. The proper time argument of §3 demonstrated that the possibility of locally privileging a temporal instant in a special relativistic spacetime with an independent metric is not prohibited by the formal temporal structure therein. Whether the same can be said for a general relativistic spacetime with a metric that is dependent upon the four dimensional matter/energy distribution remains to be shown. Exploring this possibility will lead us to the constraints that classical physics imposes on the traditional metaphysical debate.

For dynamic time to be consistent with a physical theory there must be a characterisation of time therein that allows us to privilege a present moment that flows objectively. Recall (§1.1) that flow can be construed in two different ways depending upon whether we understand the privileged present ontologically or metaphysically. In the special theory of relativity proper time is determined by a fixed background metric structure. The rate of flow of time along a worldline, being determined by the metric, is then not a function of any part of spacetime but the immediate local neighbourhood of the 'privileged' instant on the worldline in question; the local flow is determined locally. In this respect, special relativity formally precludes neither an ontologically privileged present nor a metaphysically privileged present, since the flow of time along a worldline does not force us to make an ontological commitment to any part of spacetime but the fixed background structure at a particular spacetime point. However, an ontologically privileged present *is* precluded *by definition* when we allow the metric to depend upon the matter/energy distribution across the *four dimensional spacetime ontology*, as in general relativity; the present time instant cannot be 'more real' than any other time instant (as required by an ontologically privileged present) since $T_{\mu\nu}$ at *all* instants determines the rate of any local flow. Thus any theory of classical time which remains within the scope of a naturalistic metaphysics cannot interpret flow as the existential displacement of a privileged time instant by its successor; all four dimensions of spacetime must be taken as on an ontological par.[18]

The traditional debate is thus constrained in the following way: if one wanted to maintain that there were an objectively flowing privileged time instant, then one must understand this instant to be metaphysically privileged, whereby flow is interpreted as the evolution of the property 'presentness' across consecutive time instants that are ontologically undifferentiated. Furthermore, the rate of such a metaphysical flow at any spacetime point is a function of even the most spatially and temporally distant spacetime structure. The resulting picture of reality here is then this: a temporal instant on some worldline (or worldlines) has a metaphysical property 'presentness' whose rate of flow along the worldline is determined by the global structure of spacetime. What is far from obvious is whether this picture yields a nontrivial metaphysical theory of time; for instance, in what meaningful sense is this conception of the present 'privileged' or 'objective', especially if we are simply positing a preferred temporal instant with this property to allow us to maintain that we occupy an A-theoretic reality? Whether or not there remains logical space for an A-theory of time within these constraints depends upon the way in which the A-theorist wishes to refine the notion of the privileged present.[19] As an integral element to any ensuing analysis here, I wish to point out in a sceptical spirit that the dynamic view of time seems to be beset by the imprecise and obscure nature of notions such as 'privileged', 'objective' and 'flow' and it is not entirely clear that these terms are conducive to rigorous definition in this context.[20] The implication, then, is that the A-theorist who respects naturalistic metaphysics owes us an account of the dynamic view of time that avoids the triviality of merely stipulating a spacetime point as objectively metaphysically

[18]Of course, such a constraint on the reality of four dimensional spacetime is already independently proposed within the traditional debate with respect to analyses of the special theory of relativity; for instance, by Rietdijk (1966) and Putnam (1967). The focus of the present analysis, however, is atypical. While there is discord about the metaphysical conclusions that can be drawn from arguments that rely on the relation of simultaneity in special relativity (see, for instance, Malament (1977), Dickson (1998, §8.1.2) and Brown (2005, §6.3.1)), there is no contention as to the significance of the dependent metric in Einstein's field equations.

[19]Zimmerman (forthcoming) sets out a comprehensive defence of an A-theory of time that fits explicitly within these constraints.

[20]Though see Price (forthcoming) for a recent (and not very sympathetic) analysis of flow.

distinguished.

There is a further caveat that jeopardises the viability of a dynamic view of time.[21] Even if we consider that each individual worldline in spacetime is a vehicle of objective flow, to ensure that every such worldline yields a totally ordered linear subset of the manifold we require the existence of a spacelike hypersurface $\Sigma \subset \mathcal{M}^4$ with the property that every inextendible timelike curve in $\mathcal{M}^4$ intersects Σ exactly once. We call Σ a *Cauchy surface* and note that it follows from this condition that Σ is a three dimensional spacelike submanifold of $\mathcal{M}^4$. A geometry $(\mathcal{M}^4, g_{\mu\nu})$ that admits the existence of a Cauchy surface is said to be *globally hyperbolic.* If $(\mathcal{M}^4, g_{\mu\nu})$ is globally hyperbolic then $\mathcal{M}^4$ is diffeomorphic to a manifold of the form $\Sigma \times \mathbb{R}$ (where we take Σ here to represent a diffeomorphism equivalence class of three dimensional Cauchy surfaces) (Geroch, 1970). Thus a necessary condition for the possibility of dynamic time is the requirement that our reality be represented by a manifold $\mathcal{M}^4$ that can be foliated by Cauchy surfaces.[22] A problem arises here for the dynamic view of time since only a subset of the solutions to Einstein's field equations (2) have this property; Gödel's (1949) infamous and eponymous spacetime solution, which contains closed timelike curves, is just one example of a solution to the field equations that is not globally hyperbolic.[23]

There is, however, a potential reprieve for the A-theorist in this case. The set of spacetime solutions to Einstein's field equations that can be foliated into spacelike hypersurfaces have taken on considerable significance over the last half a century[24]. The restriction to globally hyperbolic spacetime solutions is required for the Hamiltonian formulation of general relativity and this in turn is integral to using canonical quantisation techniques to develop a quantum theory of gravity. Thus, if this program turns out to be successful, the Hamiltonian formulation of general relativity may be an amenable arena for naturalistic A-theories of time. The catch, however, is that this reprieve is only plausible if it is possible to find a physical basis for fixing a preferred foliation of the spacetime manifold, which is a difficult task to say the least for a foliation invariant theory such as general relativity. A suggestion has been made in recent times, however, that the so-called *constant mean curvature* (CMC) foliation approach provides just this: a unique foliation for a reasonably large subset of spacetime solutions which is determined by constraining the possible Cauchy surfaces embeddable in $\mathcal{M}^4$ to those with constant mean extrinsic curvature.[25] Although an exhaustive discussion of foliation invariance in general relativ-

[21]The formalism here follows Belot (2007)

[22]Of course, one could argue that a total ordering of temporal instants is not essential to the dynamic view of time, i.e. dynamic time might be better characterised by the structure $D' : (\mathcal{M}^1, g, <', \varphi)$, with a partial ordering $<'$. Global hyperbolicity would not be a necessary condition for the possibility of a dynamic time represented by D'.

[23]On the other hand, just because a physical theory admits the *possibility* of a solution to the field equations of a particular sort does not imply that this solution is *necessarily* physically realisable: think of the case of a pendulum with negative length. Such argumentation may alleviate the worry an A-theorist might have with Gödel-type universes in the first place.

[24]See Dirac (1958), Bergman (1961) and Arnowitt, Deser and Misner (1962).

[25]See Wüthrich (2010), and references therein, for a more detailed discussion of the CMC approach.

ity would take as beyond the scope of the current project, a few brief comments are in order here. On the bright side for the A-theorist, Belot (2007) argues that the CMC foliation approach may be an instrumental ingredient in solving the *problem of time* in general relativity.[26] However, he also concedes that the approach "violates the spirit of general relativity" in that it reinstates a privileged distinction between time and space (2007, p. 219). On the not so bright side for the A-theorist, Wüthrich (2010) sets out a rather comprehensive and convincing argument against the possibility of using the CMC approach to support a particular A-theory of time: *presentism*. Thus while it seems that the A-theorist *may* find supporting physical structure in the Hamiltonian formulation of general relativity, there are significant obstacles still to be overcome.

6 A way forward?

There were two goals which I set out to achieve in this paper. The initial goal was to present a formal characterisation of time in special relativity, with the resulting formalism providing a new perspective on why the constraints imposed by special relativity on the traditional debate are not so restrictive as to quash the debate. The primary goal, though, was to extend this formal characterisation of time in special relativity to general relativity which, as we have seen, imposes much more restrictive constraints on the traditional debate. An overarching theme of this project has been to set out the constraints which the traditional metaphysical debate must heed to remain within the bounds of a naturalistic metaphysics with respect to classical physical theory.

The constraints imposed by the spacetime structure of general relativity on the traditional debate dictate that a metaphysical thesis about the nature of time must take all four dimensions of spacetime on an ontological par and also concede that any notion of the flow of time at a spacetime point must be determined as a function of the entire four dimensional spacetime structure (both $T_{\mu\nu}$ and $g_{\mu\nu}$, as two sides of the same coin). Whether these constraints leave room for an A-theory of time to successfully navigate the Scylla of triviality and the Charybdis of 'neo-scholastic' metaphysics remains to be seen. Either way, foliation invariance and non-globally hyperbolic spacetime solutions to Einstein's field equations loom large on the horizon.

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[26]Though see Thébault (2010) for further issues that arise in this context.

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