Large Nongaussianity in Axion Inflation

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The inflationary paradigm has enjoyed phenomenological success, however, a compelling particle physics realization is still lacking. The key obstruction is that the requirement of a suitably flat scalar potential is sensitive to Ultra-Violet (UV) physics. Axions are among the best-motivated inflaton candidates, since the flatness of their potential is naturally protected by a shift symmetry. We re-consider the cosmological perturbations in axion inflation, consistently accounting for the coupling to gauge fields $\phi F\tilde{F}$, which is generically present in these models. This coupling leads to production of gauge quanta, which provide a new source of inflaton fluctuations, $\delta\phi$. For an axion decay constant $\lesssim 10^{-2}\,M_p$, this effect typically dominates over the standard fluctuations from the vacuum and dramatically modifies phenomenological predictions. For concrete realizations that admit a UV completion (such as N-flation and axion monodromy), this can be probed in the near future. We show that: (1) a large tensor-to-scalar ratio is not generic in large field inflation, and, (2) large nongaussianity is easily obtained in very minimal and natural realizations of inflation.

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I. INTRODUCTION

Primordial inflation is the dominant paradigm in current cosmology since (i) it resolves the conceptual difficulties of the standard big bang model, and (ii) it predicts primordial perturbations with properties in excellent agreement with those that characterize the Cosmic Microwave Background (CMB) anisotropies. Despite these successes, there is still no compelling particle physics model of inflation; the key obstacle being the requirement of a sufficiently flat scalar potential, $V(\phi)$. Even generic Planck-suppressed corrections may yield unacceptably large contributions to the slow roll parameters $\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$, $\eta \equiv M_p^2 \frac{V''}{V}$, thus spoiling inflation (prime denotes derivative with respect to ϕ, while $M_p \cong 2.4 \cdot 10^{18}$GeV is the reduced Planck mass). One of the simplest solutions to this problem is to assume that the inflaton ϕ is a Pseudo Nambu Goldstone Boson (PNGB) [1]-[8]. In this case the inflaton enjoys a shift symmetry $\phi \to \phi + \text{const}$, which is broken either explicitly or by quantum effects. In the limit of exact symmetry, the ϕ direction is flat and thus dangerous corrections to ϵ, η are controlled by the smallness of the symmetry breaking. Moreover, PNGBs like the axion are ubiquitous in particle physics: they arise whenever an approximate global symmetry is spontaneously broken and are plentiful in string compactifications. Axion inflation is also phenomenologically desirable since the tensor-to-scalar ratio is typically large in such models.

The first explicit example of axion inflation was the natural inflation model [1] in which the shift symmetry is broken down to a discrete subgroup $\phi \to \phi + (2\pi)f$, resulting in a periodic potential

$$V_{\rm np}(\phi) \cong \Lambda^4 \left[1 - \cos\left(\phi/f\right)\right] \qquad (1)$$

with f the axion decay constant. For such potential, agreement with observations requires $f > M_p$, which may be problematic since it suggests a global symmetry broken above the quantum gravity scale, where effective field theory is presumably not valid. Moreover, $f > M_p$ does not seem possible in string theory [9]. More recently, several controlled realizations of axion inflation have been studied – including double-axion inflation [2], N-flation [3, 4], axion monodromy [5] and axion/4-form mixing [8] – which have $f < M_p$ but nevertheless behave effectively as large field inflation models ($\phi \gtrsim M_p$).

In axion inflation models, the inflaton couples to some gauge field as $\frac{\alpha}{f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$. The scale of this coupling is set by the axion decay constant, f; the dimensionless parameter α is typically order unity but can be ≥ 1 in multi-field [2] or extra-dimensional models [7]. It is natural to explore the implications of this generic interaction for observables. In [7] it was shown that energy dissipation into gauge fields can slow the motion of ϕ, providing a novel new inflationary mechanism that operates at very strong coupling. In this work, we point out that even in the conventional slow-roll regime, the coupling $\phi F\tilde{F}$ can have significant impact. The motion of the inflaton amplifies the fluctuations of the gauge field, which in turn produce inflaton fluctuations via *inverse decay* [10] processes: $\delta A + \delta A \to \delta\phi$. When $f \lesssim 10^{-2}\,M_p$, which is quite natural for realizations that admit an UV completion, we show that the inverse decay typically dominates over the usual vacuum fluctuations from inflation, and this has dramatic phenomenological consequences. Our results are quite general: in the spirit of effective field theory, a coupling $\phi F\tilde{F}$ should be included whenever ϕ is pseudo-scalar [11].

Recently, there has been considerable interest in nongaussian effects in the CMB (see the review [12] for references). Nongaussianity will be probed to unprecedented accuracy with the forthcoming Planck data and may provide a valuable tool to discriminate between models. Several constructions are known which can predict an observable signature; however, in the minimal cases nongaussianity is small, and obtaining an observable level usually requires either fine-tuning or unconventional field theories. Here we point out that the inverse decay contribution to $\delta\phi$ is highly nongaussian in axion models; observational bounds are easily saturated for modest values of f. Thus, the simplest and, perhaps, most natural models of inflation can lead to observable nongaussianity.

II. COSMOLOGICAL PERTURBATIONS

We consider the theory

$$S = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad (2)$$

where ϕ is the PNGB inflaton, $F_{\mu\nu}$ the field strength of the gauge field (for simplicity, a $U(1)$ gauge field is considered; the extension to non-Abelian groups is straightforward), and $\tilde{F}_{\mu\nu}$ its dual. The potential $V(\phi)$ may contain a periodic contribution of the form (1) due to non-perturbative effects and, perhaps, non-periodic contributions from other effects (such as moduli stabilization or wrapped branes). In this Section, we leave $V(\phi)$ arbitrary, except to suppose that it is sufficiently flat to support $N_e \gtrsim 60$ e-foldings of inflation. We assume an FRW geometry $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a(\tau)^2\left[-d\tau^2 + d\mathbf{x}^2\right]$.

Working in Coulomb gauge, we decompose $\vec{A}(t,\mathbf{x})$ into circular polarization modes obeying [7]

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right] A_\pm(\tau,k) = 0, \quad \xi \equiv \frac{\alpha\dot{\phi}}{2fH} \qquad (3)$$

where dot denotes differentiation with respect to t, $H \equiv \dot{a}/a$, $\xi \cong$ const. We observe that one of the polarizations of $\vec{A}$ experiences a tachyonic instability for $k/(aH) \lesssim 2\xi$. The growth of fluctuations is described by [7]

$$A_+(\tau,k) \cong \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}} \qquad (4)$$

in the interval $(8\xi)^{-1} \lesssim k/(aH) \lesssim 2\xi$ of phase space which accounts for most of the power in the produced gauge field (we take $\dot{\phi} > 0$ without loss of generality). This interval is nonvanishing only for $\xi \gtrsim \mathcal{O}(1)$, which we assume in the following. The production is uninteresting at smaller ξ.

The unstable growth of $A_+(\tau,k)$ yields an important new source of cosmological fluctuations, $\delta\phi$. The perturbations of the inflaton are described by [7, 13]

$$\left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{\nabla^2}{a^2}\right]\delta\phi(t,\mathbf{x}) = \frac{\alpha}{f}F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad (5)$$

where the source term is constructed from (4). The solution of (5) splits into two parts: the solution of the homogeneous equation and the particular solution which is due to the source. Schematically, we have

$$\delta\phi = \underbrace{\delta\phi_{\rm vac}}_{\rm homogeneous} + \underbrace{\delta\phi_{\rm inv.decay}}_{\rm particular} \qquad (6)$$

The quantity of interest is the primordial curvature perturbation on uniform density hypersurfaces, $\zeta = -\frac{H}{\dot{\phi}}\delta\phi$. We computed the two-point $\langle\zeta(\mathbf{x})\,\zeta(\mathbf{y})\rangle$ and three-point $\langle\zeta(\mathbf{x})\,\zeta(\mathbf{y})\,\zeta(\mathbf{z})\rangle$ correlation functions using the formalism of [7, 13]. The two-point function defines the power spectrum

$$\langle\zeta(\mathbf{x})\zeta(\mathbf{y})\rangle = \int\frac{dk}{k}\,\frac{\sin\left[k|\mathbf{x}-\mathbf{y}|\right]}{k|\mathbf{x}-\mathbf{y}|}\,P_\zeta(k) \qquad (7)$$

We find the result

$$P_\zeta(k) \;=\; \mathcal{P}\left(\frac{k}{k_0}\right)^{n_s-1}\left[1 + 1.1\cdot 10^{-4}\,\mathcal{P}\,\frac{e^{4\pi\xi}}{\xi^3}\right] \qquad (8)$$

$$\mathcal{P}^{1/2} \;\equiv\; \frac{H^2}{2\pi|\dot{\phi}|} \qquad (9)$$

where n_s is the spectral index, and the pivot scale is $k_0 = 0.002\,\mathrm{Mpc}^{-1}$. The two terms in (8) are the power spectra of the homogeneous and inhomogeneous parts of (6), respectively. There is no "mixed term" since the two contributions (6) are uncorrelated. (The gauge fluctuations that source $\delta\phi_{\rm inv.decay}$, and that are amplified according to (3), are not correlated with the vacuum inflaton fluctuations.) The power spectrum is probed by CMB and Large Scale Structure observations. It is found to be nearly scale invariant ($n_s \simeq 1$, the precise value depends on the data set assumed [14]; due to this approximate scale invariance the specific value of k_0 is irrelevant for our considerations), and have amplitude $P_\zeta(k) \cong 25\cdot 10^{-10}$ [15] (the so-called COBE normalization). When inverse decay fluctuations are subdominant, we have the standard result $\mathcal{P}^{1/2} = 5\cdot 10^{-5}$; however, at large ξ the value of $\mathcal{P}$ must be modified.

The three-point correlation function encodes departures from gaussianity. Nongaussian effects from inverse decays are maximal when all three modes have comparable wavelength (the equilateral configuration). The magnitude of the three-point function is conventionally quantified using the parameter f_{NL} [14]. We find that:

$$f_{NL}^{\rm equil} \cong 1.4\cdot 10^8\,\mathcal{P}^3\,\frac{e^{6\pi\xi}}{\xi^3} \qquad (10)$$

This result does not include the negligible contribution from $\delta\phi_{\rm vac}$ and is accurate as long as $|f_{NL}^{\rm equil}| \gtrsim 1$. The current WMAP bounds are $-214 < f_{NL}^{\rm equil} < 266$ (95% CL), while the Planck satellite, and planned missions, will constrain $f_{NL}^{\rm equil}$ to $\mathcal{O}(10)$ [16].

The results (8) and (10) only depend on the two dimensionless combinations ξ and $\mathcal{P}^{1/2}$, shown in Figure 1. The solid red curve indicates the parameter values which reproduce the COBE normalization of the power spectrum. In the region below, and above the dashed black line the power spectrum is dominated by $\delta\phi_{\rm vac}$ and by $\delta\phi_{\rm inv.decay}$, respectively. The COBE normalized curve crosses this boundary at $\xi \simeq 2.6$. For larger ξ, the conventionally quoted results for axion inflation are invalid. First of all, the nongaussianity of the perturbation reaches a detectable level. Moreover, $\mathcal{P}^{1/2}$ needs to be smaller than the standard $\sim 5 \cdot 10^{-5}$ value. This typically requires lowering the value of H (at least for the large-field models considered in the next Section). The decrease of H, in turn, has the effect of decreasing the power in the tensor fluctuations (gravity waves), $P_T \cong \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2$ (to leading order, $P_T(k)$ is not influenced by the unstable growth of A_+). The tensor-to-scalar ratio $r \equiv P_T/P_\zeta \cong 8.1 \cdot 10^7 H^2/M_p^2$ is an important quantity to discriminate between different inflationary models. We find that r must decrease with increasing $\xi \geq 2.6$. The current observational limit is $r \lesssim 0.2$ [14], and activity is underway to probe $r \gtrsim 0.01$ [15].

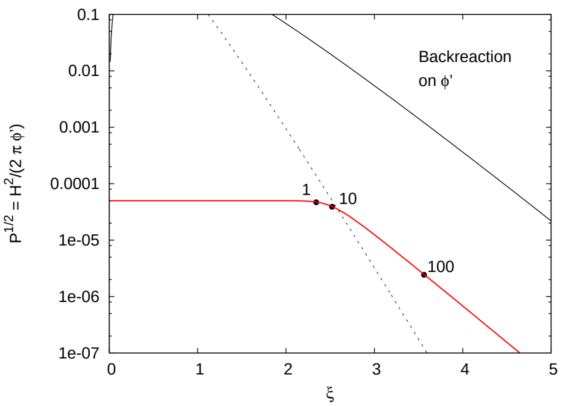

FIG. 1: Values of parameters leading to the observed COBE normalization of the power spectrum (red line), and reference values for the nongaussianity parameter $f_{NL}^{\rm equil} = 1, 10, 100$ along this curve. See the main text for details.

The results (8) and (10) have been obtained by disregarding two backreaction effects of the produced gauge quanta. Such quanta are produced at the expense of the kinetic energy of ϕ, so that, if the instability is sufficiently strong, then it will affect the inflaton dynamics. The region of parameter space where this occurs is above the black solid line ($\mathcal{P}^{1/2} > 13\xi^{3/2}\,{\rm e}^{-\pi\xi}$) shown in Figure 1. We have also disregarded the impact of the energy density of the produced quanta on the expansion rate, H. This is justified provided ${\rm e}^{2\pi\xi}/\xi^3 < 2 \cdot 10^4 M_p^2/H^2$. This constraint is not expressed in terms of ξ and $\mathcal{P}^{1/2}$, so we have not included it in Figure 1. However, it can be studied on a case-by-case basis.

III. PREDICTIONS FOR SPECIFIC MODELS

We now focus our attention on the power-law potential

$$V(\phi) = \mu^{4-p}\phi^p \tag{11}$$

which subsumes many interesting scenarios. Inflation proceeds at large field values $\phi \gtrsim M_p$ and ends when $\phi \sim M_p$. For this model, the values of H, $\dot\phi$ and n_s are uniquely determined by the number of e-foldings of observable inflation N_e, according to the standard slow roll inflaton evolution (ϵ, $\eta \ll 1$). In the following, we fix $N_e = 60$, which is the typical value taken in large field models. Once we do so, we are left with the two parameters f/α, and μ. For any given value of f/α, the mass scale μ is uniquely determined by fixing the power spectrum (8) to the COBE value. Therefore, we can plot the observational predictions as a function of f/α only. We do so in Figure 2, where we take $p = 1, 2$ for illustration. In both cases shown, backreaction effects can be neglected.

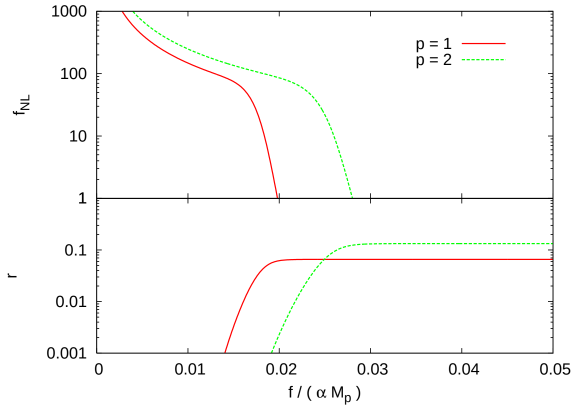

FIG. 2: Observational predictions for the large-field power-law inflation model (11) with $p = 1, 2$ and assuming $N_e \cong 60$. The spectral index is $n_s = 0.975, 0.967$ for $p = 1, 2$. At small f/α the coupling of ϕ to $F\tilde{F}$ is stronger and nongaussianity is large. The tensor-to-scalar ratio decreases at strong coupling, as discussed in the main text.

Figure 2 shows that large nongaussianity is rather generic for large-field axion inflation: $f_{NL}^{\rm equil} \gtrsim 10$ for decay constants $f/\alpha \lesssim 10^{-2} M_p$, which is natural in a model that admits a UV completion. If Planck does not detect nongaussianity, then we will have a surprising bound on the *strongest* couplings of the type $\phi F\tilde{F}$ between the inflaton and *any* gauge field. Figure 2 also shows that the model (11) need not predict observably large r, contrary to the common lore. We now consider the implications for some specific models.

Natural Inflation: The original natural inflation model [1] was based on the potential (1). If we require $n_s \gtrsim 0.95$, as suggested by recent data [14], then the model requires a large decay constant $f \gtrsim 5M_p$ [17]. In this regime inflation proceeds near the minimum $\phi = 0$ and is indistinguishable from the model (11) with $p = 2$. Large values of f weaken the coupling of ϕ to $F\tilde{F}$, hence inverse decay is negligible unless $\alpha \gtrsim 180$, whereas we expect $\alpha = \mathcal{O}(1)$ in the simplest (single-axion) scenario. On the other hand, $f \gtrsim M_p$ may be problematic and it seems that a UV completion of axion inflation requires $f < M_p$. We now turn our attention to such scenarios.

Axion Monodromy: In [5] an explicit, controlled realization of axion inflation was obtained from string theory. The potential has the form $V(\phi) = \mu^3\phi + \Lambda^4\cos(\phi/f)$ where the linear contribution arises because the shift symmetry is broken by wrapping an NS5-brane on an appropriate 2-cycle, and the periodic modulation is due to nonperturbative effects. The former typically dominates [5, 6] so we have the model (11) with $p = 1$, to first approximation. The decay constant is bounded [5] as $0.06\mathcal{V}^{-1/2}g_s^{1/4} < f/M_p < 0.9g_s$ with $g_s < 1$ the string coupling and $\mathcal{V} \gg 1$ the compactification volume in string units. From Fig. 2 we see that large nongaussianity is easily obtained for $\alpha = \mathcal{O}(1)$. Note that the periodic modulation of $V(\phi)$ can also lead to resonant nongaussianity [22] for $f \lesssim 10^{-2}M_p$ and Λ sufficiently large [6].

N-flation: In [3] it was noted that the collective motion of N axions ϕ_i, each with its own broken shift symmetry, can support inflation when $f_i < M_p$, via the assisted inflation mechanism [18]. This scenario is quite natural in string theory, where generic compactifications may contain exponentially large numbers of axions [3, 4]. For $\phi_i \lesssim f_i$ we can expand the potential near the minimum to obtain $V \cong \sum_i m_i^2\phi_i^2/2$. The dynamics of the collective field $\Phi \equiv \sqrt{\sum_i \phi_i^2}$ are well-described by a single field model of the form (11) with $p = 2$ [3, 4]. Sufficient inflation requires $\Phi > M_p$ which can be achieved for sub-Planckian ϕ_i provided $\sqrt{N}$ is sufficiently large.

Typically, the mass basis $\{\phi_i\}$ is not aligned with the interaction basis [19, 20], so that all ϕ_i will couple to a given gauge field as $\mathcal{L}_{\rm int} = -\sum_i \alpha_i\phi_i F\tilde{F}/f_i$. The coupling of the collective field Φ to $F\tilde{F}$ is highly model-dependent but can be parametrized as $\mathcal{L}_{\rm int} = -\alpha_{\rm eff}\Phi F\tilde{F}/f_{\rm eff}$, so that the result of Fig. 2 apply for the effective coupling. A precise calculation depends on the mass rotation and detailed microphysics, but we expect that an observable signal may be possible for reasonable parameters.

Axion Mixing: Ref. [8] realizes $p = 2$ via axion/4-form mixing. Here $f < M_p$ so $f_{NL}^{\rm equil} \gg 1$ is possible.

Double-Axion Inflation: Ref. [2] proposed a model characterized by two axions θ and ρ, with potential

$$V = \sum_{i=1}^{2} \Lambda_i^4 \left[1 - \cos\left(\frac{\theta}{f_i} + \frac{\rho}{g_i}\right)\right] \qquad (12)$$

which arises from the coupling of the two axions to two different gauge groups, $\frac{\theta}{f_i}F_i\tilde{F}_i$, and $\frac{\rho}{g_i}F_i\tilde{F}_i$ (up to numerical coefficients). For $f_1/g_1 = f_2/g_2$, one linear combination of the two axions becomes a flat direction of (12). This relation can be ascribed to a symmetry of the theory, and the curvature of the potential along this direction can be made controllably small if this symmetry is only slightly broken. In this case one obtains an effective large field inflaton, with a potential of the type (1), and with an effective axion constant $> M_p$, even if all the f_i, g_i are sub-Planckian. Therefore, this model can lead to large production of gauge fields and observable $f_{NL}^{\rm equil}$.

In summary, we have shown that large nongaussianity is possible for many explicit axion inflation models which admit a UV completion. Our qualitative results will carry over to any inflation model with a PNGB dynamically important during inflation, including multi-field models such as the roulette [23] or racetrack [24] scenarios. Similar effects may also be possible for higher p-form fields. It would be interesting to study the value of α in concrete string theory realizations.

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