Nonlinear Density Waves in the Single-Wave Model

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The single-wave model equations are transformed to an exact hydrodynamic closure by using a class of solutions to the Vlasov equation corresponding to the waterbag model. The warm fluid dynamic equations are then manipulated by means of the renormalization group method. As a result, amplitude equations for the slowly varying wave amplitudes are derived. Since the dispersion equation for waves has in general three roots, two cases are examined. If all three roots of the dispersion equation are real, the amplitude equations for the eigenmodes represent a system of three coupled nonlinear Schrodinger equations. In the case, where the dispersion equation possesses one real and two complex conjugate roots, the amplitude equations take the form of two globally coupled complex Ginzburg-Landau equations.

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I. INTRODUCTION

The processes of pattern and coherent structures formation in plasmas and plasma-like media have attracted attention for many years. A number of approximations and simplified models has been proposed, amongst which the single-wave model is one of the most efficient approaches to study the weakly nonlinear behavior in marginally stable plasmas. Starting from the general Vlasov-Maxwell equations for the phase space density distribution and the self-consistent field, one usually derives a self-consistent set of equations describing the evolution of the coarse-grained distribution function and a certain isolated marginally stable wave mode of the electrostatic potential.

Using the method of matched asymptotic expansions, the single-wave model equation has been recently derived[1] in the most general case. Amongst earlier pioneering work utilizing the single-wave model, the studies dedicated to the beam-plasma instability[2–4] and the bump-on-tail instability[5–7] should be mentioned.

The present paper is organized as follows. In the next section, we cast the self-consistent single-wave model equations derived by del-Castillo-Negrete[1] in an equivalent form by using a class of exact solutions to the Vlasov equation corresponding to the waterbag model[8,9]. Further, the exact closure of hydrodynamic equations are manipulated following the renormalization group (RG) approach[9]. As a result amplitude equations for the slowly varying wave envelopes are derived. Depending on the character of the solutions of the dispersion equation to cases can be distinguished. In the first case, where all three roots of the dispersion equation are real, the amplitude equations represent a system of three coupled nonlinear Schrodinger equations. If the dispersion equation possesses one real and two complex conjugate roots, the amplitude equations take the form of two globally coupled complex Ginzburg-Landau equations. Finally, in the last section, we draw some conclusions and outlook.

II. THEORETICAL MODEL AND BASIC EQUATIONS

The basic equations derived by del-Castillo-Negrete[1], which will be the starting point of our subsequent analysis can be written as

$$\partial_t f + \mathcal{V} \partial_x f + \partial_x \varphi \partial_{\mathcal{V}} f = 0, \tag{1}$$

$$\varphi = a(t) e^{ix} + a^*(t) e^{-ix}, \tag{2}$$

$$\sigma \frac{\mathrm{d}a}{\mathrm{d}t} + ila = i \langle f e^{-ix} \rangle, \tag{3}$$

where the operator averaging is denoted by

$$\langle \ldots \rangle = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \mathrm{d}\mathcal{V} \int\limits_{0}^{2\pi} \mathrm{d}x \ldots . \tag{4}$$

The independent time t and spatial x variables, as well as the dependent ones $f(x, \mathcal{V}; t)$ and $\varphi(x; t)$ entering the above equations have been properly nondimensionalized[1].

For present purposes, we restrict the analysis to a special class of exact solutions to Eq. (1) corresponding to the waterbag distribution[8,9]

$$f(x, \mathcal{V}; t) = \mathcal{C} [\mathcal{H}(\mathcal{V} - v_-(x; t)) - \mathcal{H}(\mathcal{V} - v_+(x; t))], \tag{5}$$

where $\mathcal{H}$ denotes the well-known Heaviside function, $\mathcal{C}$ is a normalization constant and $0 < x < 2\pi$ is a normalized spatial variable. It simply means that the phase space density $f(x, \mathcal{V}; t)$ remains constant within a region confined by the boundary curves $v_\pm(x; t)$, which are assumed to be single valued. The latter distort nonlinearly during the evolution of the system as specified by Eqs. (1) - (3).

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It is convenient to introduce the macroscopic fluid variables

$$\varrho = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{V} f(x, \mathcal{V}; t) = \mathcal{C}(v_+ - v_-), \tag{6}$$

$$\varrho v = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{V} \mathcal{V} f(x, \mathcal{V}; t) = \frac{\mathcal{C}}{2}\left(v_+^2 - v_-^2\right), \tag{7}$$

where $\varrho(x;t)$ and $v(x;t)$ are the density and the current velocity, respectively. It can be shown that the higher moments defining the particle pressure $\mathcal{P}(x;t)$ and the heat flow $\mathcal{Q}(x;t)$ can be expressed as

$$\mathcal{P} = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{V} (\mathcal{V} - v)^2 f(x, \mathcal{V}; t) = \frac{\mathcal{C}}{12}(v_+ - v_-)^3, \tag{8}$$

$$\mathcal{Q} = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{V} (\mathcal{V} - v)^3 f(x, \mathcal{V}; t) = 0. \tag{9}$$

In particular, the last expression (9) implies that the waterbag distribution yields an exact closure of the hydrodynamic equations in the form

$$\partial_t \varrho + \partial_x(\varrho v) = 0, \tag{10}$$

$$\partial_t v + v\partial_x v + v_T^2 \partial_x\left(\varrho^2\right) = \partial_x \varphi, \tag{11}$$

where

$$v_T^2 = \frac{1}{8\mathcal{C}^2}, \tag{12}$$

is the normalized thermal speed-squared. The macroscopic fluid equations (10) and (11) must be supplemented with the equation for the amplitude of the field mode (3), written in the form

$$\sigma \frac{\mathrm{d}a}{\mathrm{d}t} + ila = i\left\langle \varrho e^{-ix}\right\rangle, \tag{13}$$

where now the operator averaging specified by Eq. (4) involves integration on the normalized spatial variable x only, and the potential φ is given by expression (2) as before.

We further scale the hydrodynamic and field variables according to the expressions

$$\varrho = \varrho_0 + \epsilon R, \quad v = v_0 + \epsilon V, \quad a = \epsilon\alpha, \quad \varphi = \epsilon\Phi, \tag{14}$$

where ϵ is a formal small parameter, and will be set equal to one at the end of the calculations. Furthermore, $\varrho_0 = const$ and $v_0 = const$ represent the stationary solution of Eqs. (10) and (11), provided the stationary field amplitude $a_0 = 0$. The basic macroscopic fluid and electrostatic field equations can be rewritten as

$$\partial_t R + \varrho_0 \partial_x V + v_0 \partial_x R = -\epsilon\partial_x(RV), \tag{15}$$

$$\partial_t V + v_0\partial_x V + 2\varrho_0 v_T^2 \partial_x R - \partial_x \Phi = -\epsilon\partial_x\left(\frac{V^2}{2} + v_T^2 R^2\right), \tag{16}$$

$$\sigma \frac{\mathrm{d}\alpha}{\mathrm{d}t} + il\alpha = i\left\langle R e^{-ix}\right\rangle, \tag{17}$$

$$\Phi = \alpha(t)e^{ix} + \alpha^*(t)e^{-ix}. \tag{18}$$

To simplify the above system of equations, we perform a Galilean transformation specified by

$$z = x - v_0 t, \qquad \Psi(t) = \alpha(t)e^{iv_0 t}, \tag{19}$$

and cast our basic system of equations in the form

$$\partial_t R + \varrho_0 \partial_z V = -\epsilon\partial_z(RV), \tag{20}$$

$$\partial_t V + 2\varrho_0 v_T^2 \partial_z R - \partial_z \Phi = -\epsilon\partial_z\left(\frac{V^2}{2} + v_T^2 R^2\right), \tag{21}$$

$$\sigma \frac{\mathrm{d}\Psi}{\mathrm{d}t} + i\mathcal{L}\Psi = i\left\langle R e^{-iz}\right\rangle, \tag{22}$$

$$\Phi = \Psi(t)e^{iz} + \Psi^*(t)e^{-iz}, \qquad \mathcal{L} = l - \sigma v_0. \tag{23}$$

Eliminating V from the left-hand-sides of Eqs. (20) and (21), we arrive at the basic system

$$\partial_t^2 R - \lambda^2 \partial_z^2 R - \varrho_0 \Phi = -\epsilon\partial_t\partial_z(RV) + \epsilon\varrho_0\partial_z^2\left(\frac{V^2}{2} + v_T^2 R^2\right), \tag{24}$$

$$\sigma\partial_t\Psi + i\mathcal{L}\Psi = i\left\langle R e^{-iz}\right\rangle, \qquad \lambda^2 = 2\varrho_0^2 v_T^2, \tag{25}$$

for the subsequent analysis using the RG approach.

III. RENORMALIZATION GROUP REDUCTION OF THE MACROSCOPIC EQUATIONS

Following the standard procedure of the RG method[9], we represent $\widehat{\mathcal{G}}(z, Z; t)$ as a perturbation expansion

$$\widehat{\mathcal{G}}(z, Z; t) = \sum_{n=0}^{\infty} \epsilon^n \widehat{\mathcal{G}}_n(z, Z; t), \tag{26}$$

where $\widehat{\mathcal{G}} = (R, V, \Phi)$ represents all hydrodynamic and field variables, and $Z = \epsilon z$ is a slow spatial variable. Thus, the only renormalization parameter left at our disposal is the time t which will prove extremely convenient and simplify tedious algebraic manipulations in the sequel.

To zero order the perturbation equations (24) and (25) can be written as

$$\partial_t^2 R_0 - \lambda^2 \partial_z^2 R_0 = \varrho_0 \big(\Psi_0 e^{iz} + \Psi_0^* e^{-iz} \big), \qquad (27)$$

$$\sigma \partial_t \Psi_0 + i \mathcal{L} \Psi_0 = i \big\langle R_0 e^{-iz} \big\rangle. \qquad (28)$$

Since the equation for R_0 is inhomogeneous (with regard to the spatial variable z), its solution is sought in the form

$$R_0(z, Z; t) = \mathcal{F}(Z; t) e^{iz} + \mathcal{F}^*(Z; t) e^{-iz}, \qquad (29)$$

where the function $\mathcal{F}$ satisfies the equation

$$\big(\partial_t^2 + \lambda^2 \big)(\sigma \partial_t + i\mathcal{L})\mathcal{F} - i\varrho_0 \mathcal{F} = 0. \qquad (30)$$

Taking the latter into account, we can write the general solution for R_0 in the form

$$R_0(z, Z; t) = \sum_m \mathcal{A}_m(Z) e^{i\omega_m t + iz} + \sum_m \mathcal{A}_m^*(Z) e^{-i\omega_m^* t - iz}, \qquad (31)$$

where the sum spans over all roots of the characteristic equation

$$\big(\omega^2 - \lambda^2 \big)(\sigma\omega + \mathcal{L}) + \varrho_0 = 0. \qquad (32)$$

In general, the characteristic equation has three roots. However, as it will become clear from the subsequent exposition, in the cases of physical interest one of the roots is real, while the other two are complex conjugate. Note also that the arbitrary (to this end) amplitudes $\mathcal{A}_m$ are constants with respect to the fast variables z and t, but can depend on the slow spatial variable Z.

Using Eqs. (21) and (28), we find

$$V_0 = -\frac{1}{\varrho_0} \sum_m \omega_m \mathcal{A}_m e^{i\omega_m t + iz} - \frac{1}{\varrho_0} \sum_m \omega_m^* \mathcal{A}_m^* e^{-i\omega_m^* t - iz}, \qquad (33)$$

$$\Psi_0 = \frac{1}{\varrho_0} \sum_m \big(\lambda^2 - \omega_m^2 \big) \mathcal{A}_m e^{i\omega_m t}. \qquad (34)$$

In first order the basic equations (24) and (25) can be expressed as

$$\partial_t^2 R_1 - \lambda^2 \partial_z^2 R_1 - \varrho_0 \Phi_1 = 2i\lambda^2 \sum_m \nabla_Z \mathcal{A}_m e^{i\omega_m t + iz}$$

$$- \frac{1}{\varrho_0} \sum_{m,n} G_{mn} \mathcal{A}_m \mathcal{A}_n e^{i(\omega_m + \omega_n)t + 2iz} + c.c., \qquad (35)$$

$$\sigma \partial_t \Psi_1 + i\mathcal{L}\Psi_1 = i \big\langle R_1 e^{-iz} \big\rangle. \qquad (36)$$

Here ∇_Z denotes differentiation with respect to the slow variable Z. Moreover, G_{mn} is a symmetric matrix given by the expression

$$G_{mn} = (\omega_m + \omega_n)^2 + 2\big(\omega_m \omega_n + \lambda^2 \big). \qquad (37)$$

It can be verified in a straightforward manner that the general solution of the first order equations is

$$R_1 = t \sum_m V_{gm} \nabla_Z \mathcal{A}_m e^{i\omega_m t + iz}$$

$$+ \frac{1}{\varrho_0} \sum_{m,n} F_{mn} \mathcal{A}_m \mathcal{A}_n e^{i(\omega_m + \omega_n)t + 2iz} + c.c., \qquad (38)$$

where

$$F_{mn} = \frac{(\omega_m + \omega_n)^2 + 2\big(\omega_m \omega_n + \lambda^2 \big)}{(\omega_m + \omega_n)^2 - 4\lambda^2}. \qquad (39)$$

The quantity V_{gm} is the group velocity, defined as follows. Let us introduce the dispersion function[10]

$$\mathcal{D}(k, \omega) = \big(\lambda^2 k^2 - \omega^2 \big)(\sigma\omega + \mathcal{L}) - \varrho_0, \qquad (40)$$

corresponding to a general solution of Eq. (24) proportional to $e^{i\omega t + ikz}$, where k is the wave number. Obviously, the dispersion equation for $k = 1$ that is, $\mathcal{D}(1, \omega) = 0$ reduces to the characteristic equation (32). The group velocity is given by the expression[10]

$$V_{gm} = \left. \frac{\mathrm{d}\omega_m}{\mathrm{d}k} \right|_{k=1} = - \left. \frac{\partial \mathcal{D}}{\partial k} \left(\frac{\partial \mathcal{D}}{\partial \omega_m} \right)^{-1} \right|_{k=1}, \qquad (41)$$

or in an explicit form

$$V_{gm} = \frac{2\varrho_0 \lambda^2}{\big(\omega_m^2 - \lambda^2 \big)\big(\sigma\lambda^2 - 3\sigma\omega_m^2 - 2\mathcal{L}\omega_m \big)}. \qquad (42)$$

For the first order current velocity V_1 we obtain

$$V_1 = -\frac{1}{\varrho_0^2} \sum_{m,n} \mathcal{V}_{mn} \mathcal{A}_m \mathcal{A}_n e^{i(\omega_m + \omega_n)t + 2iz} + c.c. + \ldots, \qquad (43)$$

where

$$\mathcal{V}_{mn} = \frac{\omega_m \omega_n + \lambda^2 + 2\lambda^2 F_{mn}}{\omega_m + \omega_n}, \qquad (44)$$

and the dots on the right-hand-side of Eq. (43) represent additional terms, which do not contribute to the secular terms in second order.

The final step in our perturbative procedure is to obtain the secular second-order solution. Retaining terms giving rise to secular contributions in the second-order solution, we can express the constitutive equations as follows

$$\partial_t^2 R_2 - \lambda^2 \partial_z^2 R_2 - \varrho_0 \Phi_2$$

$$= 2it\lambda^2 \sum_m V_{gm} \nabla_Z^2 \mathcal{A}_m e^{i\omega_m t + iz} + \lambda^2 \sum_m \nabla_Z^2 \mathcal{A}_m e^{i\omega_m t + iz}$$

$$- \frac{2}{\varrho_0^2} \sum_{m,n} \Gamma_{mn} \mathcal{A}_m \left| \widetilde{\mathcal{A}}_n \right|^2 e^{i\omega_m t + iz} + c.c., \tag{45}$$

$$\sigma \partial_t \Psi_2 + i\mathcal{L}\Psi_2 = i\langle R_2 e^{-iz} \rangle, \tag{46}$$

where the coupling matrix Γ_{mn} is given by the expression

$$\Gamma_{mn} = \omega_m \omega_n + \lambda^2 + 3\lambda^2 F_{mn}$$

$$+ (\omega_m + 2i\gamma_n)(\omega_n - 2i\gamma_n) F_{mn}, \tag{47}$$

and

$$\gamma_m = \mathrm{Im}(\omega_m), \qquad \widetilde{\mathcal{A}}_m = \mathcal{A}_m e^{-\gamma_m t}. \tag{48}$$

It is straightforward to verify that the secular second-order solution can be expressed as

$$R_2 = \sum_m \left(\frac{V_{gm}^2 t^2}{2} - \frac{it}{2} \mathcal{G}_m \right) \nabla_Z^2 \mathcal{A}_m e^{i\omega_m t + iz}$$

$$- \frac{it}{\varrho_0^2 \lambda^2} \sum_{m,n} V_{gm} \Upsilon_{mn} \mathcal{A}_m \left| \widetilde{\mathcal{A}}_n \right|^2 e^{i\omega_m t + iz} + c.c., \tag{49}$$

where

$$\Upsilon_{mn} = \Gamma_{mn} \frac{\sigma(\omega_m + 2i\gamma_n) + \mathcal{L}}{\sigma\omega_m + \mathcal{L}}, \tag{50}$$

$$\mathcal{G}_m = V_{gm} \left[1 + \frac{V_{gm}^2}{\varrho_0 \lambda^2} \left(\omega_m^2 - \lambda^2 \right) (3\sigma\omega_m + \mathcal{L}) \right]. \tag{51}$$

The last step is to collect all terms corresponding to increasing orders, which contribute to $R(z, Z; t)$ and perform a resummation such as to absorb secular terms (proportional to various powers of the time variable t) present in R_1 and R_2. Since this approach is standard[9], we omit details here.

IV. THE RENORMALIZATION GROUP EQUATION

Following the standard procedure[9,10] of the RG method, we finally obtain the desired RG equation

$$i\partial_t \widetilde{\mathcal{A}}_m - iV_{gm} \partial_z \widetilde{\mathcal{A}}_m$$

$$= \frac{\mathcal{G}_m}{2} \partial_z^2 \widetilde{\mathcal{A}}_m + \frac{V_{gm}}{\varrho_0^2 \lambda^2} \sum_n \Upsilon_{mn} \widetilde{\mathcal{A}}_m \left| \widetilde{\mathcal{A}}_n \right|^2, \tag{52}$$

where now $\widetilde{\mathcal{A}}_m$ denotes the renormalized complex amplitude of the type (48). In terms of the renormalized wave envelopes, the macroscopic density $\varrho(z; t)$ can be expressed as

$$\varrho(z; t) = \varrho_0 + \sum_m \widetilde{\mathcal{A}}_m(z; t) e^{i\mathrm{Re}(\omega_m)t + iz}$$

$$+ \sum_m \widetilde{\mathcal{A}}_m^*(z; t) e^{-i\mathrm{Re}(\omega_m)t - iz}. \tag{53}$$

Similar expressions hold for the current velocity V [compare with Eq. (33)] and for the electrostatic potential Ψ [compare with Eq. (33)].

It was mentioned in the previous section that being an algebraic equation of third order, the characteristic equation (32) possesses three roots in general. Thus, we can distinguish the following two cases of physical interest.

All three roots ω_1, ω_2 and ω_3 are real. Therefore, the renormalization group equation (52) comprises a system of three coupled nonlinear Schrodinger equations

$$i\partial_t \mathcal{A}_m - iV_{gm} \partial_z \mathcal{A}_m$$

$$= \frac{\mathcal{G}_m}{2} \partial_z^2 \mathcal{A}_m + \frac{V_{gm}}{\varrho_0^2 \lambda^2} \sum_{n=1}^{3} \Gamma_{mn} \mathcal{A}_m |\mathcal{A}_n|^2, \tag{54}$$

where $m = 1, 2, 3$.

One real ω_1 root and two ω_2 and ω_2^* complex conjugate roots. In this case the renormalization group equation (52) represents a system of two coupled complex Ginzburg-Landau equations of the form

$$i\partial_t \mathcal{A}_1 - iV_{g1} \partial_z \mathcal{A}_1 = \frac{\mathcal{G}_1}{2} \partial_z^2 \mathcal{A}_1$$

$$+ \frac{V_{g1}}{\varrho_0^2 \lambda^2} \left[\Gamma_{11} |\mathcal{A}_1|^2 + \Gamma_{12}(1 + 2i\gamma_{12}) \left| \widetilde{\mathcal{A}}_2 \right|^2 \right] \mathcal{A}_1, \tag{55}$$

$$i\partial_t \widetilde{\mathcal{A}}_2 - iV_{g2} \partial_z \widetilde{\mathcal{A}}_2 = \frac{\mathcal{G}_2}{2} \partial_z^2 \widetilde{\mathcal{A}}_2$$

$$+ \frac{V_{g2}}{\varrho_0^2 \lambda^2} \left[\Gamma_{21} |\mathcal{A}_1|^2 + \Gamma_{22}(1 + 2i\gamma_{22}) \left| \widetilde{\mathcal{A}}_2 \right|^2 \right] \widetilde{\mathcal{A}}_2, \tag{56}$$

where the quantities γ_{12} and γ_{22} are given by the expressions

$$\gamma_{12} = \frac{\sigma\gamma_2}{\sigma\omega_1 + \mathcal{L}}, \qquad \gamma_{22} = \frac{\sigma\gamma_2}{\sigma\omega_2 + \mathcal{L}}. \tag{57}$$

The complexity of the coupling coefficient in Eq. (55) prevents the system (55) - (56) from possessing a solution in the form of a plane wave. However, if the imaginary part of the complex coupling coefficient in Eq. (55) is small and can be neglected, the above system reduces to a coupled set of a nonlinear Schrodinger and a complex Ginzburg-Landau equation. Since the second equation (56) generally possesses a solution in the form of a planar wave with constant amplitude, the first equation (55) can be transformed approximately to a dissipative nonlinear Schrodinger equation. Finally, in the third limiting case, where $\mathcal{A}_1 = 0$, we end up with a single complex Ginzburg-Landau equation (56) for the $\widetilde{\mathcal{A}}_2$ amplitude only.

V. CONCLUDING REMARKS

Using a class of waterbag phase space density distributions, which is an exact solutions to the Vlasov equation, we have cast the single-wave model equations to a fluid dynamic form with nonzero thermal velocity. Since the continuity and momentum balance equations comprise an exact hydrodynamic closure, the hydrodynamic representation is fully equivalent to the original Vlasov-Maxwell system.

Based on the renormalization group method, a system of coupled nonlinear equations for the slowly varying amplitudes of interacting plasma density waves has been derived. Depending on the solution of the dispersion equation the system mentioned above takes the form of either three coupled nonlinear Schrodinger equations in the case, where all three roots are real, or two coupled complex Ginzburg-Landau equations if the dispersion equation possesses one real and two complex conjugate roots.

An interesting continuation of the present study would be the consideration of large-scale hydrodynamic fluctuations and their influence on the dynamics of perturbations. This we plan to complete in a future publication.

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