Fundamental physics and cosmology with LISA

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In this article we give a brief review of the fundamental physics that can be done with the future space-based gravitational wave detector LISA. This includes detection of gravitational wave bursts coming from cosmic strings, measuring a stochastic gravitational wave background, mapping space-time around massive compact objects in galactic nuclei with extreme-mass-ratio inspirals and testing the predictions of General Relativity for the strong dynamical fields of inspiralling binaries. We give particular attention to new results which show the capability of LISA to constrain cosmological parameters using observations of coalescing massive Black Hole binaries.

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I. INTRODUCTION

LISA is a proposed joint ESA-NASA mission which will be launched around 2025 and which will be sensitive to gravitational waves (GW) of low-frequency (between 10^{-4} and 0.1 Hz). The number of astrophysical sources that emit gravitational waves in this frequency range is significantly larger than the number that emit in the high frequency range (above a few Hz) that is accessible to ground-based detectors. In our own Galaxy there are $\sim 10^6 - 10^7$ ultra-compact white dwarf binaries which will be emitting gravitational waves in the LISA band. The majority of these will not be individually resolved and will form a gravitational wave foreground below $\sim$ 3mHz [1] [2]. Coalescing massive black hole (MBH) binaries will be detectable throughout the Universe. The parameters of those binaries will be measured with an accuracy not achievable by any other means. The distribution of the parameters as a function of redshift will provide valuable insights into the process of galaxy formation [3]. Extreme-mass-ratio inspirals (EMRIs) result from the capture of a compact stellar mass object (a white dwarf, neutron star or black hole) by a MBH from a surrounding cusp of stars in galactic nuclei. Those sources will provide us with understanding of the stellar dynamics and evolution of the central parsec of galaxies. All of this astrophysical information is very difficult (if possible at all) to obtain in any other way but through the GW observations with LISA [4].

Besides astrophysics, LISA will be a laboratory for a fundamental physics. LISA will be able to detect gravitational wave bursts coming from cusps forming on cosmic strings if they exist. Strings are one dimensional topological defects produced as a result of braking a $U1$ symmetry. This symmetry braking and (therefore cosmic string formation) is a prediction of the grand unification theory (GUT) [5] and of string-inspired inflationary models [6, 7]. The low frequency operational band of LISA makes it more sensitive to these bursts than are ground based GW detectors.

Similar to the Cosmic Microwave Background, a stochastic gravitational wave background may have been formed in the early Universe as a result of the parametric amplification of "zero-point" quantum oscillations [8] or during the first order phase transition at the end of inflation [9–11]. The present energy density of the stochastic GW background is very uncertain but if the density is close to the more optimistic predictions of its value then it could be probed with LISA. Even if it is strong enough, we will need to distinguish the relic stochastic GW background from the GW foreground discussed above. This might be possible if we take into account the cyclo-stationarity of the Galactic foreground, i.e., the fact that the level of the galactic foreground should vary over a year as the LISA antenna pattern sweeps over the galactic centre and then away again.

Coalescing MBH binaries can serve as standard sirens (as opposed to "candles" in the electromagnetic spectrum) [12, 13]. GW measurements can give us an accurate estimate of the luminosity distance to a MBH merger. If we can identify an electromagnetic counterpart to a GW event, then we can measure the redshift of the source and get an independent point on the luminosity-distance/redshift relation that can be used to estimate cosmological parameters (like the cosmological constant, matter density, Hubble constant, spatial curvature etc.). In this paper (as in the original proposal [12]), we will not assume that electromagnetic (e/m) identification of the host is possible. Instead, we will use the fact that LISA will observe about 30 events per year and use a statistical technique to derive constraints on cosmological parameters. In particular, we will assume that all parameters are known besides the effective equation of state for the dark energy and focus on the precision with which LISA could determine this.

The study of stellar dynamics (especially the dynamics of the S-stars) in the Galactic center tells us that there is a very compact dark object with a mass $\sim 4 \times 10^6 M_\odot$ in the Galactic nucleus [14]. We expect the presence of a dark massive object (DMO) in the nuclei of almost all galaxies. The common belief is that DMO should be a MBH, but we do not have direct observational evidence for the presence of an event horizon in these systems. There is an intensive ongoing program to scope out the possibility of testing the nature of the DMO using GW observations of extreme-mass-ratio inspirals. A compact object captured by a DMO with mass in the LISA range ($\sim 10^6 M_\odot$) generates $10^4 - 10^6$ cycles of gravitational waves while it is orbiting in the strong gravitational field of the DMO. The emitted GWs encode an imprint of the spacetime surrounding the DMO and the hope is that we can extract information from the GW signal which will allow us to test the nature of the DMO. The most promising approach is to measure the multipoles of the DMO (mass, spin and quadrupole moment) and compare them with the corresponding values for the Kerr metric.

In this article we review the current status of our understanding of these various topics. We will start, in Section II, with a description of LISA's ability to detect GW bursts from cosmic strings before moving on to a discussion of how to detect a cosmological stochastic GW background in Section III. In Section IV we will present new results on constraining cosmological parameters with LISA using statistical method. We then review the program of "mapping spacetime" with EMRIs in Section V before concluding the article with a summary in Section VI.

II. DETECTING GRAVITATIONAL BURSTS FROM THE COSMIC STRINGS

As mentioned in the Introduction, a string soliton solution will arise in a gauge theory with a broken $U(1)$ symmetry or in any phase transition in which $U(1)$ becomes broken during the evolution. The original interest in cosmic strings was in the context of grand unification theories, but it was later [6, 15] suggested that strings would be produced by D-brane annihilation at the end of brane inflation. A fraction of these strings will be in the form of finite size loops, and other strings will be infinite. The latter are particularly important as they stretch with the expansion of the Universe while the small loops quickly decay. When two strings collide they may reconnect with a probability P_{rec}. The main parameter characterizing a string is its tension μ. The two parameters (μ, P_{rec}) characterize the properties of a network of strings [16]. We refer readers to [17, 18] for nice reviews on the mechanisms for production of cosmic strings and their detection in [19].

When a long string intersects itself, a loop breaks off. If the size of the loop is smaller than the horizon, it will not expand and behaves as a massive object. The closed loops decay through emission of gravitational radiation. The spectrum of the resulting GW background is roughly flat over LISA's operational frequency band and can be characterized by the energy density of the gravitational waves (GW) ρ_{GW}:

$$\Omega_{GW}(f) \equiv \frac{d\rho_{GW}/d\ln f}{\rho_c}, \tag{1}$$

where ρ_c is the Universe critical density. This quantity depends on the tension of the strings: $\Omega_{GW} \sim \sqrt{G\mu}$ [5, 18]. The proportionality coefficient depends on the scaling factor at the moment of loop formation α and on Γ, the scaling factor for the decay time of a loop (of length l) $t_d = l/\Gamma G\mu$. The limit on Ω_{GW} from ground-based gravitational wave detectors LIGO-VIRGO is $\Omega_{GW}(f \sim 100Hz) \leq 6.9 \times 10^{-6}$ [20]. The best current limit comes from pulsar timing. Analysis of the timing residuals attributed to the effect of propagation of electromagnetic pulses in a stochastic gravitational field yields an upper limit $\Omega_{GW}(f \sim 10^{-8}Hz) \leq 4 \times 10^{-8}$ which corresponds to a limit on the tension $G\mu < 1.5 \times 10^{-8}$ [21, 22]. This limit will get improved as the observation time increases. The prospects with LISA are even better. It was shown in [22] that LISA would be able to detect a background from cosmic strings with corresponding tension $G\mu > 10^{-16}$.

When two long strings reconnect they produce two long kinked strings. These kinks propagate along the strings and tend to straighten in time which reduces the total strength and the energy is released via gravitational wave radiation. Kinks produce short GW bursts. Another source of GW burst are cusps, which are created when a tiny part of the string propagates with a speed close to the speed of light. It was shown in [23] that GW bursts coming from cusps are more detectable than those coming from kinks in GW observations. The GW radiation from cusps is linearly polarized and highly beamed in the direction of propagation of the cusp. The shape of the burst in the time domain has very simple form: $h(t) \sim |t - t_c|^{1/3}$ where t_c is the time of arrival of the burst at an observer. The cusp at $t = t_c$ exists only for an observer who lies *exactly* along the direction of the cusps's velocity. For an observer who is at an angle θ to the center of the radiation cone, there is an exponential decay of the radiation at frequencies above $f_{max} \sim 1/\theta^3$. This upper frequency cut off in return smoothes $h(t)$ at $t = t_c$ [23, 24]. The GW waveform in the frequency domain scales as $\tilde{h}(f) \sim f^{-4/3}$ (as compared to the GW bursts from the kinks which scales as $\tilde{h}(f) \sim f^{-5/3}$). A simple estimation in [25] suggested that LISA will be a factor of ten more sensitive to GWs from cusps than the advanced ground based detectors. The square of the signal-to-noise ratio (SNR) per infinitesimal logarithmic frequency band $(d\ln f)$ is approximately $|f\tilde{h}(f)|^2/fS_h(f)$, where $S_h(f)$ is the noise power spectral density. Then the SNR for a GW burst from a cusp is roughly $f_b^{-1/3}/(f_b S_h(f_b))^{1/2}$, where f_b is the frequency where the detector has the best sensitivity (this is ~ 100Hz for advLIGO and ~ 0.003Hz for LISA). The denominator $((f_b S_h(f_b))^{1/2})$ for LISA is few times higher than the corresponding number for advanced LIGO while the numerator $f_b^{-1/3}$ is approximately 30 times larger for LISA. This implies a factor of $\sim 10^3$ improvement in the observable volume for a given SNR threshold. We expect that a string network will produce quite a few strong bursts standing above the background. The distribution of SNRs of detected GW bursts will provide us with valuable information on the sting network.

Since we know the form of the GW signal from a cosmic string cusp, we will be able to use matched filtering techniques to search for them in the LISA data stream. In order to assess existing data analysis techniques there have been a sequence of Mock LISA data challenges, and the last two of these included bursts from cosmic string cusps in the data sets. These challenges involve blind searches for multiple signals in simulated LISA data [26, 27]. For the cosmic string cusps, the exact number of bursts in the data was not known (it was drawn from a Poisson distribution with mean 5).The injections used the following waveform model for the GW signal [27]:

$$\tilde{h}_{ij}(f) = (A^+_{ij} + A^\times_{ij})A(f)e^{2\pi i f t_c}, \tag{2}$$

$$A^+_{ij} + A^\times_{ij} = \mathcal{A}\Lambda(f)(e^+_{ij}cos(\psi) + e^\times_{ij}sin(\psi)) \tag{3}$$

$$\Lambda(f) = \begin{cases} f^{-4/3} & f < f_{max} \\ f^{-4/3} e^{1-f/f_{max}} & f > f_{fmax} \end{cases} \quad (4)$$

The overall amplitude $\mathcal{A} \sim \frac{G\mu l^{2/3}}{D_L}$ depends on the luminosity distance to the source D_L, the string tension and the characteristic scale of the string. The polarization angle ψ and the polarization tensors are defined with respect to the fixed direction of GW propagation.

The results of the blind search [25, 28, 29] (see also [27] for the overall picture) showed that current methods allow us to find GW bursts from cusps rather easily. Parameter estimation for these systems is a rather different issue, however, as the short duration of the signal means there are strong degeneracies in the parameter space. In particular the sky localization is rather poorly constrained (more than one radian) even if we are at the right maximum of the likelihood. Other parameters like polarization angle and the amplitude strongly correlate with the sky location and are thus also poorly determined. The parameter which can be determined the best is the time that the burst passes through the detector (t_c). Based on these results, we have very good prospects of detecting GW bursts from cosmic cusps with LISA but this will be with rather poor determination of the parameters.

III. STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

There are different mechanisms which lead to the production of a cosmological gravitational wave background. The main one is the adiabatic amplification of quantum fluctuations [8], which is most efficient during inflation and is often referred to as the inflationary “relic” GW background. Other mechanisms deal with classical (as opposed to quantum) sources which potentially can be much stronger in some frequency bands. One such background is that created by decaying cosmic string loops, as we discussed in the previous section. There is a vast literature reviewing this topic [30–34] and as we do not have anything new to add we will just mention the various sources and the prospects for their detection with LISA.

A phase transition corresponding to symmetry breaking of the fundamental interactions could have happened in the early Universe. In the first order phase transition the Universe is initially trapped in a metastable phase (with unbroken symmetries). The transition from the metastable phase to the ground state occurs through the quantum tunneling across the barrier of potential energy of a scalar field. This transition nucleates randomly in bubbles. The size of these bubbles increases as the temperature of the Universe drops and large bubbles then collide bringing the Universe to a broken symmetry phase. Gravitational radiation is produced as soon as the spherical symmetry of an individual bubble is broken. In addition the growth of bubbles converts internal energy into relativistic flows which generate high bulk turbulent velocities and accelerates the fluid leading to further generation of GWs. The spectrum of GW from the first order phase transition is peaked around the frequency determined by the typical temperature at which the transition takes place: $f_{peak} \sim 100\,\mathrm{Hz}(\mathrm{T}/10^5\mathrm{TeV})$ [35, 36]. The electroweak phase transition happened at the energy scale of 100 GeV, which gives a peak at ~ 0.1mHz, right in LISA’s band.

Other possible sources of stochastic GWs are reheating and the dynamics of extra dimensions. There is an uncertainty as to when inflation finished and consequently gravitational radiation generated during the reheating epoch could be in the right band for LISA with a detectable amplitude. Brane-world scenarios further boost the ratio of tensor-to-scalar perturbations and correspondingly increase the expected GW background.

How do we detect GWs with LISA? In order to cancel laser phase noise, six phase measurements at the spacecraft are combined with a technique known as Time Delay Interferometry [37]. One can derive three TDI streams (A, E, T) in which the noise is uncorrelated (even for an unequal arm LISA [38]). The stochastic GW signal is present in the A and E channels, but it is strongly suppressed at low frequencies in the T channel, which means a correlated stochastic signal is present in A and E, while only instrumental noise is present in T. Taking into account the correlation between A and E and the “null” signal in T at low frequencies allows the detection of a stochastic GW signal. The authors in [38] demonstrated that using Bayesian techniques LISA would be able to detect a GW background as low as $\Omega_{GW} = 6 \times 10^{-13}$.

IV. CONSTRAINING COSMOLOGICAL PARAMETERS WITH LISA

The typical gravitational wave signals that we expect to be present in the LISA data will be generally quite strong which allows very precise measurements to be made of the source parameters. In particular, we expect to be able to determine the luminosity distance of a coalescing MBH binary at redshift $z = 1$ to a precision better than 1% and to localize it on the sky to within $10 - 100$ arcminutes [39]. If the binary is embedded in the circumnuclear disk there might be a transient electromagnetic signal associated with the GW event. For some nearby events, this e/m counterpart could be detectable (if the GW measurement provides good localisation of the source) which would allow us to measure the redshift of the source in addition to the luminosity distance D_L. Measurements of $D_L(z)$ tell us about the cosmological parameters. For this reason, coalescing MBH binaries are referred to as standard sirens [12, 13]. A lot of effort is currently being put into modelling possible e/m signals which could accompany the merger of two MBHs (see for example [40] and references therein).

In this we describe results that do not rely on the existence of redshift measurements using e/m observations, but are instead based on a statistical approach to this problem. The motivation for this analysis is that e/m counterparts may be very weak or indeed not exist at all. We must then ask whether it is still possible to constrain the cosmological parameters by combining GW measurements from a few dozen sources. While the other sections of this article are reviewing previous work on the capabilities of LISA, the results in this section are new and appear here for the first time.

We simulate the LISA data set by adopting the model for the hierarchical formation of MBHs described in [41]. This model predicts the number of MBH binary merger events that LISA will see, and the SNR and MBH mass distribution for these events. We assume three years of LISA observations and generated many realizations of the set of LISA observations. There are typically about thirty events in three years of observation up to a redshift of $z \leq 3$. We assume that the BHs are spinning and the spins point in random directions and have arbitrary magnitude. We assume that the errors in the parameters determined from the GW observation have a multivariate Gaussian distribution with variance-covariance as predicted by the Fisher information matrix. This is a reasonable assumption due to the high strength of the GW signal. We are interested in the errors that are present in the estimation of the luminosity distance and the sky location. To each gravitational wave event we associate an error box (cylinder). The radius of the error cylinder is determined by the accuracy of localization of the GW source on the sky. The length of the cylinder along the line of sight is determined by the error associated with measurements of D_L and uncertainties in the cosmological parameter, which are taken as priors. The error in D_L coming from the GW measurements alone is quite low and dominates the error only at low redshift ($z < 0.5$). At higher redshifts, the main source of distance error is weak lensing. Dark and bright matter between us and the source (de)magnifies the GW signal making it appear (further) closer than it is in reality. We model weak lensing using the mean errors quoted in [42] and assumed these are normally distributed, which is streakily speaking not correct [43]. In Figure 1 we show the (median) error in the GW measurements due to instrumental noise as a black (solid) line, the errors due to weak lensing as (red) circles and the combined error as a (blue) circle-solid line.

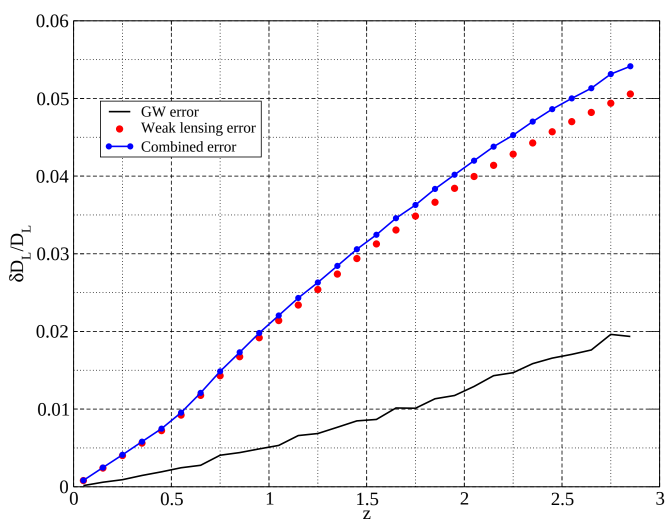

FIG. 1. Relative error in the luminosity distance due to instrumental noise (black solid line), due to weak lensing (red circles) and the combined error (blue circle-solid line). The weak lensing error is taken from [42].

The next step is to model the Universe. We have used the Millennium simulation [44] to mimic the large scale structure of the Universe at different redshifts. The Millennium simulation assumes a lambda cold dark matter model of the Universe and so do we. In particular, we model the evolution of the Hubble parameter as

$$H^2 = H_0^2 \left[\Omega_m^0 (1+z)^3 + \Omega_{de}^0 \exp\left(3 \int_0^z dz \frac{1+\omega(z)}{1+z} \right) \right]. \tag{5}$$

with $H_0 = 73.0\,\mathrm{km/(sec\,Mpc)}$, $\Omega_{\mathrm{m}}^0 = 0.25$, $\Omega_{\mathrm{de}}^0 = 0.75$, $\omega(\mathrm{z}) = -1$. Using the semi-analytic model described in [45], one can attach masses to the components of each galaxy and luminosity. For each GW event in our realisation of the

LISA data, we choose the nearest snap shot of the Millennium simulation and associate one of the directions in the comoving volume as the direction of the line of sight (redshift). Then we chose the galaxy that hosts the merging MBHs. The probability of the galaxy to be a host is proportional to the local density. We note, however, that the probability that the host is in a low density region is not small (about 50%), since although the probability that the merger happens in a low density part of the universe is low, there are many such parts with low local density. Once the host is chosen it is surrounded by the error box (cylinder) discussed above. In practice we need to place an error box in the sky coordinates and in the redshift and in order to go from the measured luminosity distance to the redshift we must assume a cosmological model. As a result the size of the error box is governed not only by the errors associated with the determination of the luminosity distance but also by the cosmological uncertainty in translating the D_L to a redshift. We assume that all cosmological parameters are known but the effective equation of state for dark energy, which is modelled as $\omega(z) = -1 + w$. We take the currently estimated range for $w \in [-0.3 : 0.3]$. Our main objective is to show that combining all GW observations can tighten these constraints on w. A similar statistical approach was taken in [46] to reduce the uncertainty in H_0 as derived from GW observations of extreme-mass-ratio inspirals with LISA. The idea is to measure the redshift of all galaxies which are potential hosts of the merger, i.e., which lie within in the error box, combine these statistically into an estimate of the source redshift, and correlate the results across all GW events. We emphasize several important points: (i) we assume that, by the time LISA flies, we will be able to measure the redshift to all galaxies with an apparent magnitude $m \leq 24$, therefore there is a strong selection effect at high redshifts, at which we will see only very massive galaxies, whereas LISA mergers are most likely to be associated with moderate mass $\sim 10^{10} M_{\odot}$ merging galaxies. Therefore, the host may not be included in the set of galaxies used. (ii) The method relies on the fact that the distribution density within the error box is not uniform. The larger the contrast in density along the line of sight the better our method works. For this reason we do not consider GW events at redshifts $z \geq 3$ (iii) Since we might not be able to observe hosts at moderate to high redshifts, we rely on the self-similarity in the density profile across different mass ranges, i.e., that the distribution of the lighter galaxies which host LISA sources is similar to the distribution of the more massive galaxies that we can observe. We did check this self-similarity using the Millennium data. In our simulations we choose the host from all galaxies in the snap shot but for measuring the redshift we use only the galaxies with apparent magnitude $m \leq 24$. We also use only the bright galaxies to estimate the local density. If the chosen host close to the boundary of the snap shot' box we assume periodic boundary conditions for the data (as was done in the Millennium simulation itself).

The posterior (marginalized) density distribution for w from a single GW event is

$$P_j(w) = p_0(w) \int \Lambda_j(D_L(z,w), \theta, \phi) p_j(\theta, \phi, z) \, d\theta \, d\phi \, dz, \tag{6}$$

where $p_0(w)$ is our prior on w, $\Lambda_j(D_L(z,w), \theta, \phi)$ is the likelihood marginalized over the other parameters of the MBH binary so it depends only on D_L and the sky position (θ, ϕ), and $p_j(\theta, \phi, z)$ is the astrophysical prior for a given galaxy to be the host, which is proportional to the square of the local density. The final distribution for w is the product of the probabilities for individual GW events: $P(w) = \prod_j P_j(w)$.

We considered 30 realisations of the LISA data and used a flat prior on w. The final probability density $P(w)$ could be well approximated in most of the realisations by a single gaussian and in the few other case by a sum of several gaussian profiles. The peak of the distribution was usually found to be close to zero (which was the true value used to generate the data set) and the width of the gaussian was a factor 2-8 smaller than the prior range. A summary of the results of the simulations are shown in Figure 2. In this figure, we have fit the posterior in each realisation with a Gaussian and show the mean (as a circle) and the variance (as an error bar) in each realisation in the left panel. We also show the χ^2 value for each of the gaussian fits to $P(w)$ in the right panel. In a few cases there was a nearby merger which could be very well localized on the sky, and there was then only one cluster of galaxies in the error box, which is essentially equivalent to having an e/m counterpart. The GW events for which we could identify the e/m counterpart in this way were removed from the analysis to allow us to show how well we can constrain w without an e/m identification of the host. The figure shows that we have very good prospects for constraining the effective equation of state of the dark energy provided we know all the other cosmological parameters (which could be the case by the time LISA is launched). In a separate publication [47] we will give a more detailed description of our simulation and analyze more realizations with varying parameters.

V. MAPPING SPACETIME AROUND COMPACT MASSIVE OBJECTS IN GALACTIC NUCLEI

Now we turn our attention to probing the nature of the DMOs that are observed to reside in galactic nuclei. The common belief is that these DMOs are Kerr BHs, but can we test this assumption? One of the ways to answer this question is through observations of GWs from extreme-mass-ratio inspirals. In such systems, the inspiralling compact object (CO) can spend a significant amount of time in the strong field region of spacetime close to the central object

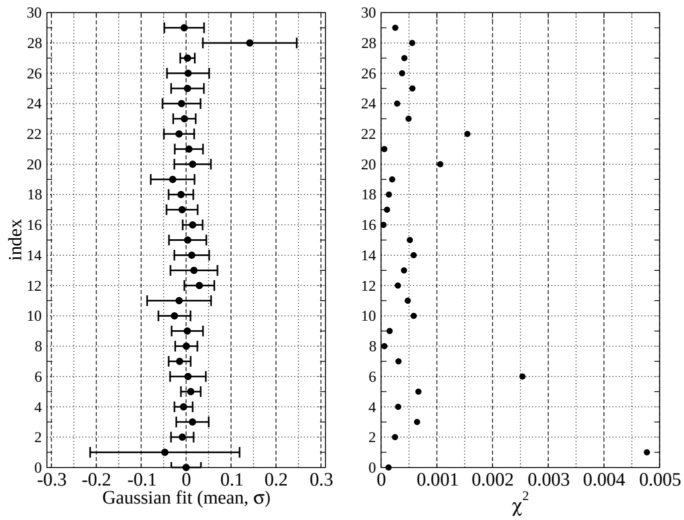

FIG. 2. Left panel: parameters of the gaussian fit for the final probability density $P(w)$ for each of the 30 realizations; Right panel: goodness of the gaussian fit, χ^2.

before it plunges. Detailed information about the structure of the spacetime that the CO is orbiting in will be encoded in the GW signals that are received by LISA. If a strong EMRI signal is detected, then we will first need to extract this information, and, if a deviation from "Kerriness" is detected, we will need to interpret it. We refer readers to [48] for a review on EMRI waveforms and to [49] for related astrophysics and concentrate here only on the use of the EMRI signals for mapping spacetime.

The question is, what kind of information in a GW signal would indicate a deviation from the Kerr spacetime? GW measurements with LISA will rely on matched filtering and will therefore be sensitive to effects that lead to a mismatch in the phase between the signal and the model. We will detect GWs over $10^4 - 10^6$ orbits of the CO, and so we will be sensitive to relatively small changes in the orbital motion. Most of the research to date has focussed on exploring how orbits differ in spacetimes that deviate from Kerr. We review below the various attempts that have been made to construct and characterize such spacetimes.

The first approach to spacetime mapping was proposed in [50]. This relied on using the multipole decomposition of the metric outside a stationary axisymmetric central body in general relativity, which can be expressed in terms of mass M_l and current S_l multipole moments. For a Kerr BH these moments are determined by two parameters — $M_0 = M_{BH}$, the total mass of the BH and $S_1 = M_0 a$, the spin — and all higher moments are determined from these through the relation $M_l + iS_l = M_{BH}(ia)^l$. If more than two moments of the spacetime are measured through GW observations, these can therefore be tested for consistency with the Kerr solution. It was shown that the different multipole moments enter at different orders in a decomposition of the vertical and radial epicyclic frequencies as a function of orbital frequency for circular-equatorial orbits [50]. These epicyclic frequencies can, in principle, be determined from a gravitational wave observation, allowing the multipole moments to be measured. However, the higher multipoles enter the phasing of the GWs waves with a strength that decreases with multipole number, so in practice we would expect only to be able measure the first few moments. Nonetheless, for an EMRI with masses $10\,M_\odot, 10^6\,M_\odot$ and with SNR ~ 100 it is estimated that we will be able to measure the quadrupole moment of the central object with a precision 10^{-3} [51] while simultaneously measuring the mass and spin of the black hole to $\sim 10^{-4}$.

The multipole decomposition is not a convenient way to characterize nearly-Kerr spacetimes, as Kerr itself has an infinite number of non-zero multipoles. For that reason, several authors have instead looked at various "bumpy" BHs, which are solutions to the Einstein field equations that are close to Kerr and depend on a deviation parameter, ϵ, such that $\epsilon = 0$ is the Kerr metric. This deviation parameter can in principle be measured or bounded by a GW observation.

- The first such attempt was in [52]. They constructed a bumpy BH using a perturbation of Schwarzschild. This was later extended to rotating objects using an elegant new algorithm in [53]. In these papers the authors analyzed how the deviations from the Kerr solution modified the fundamental orbital frequencies of geodesics.

- A different approach was taken in [54] where the bumpy BH was inspired by the Hartle-Thorne metric for slowly rotating bodies. A kludge waveform (without radiation reaction) was constructed and for the first time the confusion problem was considered, i.e., that there could exist a Kerr waveform which matches near-perfectly the waveform of a quasi-Kerr spacetime with slightly different parameters, if radiation reaction is not taken into account. This "quasi-Kerr" metric was used in [55] and follow up papers to analyze the propagation of null geodesics and to consider the imprint of non-Kerrness in electromagnetic observations.

- In [56] the authors considered an exact solution of GR (Manko-Novikov) describing an arbitrary axisymmetric body which can deviate from Kerr at any multipole moment. They again studied the frequencies of geodesic motion and the existence of a third integral of the motion (which is the Carter constant in Kerr spacetime) as well as precession rates in this general spacetime. It was found that the third integral could be lost under certain circumstances, which would be a "smoking-gun" for a deviation from Kerr.

In each of the above papers, the main observable considered was the frequency of geodesic orbits, which would show up in GW observations through small phase-shifts over the observation. The loss of the third integral considered in [56] is a more robust signature of a deviation from the Kerr metric as that behaviour is qualitatively different from what is expected in the Kerr spacetime. Another example of a feature that is qualitatively different in a perturbed spacetime was suggested in [57]. When an integrable system is perturbed, resonant points become smeared out into resonant chains of islands (Poincare-Birkhoff theorem). Such deviations would therefore show up as a persistent resonance in the observed GWs, i.e., by a persistent period of time in which two of the fundamental frequencies of the orbit were commensurate.

In all the above cases, the authors dealt with vacuum solutions of GR, but it is clear that astrophysical black holes will not be pure vacuum solutions. There have also been several attempts to describe a deviation from Kerr due to the presence of matter, for instance by a scalar field with considerable self-interaction (a so called massive Boson star) [58]. The non-rotating Boson star has no horizon and is not as compact as a BH, so the orbit of an inspiralling compact object could pass inside the Boson star, where the metric deviates from Schwarzschild. This will leave an imprint on the GW signal, as the GW emission will persist after the plunge should have been observed. Another possibility is that the Kerr BH is surrounded by an accretion disk. The authors in [59] considered the gravitational influence of a massive self-gravitating torus around the Kerr MBH on the orbit of a CO that did not intersect the disc. If radiation reaction is not taken into account, one can readjust the orbital parameters, e.g., the mass and spin of the MBH, in the presence of the torus so that the confusion problem (i.e., that we cannot distinguish the waveform from a pure Kerr case) discussed earlier is seen. The amount of material required to leave an imprint on the signal was found to be unreasonably large, suggesting that the gravitational influence of an accretion disc would be undetectable even when radiation reaction was taken into account. However, in a follow up paper [60], the effect of the hydrodynamical drag force on the particle was considered when the CO orbit intersects the disc. In this case, a small amount of energy will be dissipated due to the interaction of the body with the gas on each passage through the disc (possibly several times per orbit). This interaction leads to a decrease in the orbital inclination, which is qualitatively different to the pure-GR case in which radiation-reaction drives increasing inclination, and small changes in the other orbital parameters. However, for the drag force effect to be comparable to the radiation reaction force (self-force) one would again need a very massive accretion disk and a lower mass MBH ($\sim 10^5\,M_\odot$). Recently it was shown [61] that the presence of a second MBH within a few tenths of a parsec could also leave a measurable imprint on the EMRI waveform. This comes from the fact that the center of EMRI system is then no longer an inertial frame due to the acceleration exerted by the perturber and this leads to a Doppler phase shift of the GW signal.

All the above situations were considered within the theory of GR. However, deviations in EMRI observations might also arise if the true theory of gravity was *not* GR. In [62], the authors considered BHs in Chern-Simons theory, which deviate from Kerr BHs in the fourth multipole moment. This affects geodesic motion and correspondingly the phase of the GW signal. The authors found that deviations in geodesics could be significant, but more work is required to estimate the precision with which EMRI observations will be able to constrain a Chern-Simons modification to GR. EMRI observations can also be used to test theories of massive gravity. In these theories the GW have additional polarizations [63], and propagate with a speed different from the speed of light. One can thus see dispersion by comparing different harmonics of the EMRI signal. If we are lucky enough to see a disruption of a WD by $\sim 10^5\,M_\odot$ MBH [64], then we could also compare the time delay between the GW and electromagnetic signals. EMRIs can also be used to constrain scalar-tensor theories, but this requires the inspiralling object to be a neutron star rather than a black hole. In [65] it was estimated that observations of a neutron star inspiralling into a $10^4\,M_\odot$ spinning BH with SNR ~ 10 could bound the Brans-Dicke parameter, ω_{BD}, to $\omega_{BD} >$few$\times 10^3$, which is slightly better than current Solar System bounds from observations of the Cassini satellite.

While most research to date on testing GR using LISA has focussed on EMRIs, test will also be possible using observations of coalescing MBH binaries of comparable mass. In such systems, the deviation from GR will again leave an imprint on the inspiral and correspondingly on the phase of GWs. For instance, in [65] the authors estimated the precision with which effects coming from massive gravity could be measured with LISA. They showed that by observing a spinning $M \sim 10^6\,M_\odot$ comparable mass MBH binary LISA would be able to constrain the Compton wavelength of the graviton down to $\sim 10^{-16}$ (in units M). In addition, the post merger GW signal (quasi normal mode ring-down) depends on the final MBH which is characterized by its mass and spin. If we manage to detect more than two harmonics of the ringdown waveform, then we will be able to test the no-hair theorem by making a consistency check [66] that the mass and spins measured from all the harmonics are consistent. This test will only be practical for comparable-mass systems, as the amplitude of ringdown radiation after an EMRI will be too small to allow detection.

VI. SUMMARY

In this review article we have shown the capability of LISA to serve as a lab for testing fundamental physics. LISA, operating in a low GW frequency band, is sensitive enough to detect or to set upper bounds on cosmic strings and stochastic gravitational wave backgrounds that are much better than those currently available. By analyzing the phasing of the GWs emitted during extreme-mass-ratio inspirals and the coalescence of MBHs we can potentially deduce information about the deviation of the space time from the pure (vaccum) Kerr solution. Such deviations could arise from the presence of a non-Kerr object in the spacetime or perturbing matter around the black hole or due to a deviation from GR.

We have also presented new results on constraining cosmological parameters with GW observations of MBH binaries to high ($z \leq 3$) redshift. In particular, if we assume that the only unknown parameter is the effective equation of state for the dark energy, w, then using a statistical method and observing about 30 MBH binaries we will be able to reduce the current uncertainty $|\delta w| < 0.3$ by a factor of 2 to 8.

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