Dark Matter as a Relativistic Inertial Effect in Einstein Canonical Gravity?

Luca Lusanna

Sezione INFN di Firenze
Polo Scientifico
Via Sansone 1
50019 Sesto Fiorentino (FI), Italy
Phone: 0039-055-4572334
FAX: 0039-055-4572364
E-mail: lusanna@fi.infn.it

Abstract

After the study of non-inertial frames in special relativity with emphasis on the problem of clock synchronization (i.e. of how to define 3-space), an overview is given of Einstein canonical gravity in the York canonical basis and of its Hamiltonian Post-Minkowskian (PM) linearization in 3-orthogonal gauges. It is shown that the York time (the trace of the extrinsic curvature of 3-spaces) is the inertial gauge variable describing the general relativistic remnant of the clock synchronization gauge freedom. The dark matter implied by the rotation curves of galaxies can be explained with a choice of the York time implying a PM extension of the Newtonian celestial frame ICRS.

Talk at "Group 28: The XXVIII International Colloquium on Group-Theoretical Methods in Physics", Northumbria University, Newcastle, 26-30 July 2010.

November 15, 2010

We can speak of predictability in physics when there is a well-posed Cauchy problem for the partial differential equations involved in the problem under investigation. A pre-requisite for the Cauchy problem is a sound definition of an instantaneous 3-space where the Cauchy data are given. While in the Galilei space-time of Newtonian physics this is not a problem due to the absolute nature of time and of Euclidean 3-space, no intrinsic notion of 3-space exists in special and general relativistic theories. In special relativity (SR), where there is the absolute Minkowski space-time, only the conformal structure (the light-cone) is intrinsically given. The standard way out from the problem of 3-space is to choose the Euclidean 3-space of an inertial frame centered on an inertial observer and then use the kinematical Poincare' group to connect different inertial frames. However, this is not possible in general relativity (GR), where there is no absolute notion since also space-time becomes dynamical (with the metric structure satisfying Einstein's equations). The equivalence principle implies the absence of global inertial frames: in the restricted class of globally hyperbolic, asymptotically Minkowskian at spatial infinity, space-times the best we can have are global non-inertial frames connected by 4-diffeomorphisms (the gauge group of GR).

As a consequence, also in SR we have to face the problem of reformulating physics in non-inertial frames centered on accelerated observers [1] as a first step before facing GR.

In Minkowski space-time the Euclidean 3-spaces of the inertial frames centered on an inertial observer A are identified by means of Einstein convention for the synchronization of clocks: the inertial observer A sends a ray of light at x^o_i towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at x^o_f; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x^o_P = x^o_i + \frac{1}{2}(x^o_f - x^o_i) = \frac{1}{2}(x^o_i + x^o_f)$. Therefore the description of non-inertial frames must replace Einstein convention with a more general one allowing non-Euclidean 3-spaces.

Since in GR the gauge freedom is the arbitrariness in the choice of the 4-coordinates, in the non-inertial frames of SR a similar arbitrariness is expected. However, the experimental description of matter in both theories is based on *metrological conventions*: a) an atomic clock as a standard of time; b) the velocity of light in place of a standard of length; c) a conventional reference frame centered on a given observer as a standard of space-time (think to GPS!). The description of satellites around the Earth is done by means of NASA coordinates [2] either in ITRS2003 [3] (frame fixed on the Earth surface) or in GCRS (the geocentric frame, centered on the Earth center, of IAU2000 [4]). The description of planets and spacecrafts uses the BCRS of IAU2000 [4] centered on the barycenter of the Solar System (a quasi-inertial frame in a nearly Minkowski space-time due to gravity) Both GCRS and BCRS are connected to Post-Newtonian solutions of Einstein equations in special harmonic gauges. Instead in astronomy the positions of stars and galaxies are determined from the data (luminosity, light spectrum, angles) on the sky as living in a 4-dimensional nearly-Galilei space-time with the celestial ICRS [5] frame considered as a "quasi-inertial frame" (all galactic dynamics is Newtonian gravity), in accord with the assumed validity of the cosmological and Copernican principles. Namely one assumes a homogeneous and isotropic cosmological Friedmann-Robertson - Walker solution of Einstein equations (the standard ΛCDM cosmological model [6]). In it the constant intrinsic 3-curvature of instantaneous 3-spaces is nearly zero as implied by the CMB data [6], so that Euclidean 3-spaces (and

Newtonian gravity) can be used. However, to reconcile all the data with this 4-dimensional reconstruction one must postulate the existence of dark matter and dark energy as the dominant components of the classical universe after the recombination 3-surface!

As a consequence, we need a metrology-oriented description of non-inertial frames already in SR [1, 7]. This can be done with the *3+1 point of view* and the use of observer-dependent Lorentz scalar radar 4-coordinates. Let us give the world-line $x^\mu(\tau)$ of an arbitrary time-like observer carrying a standard atomic clock: τ is an arbitrary monotonically increasing function of the proper time of this clock. Then we give an admissible 3+1 splitting of Minkowski space-time, namely a nice foliation with space-like instantaneous 3-spaces Σ_τ: it is the mathematical idealization of a protocol for clock synchronization (all the clocks in the points of Σ_τ sign the same time of the atomic clock of the observer). On each 3-space Σ_τ we choose curvilinear 3-coordinates σ^r having the observer as origin. These are the radar 4-coordinates $\sigma^A = (\tau; \sigma^r)$. If $x^\mu \mapsto \sigma^A(x)$ is the coordinate transformation from the Cartesian 4-coordinates x^μ of a reference inertial observer to radar coordinates, its inverse $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the *embedding* functions $z^\mu(\tau, \sigma^r)$ describing the 3-spaces Σ_τ as embedded 3-manifold into Minkowski space-time. The induced 4-metric on Σ_τ is the following functional of the embedding ${}^4g_{AB}(\tau, \sigma^r) = [z^\mu_A\, \eta_{\mu\nu}\, z^\nu_B](\tau, \sigma^r)$, where $z^\mu_A = \partial\, z^\mu / \partial\, \sigma^A$ and ${}^4\eta_{\mu\nu} = \epsilon\, (+ - --)$ is the flat metric ($\epsilon = \pm 1$ according to either the particle physics $\epsilon = 1$ or the general relativity $\epsilon = -1$ convention). While the 4-vectors $z^\mu_r(\tau, \sigma^u)$ are tangent to Σ_τ, so that the unit normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma}\, [z^\alpha_1\, z^\beta_2\, z^\gamma_3](\tau, \sigma^u)$, we have $z^\mu_\tau(\tau, \sigma^r) = [N\, l^\mu + N^r\, z^\mu_r](\tau, \sigma^r)$ ($N(\tau, \sigma^r) = \epsilon\, [z^\mu_\tau\, l_\mu](\tau, \sigma^r)$ and $N_r(\tau, \sigma^r) = -\epsilon\, g_{\tau r}(\tau, \sigma^r)$ are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions [1]:
1) $N(\tau, \sigma^r) > 0$ in every point of Σ_τ (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
2) $\epsilon\, {}^4g_{\tau\tau}(\tau, \sigma^r) > 0$, so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon\, {}^4g_{rs}(\tau, \sigma^u)$ having three positive eigenvalues (these are the Møller conditions [1, 8]);
3) all the 3-spaces Σ_τ must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

In the 3+1 point of view the 4-metric ${}^4g_{AB}(\tau, \vec{\sigma})$ on Σ_τ has the components $\epsilon\, {}^4g_{\tau\tau} = N^2 - N_r\, N^r$, $-\epsilon\, {}^4g_{\tau r} = N_r = {}^3g_{rs}\, N^s$, ${}^3g_{rs} = -\epsilon\, {}^4g_{rs} = \sum_{a=1}^3 {}^3e_{(a)r}\, {}^3e_{(a)s} = \tilde{\phi}^{2/3} \sum_{a=1}^3 e^{2\, \sum_{\bar{b}=1}^2 \gamma_{\bar{b}a}\, R_{\bar{b}}}\, V_{ra}(\theta^i)\, V_{sa}(\theta^i))$, where ${}^3e_{(a)r}(\tau, \sigma^u)$ are cotriads on Σ_τ, $\tilde{\phi}^2(\tau, \sigma^r) = det\, {}^3g_{rs}(\tau, \sigma^r)$ is the 3-volume element on Σ_τ, $\lambda_a(\tau, \sigma^r) = [\tilde{\phi}^{1/3}\, e^{\sum_{\bar{b}=1}^2 \gamma_{\bar{b}a}\, R_{\bar{b}}}](\tau, \sigma^r)$ are the positive eigenvalues of the 3-metric ($\gamma_{\bar{a}a}$ are suitable numerical constants) and $V(\theta^i(\tau, \sigma^r))$ are diagonalizing rotation matrices depending on three Euler angles. The components ${}^4g_{AB}$ or the quantities N, N_r, γ, $R_{\bar{a}}$, θ^i, play the role of the *inertial potentials* generating the relativistic apparent forces in the non-inertial frame. It can be shown [1, 8] that the Newtonian inertial potentials are hidden in the functions N, N_r and θ^i. The extrinsic curvature

[1] These conditions imply that global *rigid* rotations are forbidden in relativistic theories. In Ref.[1, 8] there is the expression of the admissible embedding corresponding to a 3+1 splitting of Minkowski space-time with parallel space-like hyper-planes (not equally spaced due to a linear acceleration) carrying differentially rotating 3-coordinates without the coordinate singularity of the rotating disk. It is the first consistent global non-inertial frame of this type.

${}^3K_{rs}(\tau,\sigma^u) = [\frac{1}{2N}(N_{r|s} + N_{s|r} - \partial_\tau\, {}^3g_{rs})](\tau,\sigma^u)$, describing the *shape* of the instantaneous 3-spaces of the non-inertial frame as embedded 3-manifolds of Minkowski space-time, is a secondary inertial potential functional of the independent inertial potentials ${}^4g_{AB}$.

Instead in GR the 4-metric is described by ten dynamical fields ${}^4g_{\mu\nu}(x)$: it is not only a (pre)potential for the gravitational field but also determines the chrono-geometrical structure of space-time through the line element $ds^2 = {}^4g_{\mu\nu}\, dx^\mu\, dx^\nu$ (it teaches relativistic causality to the other fields) [2]. To get its Hamiltonian description in the quoted restricted class of space-times the same 3+1 point of view and the radar 4-coordinates employed in SR can be used: this allows to separate the inertial (gauge) degrees of freedom of the gravitational field (playing the role of inertial potentials) from the dynamical tidal ones. But now the admissible embeddings $x^\mu = z^\mu(\tau,\sigma^r)$ are not dynamical variables: instead their gradients $z^\mu_A(\tau,\sigma^r)$ give the transition coefficient from radar to world 4-coordinates, ${}^4g_{AB}(\tau,\sigma^r) = [z^\mu_A\, z^\nu_B](\tau,\sigma^r)\, {}^4g_{\mu\nu}(z(\tau,\sigma^r))$. As shown in Ref.[10], the dynamical nature of space-time implies that each solution of Einstein's equations dynamically selects a preferred 3+1 splitting of the space-time, namely in GR the instantaneous 3-spaces (and therefore the associated clock synchronization convention) are dynamically determined. Now the extrinsic curvature of the 3-spaces will be a mixture of dynamical (tidal) pieces and inertial gauge variables playing the role of inertial potentials.

The description of isolated systems (particles, strings, fields, fluids) admitting a Lagrangian formulation in the non-inertial frames of SR is done by means of *parametrized Minkowski theories* [1, 7]. The matter variables are replaced with new ones knowing the 3-spaces Σ_τ. For instance a Klein-Gordon field $\tilde{\phi}(x)$ will be replaced with $\phi(\tau,\sigma^r) = \tilde{\phi}(z(\tau,\sigma^r))$; the same for every other field. Instead for a relativistic particle with world-line $x^\mu(\tau)$ we must make a choice of its energy sign: then it will be described by 3-coordinates $\eta^r(\tau)$ defined by the intersection of the world-line with Σ_τ: $x^\mu(\tau) = z^\mu(\tau,\eta^r(\tau))$. Differently from all the previous approaches to relativistic mechanics, the dynamical configuration variables are the 3-coordinates $\eta^r_i(\tau)$ and not the world-lines $x^\mu_i(\tau)$ (to rebuild them in an arbitrary frame we need the embedding defining that frame!). Then the matter Lagrangian is coupled to an external gravitational field and the external 4-metric is replaced with the 4-metric $g_{AB}(\tau,\sigma^r)$ of an admissible 3+1 splitting of Minkowski space-time. With this procedure we get a Lagrangian depending on the given matter and on the embedding $z^\mu(\tau,\sigma^r)$, which is invariant under *frame-preserving diffeomorphisms*. As a consequence, there are four first-class constraints (an analogue of the super-Hamiltonian and super-momentum constraints of canonical gravity) implying that the embeddings $z^\mu(\tau,\sigma^r)$ are *gauge variables*, so that all the admissible non-inertial or inertial frames are gauge equivalent, namely physics does *not* depend on the clock synchronization convention and on the choice of the 3-coordinates σ^r: only the appearances of phenomena change by changing the notion of instantaneous 3-space. Even if the gauge group is formed by the frame-preserving diffeomorphisms, the matter energy-momentum tensor allows the determination of the ten conserved Poincare' generators P^μ and $J^{\mu\nu}$ (assumed finite) of every configuration of the system.

[2] The ACES mission of ESA [9] will give the first precision measurement of the gravitational redshift of the geoid, namely of the $1/c^2$ deformation of Minkowski light-cone caused by the geo-potential. In every quantum field theory, where the definition of the Fock space requires the use of the fixed light-cone of the background, this is a non-perturbative effect requiring the resummation of the perturbative expansion.

If we restrict ourselves to inertial frames, we can define the *inertial rest-frame instant form of dynamics for isolated systems* by choosing the 3+1 splitting corresponding to the intrinsic inertial rest frame of the isolated system centered on an inertial observer: the instantaneous 3-spaces, named Wigner 3-space due to the fact that the 3-vectors inside them are Wigner spin-1 3-vectors [7], are orthogonal to the conserved 4-momentum P^μ of the configuration. In Ref.[1] there is the extension to admissible *non-inertial rest frames*, where P^μ is orthogonal to the asymptotic space-like hyper-planes to which the instantaneous 3-spaces tend at spatial infinity. This non-inertial family of 3+1 splittings is the only one admitted by the asymptotically Minkowskian space-times covered by canonical gravity formulation discussed below.

The framework of the inertial rest frame allowed the solution of the following old open problems:

A) The explicit form of the Lorentz boosts for some interacting systems [11, 12].

B) The classification of the relativistic collective variables, replacing the Newtonian center of mass, that can be built in terms of the Poincare' generators and their non measurability due to the non-local character of such generators (they know the whole 3-space Σ_τ) [1, 13].

C) The description of every isolated system as a decoupled (non-measurable) canonical non-covariant (Newton-Wigner) center of mass carrying a pole-dipole structure: the invariant mass and the rest spin of the system expressed in terms of Wigner-covariant relative variables inside the Wigner 3-spaces [1, 13–15].

D) The formulation of classical relativistic atomic physics [11, 14] (the electro-magnetic field in the radiation gauge plus charged scalar particles with Grassmann-valued electric charges to regularize the self-energies) and the identification of the Darwin potential at the classical level by evading Haag's theorem.

E) A new formulation of *relativistic quantum mechanics* [15] englobing all the known results about relativistic bound states and a first formulation of *relativistic entanglement* taking into account the *non-locality and spatial non-separability coming from the Poincare' group*.

Let us now consider Einstein's general relativity where space-time is dynamical. Since all the properties of the standard model of elementary particles are connected with properties of the representations of the Poincare' group in inertial frames of Minkowski space-time, we shall restrict ourselves to globally hyperbolic, asymptotically Minkowskian at spatial infinity, topologically trivial space-times, for which a well defined Hamiltonian formulation of gravity is possible if we replace the Hilbert action with the ADM one. The 4-metric tends in a suitable way to the flat Minkowski 4-metric ${}^4\eta_{\mu\nu}$ at spatial infinity: having an *asymptotic* Minkowskian background we can avoid to split the 4-metric in the bulk in a background plus perturbations in the weak field limit. In these space-times we can use admissible 3+1 splittings and observer-dependent radar 4-coordinates. Since tetrad gravity is more natural for the coupling of gravity to the fermions, the 4-metric is decomposed in terms of cotetrads, ${}^4g_{AB} = E^{(\alpha)}_A \, {}^4\eta_{(\alpha)(\beta)} \, E^{(\beta)}_B$ [3], and the ADM action, now a functional of

[3] (α) are flat indices; the cotetrads $E^{(\alpha)}_A$ are the inverse of the tetrads $E^A_{(\alpha)}$ connected to the world tetrads by $E^\mu_{(\alpha)}(x) = z^\mu_A(\tau, \sigma^r)\, E^A_{(\alpha)}(z(\tau, \sigma^r))$.

the 16 fields $E^{(\alpha)}_A(\tau,\sigma^r)$, is taken as the action for ADM tetrad gravity. In tetrad gravity the diffeonorphism group is enlarged with the O(3,1) gauge group of the Newman-Penrose approach (the extra gauge freedom acting on the tetrads in the tangent space of each point of space-time and reducing from 16 to 10 the number of independent fields like in metric gravity). This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer's gyroscopes. This framework was developed in the works in Refs.[16].

In these space-times we assume direction-independent boundary conditions on the 4-metric and its conjugate momenta able to kill *super-translations* [17], so that the SPI group of asymptotic symmetries [18] is reduced to the ADM Poincare' group with the generators P^A_{ADM}, J^{AB}_{ADM} given as boundary conditions. It turns out that the admissible 3+1 splittings are the *non-inertial rest frames* (with the 3-spaces asymptotically orthogonal to the ADM 4-momentum; $P^r_{ADM} \approx 0$ is the rest-frame condition) of the 3-universe with a mass and a rest spin fixed by the boundary conditions [4]. In absence of matter Christodoulou - Klainermann space-times [19] are compatible with this description. With this kind of formalism we can get a deparametrization of general relativity: if we switch off the Newton constant and we choose the flat Minkowski 4-metric in Cartesian coordinates as solution of Einstein's equations, we get the description of the matter present in the 3-universe in the non-inertial rest frames of Minkowski space-time with the weak ADM Poincare' group collapsing in the Poincare' group of particle physics. As shown in Refs.[16], with the previous boundary conditions the DeWitt surface term at spatial infinity in the Dirac Hamiltonian turns out to be the *strong* ADM energy (a flux through a 2-surface at spatial infinity), which is equal to the *weak* ADM energy (expressed as a volume integral over the 3-space) plus constraints. Therefore in this family of space-times there is *not a frozen picture*, like in the family of spatially compact without boundary space-times considered in loop quantum gravity, where the Dirac Hamiltonian is a combination of constraints.

In this framework the configuration variables are cotetrads, which are connected to cotetrads adapted to the 3+1 splitting of space-time (so that the adapted time-like tetrad is the unit normal to the 3-space Σ_τ) by standard Wigner boosts for time-like vectors of parameters $\varphi_{(a)}(\tau,\sigma^r)$, $a=1,2,3$: $E^{\alpha)}_A = L^{(\alpha)}{}_{(\beta)}(\varphi_{(a)})\, \overset{o}{E}{}^{(\beta)}_A$. The adapted cotetrads have the following expression in terms of cotriads ${}^3e_{(a)r}$ on Σ_τ and of the lapse $N = 1+n$ and shift $n_{(a)} = N^r\, {}^3e_{(a)r}$ functions: $\overset{o}{E}{}^{(o)}_\tau = 1+n$, $\overset{o}{E}{}^{(o)}_r = 0$, $\overset{o}{E}{}^{(a)}_\tau = n_{(a)}$, $\overset{o}{E}{}^{(a)}_r = {}^3e_{(a)r}$. The 4-metric becomes ${}^4g_{\tau\tau} = \epsilon\,[(1+n)^2 - \sum_a n^2_{(a)}]$, ${}^4g_{\tau r} = -\epsilon \sum_a n_{(a)}\, {}^3e_{(a)r}$, ${}^4g_{rs} = -\epsilon\, {}^3g_{rs} = -\epsilon \sum_a {}^3e_{(a)r}\, {}^3e_{(a)s}$. The 16 configurational variables in the ADM action are $\varphi_{(a)}$, $1+n$, $n_{(a)}$, ${}^3e_{(a)r}$. There are ten primary constraints (the vanishing of the 7 momenta of boosts, lapse and shift variables plus three constraints describing the rotation on the flat indices (a) of the cotriads) and four secondary ones (the super-Hamiltonian and super-momentum constraints): all of them are first class in the phase space spanned by 16+16 fields. This implies that there are 14 gauge variables describing *inertial effects* and 2 canonical pairs of physical degrees of freedom describing the *tidal effects* of the gravitational field (namely gravitational waves in the weak

[4] Therefore there are asymptotic inertial observers to be identified with the fixed stars of star catalogues. If ϵ^μ_A are a set of asymptotic flat tetrads, the simplest embedding adapted to the 3+1 splitting of space-time is $x^\mu = z^\mu(\tau,\sigma^r) = x^\mu(\tau) + \epsilon^\mu_r\,\sigma^r = x^\mu_o + \epsilon^\mu_A\,\sigma^A$ and we have ${}^4g_{AB}(\tau,\sigma^r) = \epsilon^\mu_A\,\epsilon^\nu_B\, {}^4g_{\mu\nu}(x)$.

field limit). In this canonical basis only the momenta ${}^3\pi^r_{(a)}$ conjugated to the cotriads are not vanishing.

Then in Ref.[18] we have found a canonical transformation to a canonical basis adapted to ten of the first class constraints. It implements the York map of Ref.[20] and diagonalizes the York-Lichnerowicz approach [21]. Its final form is ($\alpha_{(a)}(\tau,\sigma^r)$ are angles)

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$	θ^r	$\tilde{\phi}$	$R_{\bar{a}}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi^{(\alpha)}_{(a)} \approx 0$	$\pi_n \approx 0$	$\pi_{\bar{n}_{(a)}} \approx 0$	$\pi^{(\theta)}_r$	$\pi_{\tilde{\phi}} = \frac{c^3}{12\pi G}\,{}^3K$	$\Pi_{\bar{a}}$

$$
\begin{aligned}
&{}^3e_{(a)r} = R_{(a)(b)}(\alpha_{(c)})\,{}^3\bar{e}_{(b)r} = R_{(a)(b)}(\alpha_{(c)})\,V_{rb}(\theta^i)\,\tilde{\phi}^{1/3}\,e^{\sum_{\bar{a}}^{1,2}\gamma_{\bar{a}a}\,R_{\bar{a}}},\\
&{}^4g_{\tau\tau} = \epsilon\,[(1+n)^2 - \sum_a \bar{n}^2_{(a)}], \qquad {}^4g_{\tau r} = -\epsilon\,\bar{n}_{(a)}\,{}^3\bar{e}_{(a)r},\\
&{}^4g_{rs} = -\epsilon\,{}^3g_{rs} = -\epsilon\,\tilde{\phi}^{2/3}\sum_a V_{ra}(\theta^i)\,V_{sa}(\theta^i)\,e^{2\sum_{\bar{a}}^{1,2}\gamma_{\bar{a}a}\,R_{\bar{a}}},
\end{aligned}
\tag{0.1}
$$

In this York canonical basis the *inertial effects* are described by the arbitrary gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$, $1+n$, $\bar{n}_{(a)}$, θ^i, ${}^3K$, while the *tidal effects*, i.e. the physical degrees of freedom of the gravitational field, by the two canonical pairs $R_{\bar{a}}$, $\Pi_{\bar{a}}$, $\bar{a}=1,2$. The momenta $\pi^{(\theta)}_r$ and the 3-volume element $\tilde{\phi} = \sqrt{det\,{}^3g_{rs}}$ have to be found as solutions of the super-momentum and super-hamiltonian (i.e. the Lichmerowicz equation) constraints, respectively. The gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$ parametrize the extra O(3,1) gauge freedom of the tetrads (the gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the world-line). The gauge angles θ^i (i.e. the director cosines of the tangents to the three coordinate lines in each point of Σ_τ) describe the freedom in the choice of the 3-coordinates σ^r on each 3-space: their fixation implies the determination of the shift gauge variables $\bar{n}_{(a)}$, namely the appearances of gravito-magnetism in the chosen 3-coordinate system.

The final basic gauge variable is a momentum, namely the trace ${}^3K(\tau,\sigma^r)$ of the extrinsic curvature (also named the *York time*) of the non-Euclidean 3-space Σ_τ. The Lorentz signature of space-time implies that ${}^3K$ is a momentum variable: it is a time coordinate, while θ^i are spatial coordinates. Differently from SR ${}^3K$ is *an independent inertial gauge variable describing the remnant in GR of the freedom in clock synchronization*! The other components of the extrinsic curvature are dynamically determined. This gauge variable has no Newtonian counterpart (the Euclidean 3-space is absolute), because its fixation determines the final shape of the non-Euclidean 3-space. Moreover this gauge variable gives rise to a negative kinetic term in the weak ADM energy $\hat{E}_{ADM}$, vanishing only in the gauges ${}^3K(\tau,\vec{\sigma}) = 0$ [18].

In the York canonical basis the Hamilton equations generated by the Dirac Hamiltonian $H_D = \hat{E}_{ADM} + (constraints)$ are divided in four groups: A) the contracted Bianchi identities, namely the evolution equations for $\tilde{\phi}$ and $\pi^{(\theta)}_i$ (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times); B) the evolution equation for the four basic gauge variables θ^i and ${}^3K$: these equations determine the lapse and the shift functions once the basic gauge variables are fixed; C) the evolution equations for the tidal variables $R_{\bar{a}}$, $\Pi_{\bar{a}}$; D) the Hamilton equations for matter, when present. Once a gauge is completely fixed, the Hamilton equations become deterministic. Given a solution of the super-momentum and super-Hamiltonian constraints and the Cauchy data for the tidal variables on an initial 3-space, we can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer.

In the first paper of Ref.[22], we studied the coupling of N charged scalar particles plus the electro-magnetic field to ADM tetrad gravity in this class of asymptotically Minkowskian space-times without super-translations. To regularize the self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued. The introduction of the non-covariant radiation gauge allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the action-at-a- distance Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames. After the reformulation of the whole system in the York canonical basis, we give the restriction of the Hamilton equations and of the constraints to the family of *non-harmonic 3-orthogonal* Schwinger time gauges, in which the instantaneous Riemannian 3-spaces have a non-fixed trace ${}^3K$ of the extrinsic curvature but a diagonal 3-metric. This family of gauges is determined by the gauge fixings $\theta^i(\tau, \sigma^r) \approx 0$ and ${}^3K(\tau, \sigma^r) \approx (arbitrary\, numerical\, function)$.

In the second paper of Ref.[22] it was shown that in this family of non-harmonic 3-orthogonal Schwinger gauges it is possible to define a consistent *linearization* of ADM canonical tetrad gravity plus matter in the weak field approximation, to obtain a formulation of Hamiltonian Post-Minkowskian gravity with non-flat Riemannian 3-spaces and asymptotic Minkowski background. This means that the 4-metric tends to the asymptotic Minkowski metric at spatial infinity, ${}^4g_{AB} \rightarrow {}^4\eta_{AB}$. The decomposition ${}^4g_{AB} = {}^4\eta_{AB} + {}^4h_{(1)AB}$, with a first order perturbation ${}^4h_{(1)AB}$ vanishing at spatial infinity, is only used for calculations, but has no intrinsic meaning. Moreover, due to the presence of a ultra-violet cutoff for matter, we can avoid to make Post-Newtonian expansions, namely we get fully relativistic expressions. We have found solutions for the first order quantities $\pi^{(\theta)}_{(1)r}$, $\tilde{\phi} = 1 + 6\,\phi_{(1)}$, $1 + n_{(1)}$, $\bar{n}_{(1)(a)}$ (the action-at-a-distance part of the gravitational interaction). Then we can show that the tidal variables $R_{\bar{a}}$ satisfy a wave equation $\Box\, R_{\bar{a}} = (known\, functional\, of\, matter)$ with the D'Alambertian associated to the asymptotic Minkowski 4-metric. Therefore, by using a no-incoming radiation condition based on the asymptotic Minkowski light-cone, we get a description of gravitational waves in these non-harmonic gauges, which can be connected to generalized TT(transverse traceless) gauges, as retarded functions of the matter. These gravitational waves do not propagate in inertial frames of the background (like it happens in the standard harmonic gauge description), but in non-Euclidean instantaneous 3-spaces differing from Euclidean 3-spaces at the first order (their intrinsic 3-curvature is determined by the gravitational waves) and dynamically determined by the linearized solution of Einstein equations. These 3-spaces have a first order extrinsic curvature (with ${}^3K_{(1)}(\tau, \sigma^r)$ describing the clock synchronization convention) and a first order modification of Minkowski light-cone.

We can write explicitly the linearized Hamilton equations for the particles and for the electro-magnetic field: among the forces there are both the inertial potentials and the gravitational waves. In the third paper of Ref.[22] we disregarded electro-magnetism and we studied the non-relativistic limit of the particle equations. We found that the particle 3-

coordinates $\eta^r_i(\tau = ct) = \tilde{\eta}^r_i(t)$ satisfy the equation $m\,\frac{d^2\tilde{\eta}^r_i(t)}{dt^2} = \sum_{j\neq i} F^r_{Newton}(\vec{\tilde{\eta}}_i(t) - \vec{\tilde{\eta}}_j(t)) + \frac{1}{c}\,\frac{d\tilde{\eta}^r_i(t)}{dt}\,\Big(\frac{1}{\triangle}\,c^2\,\partial^2_\tau\,{}^3K_{(1)}(\tau = ct, \vec{\sigma})\Big)|_{\vec{\sigma}=\vec{\tilde{\eta}}_i(t)}$, where $\vec{F}_{Newton}$ is the Newton gravitational force. Therefore the (arbitrary in these gauges) double rate of change in time of the trace of the extrinsic curvature creates a post-Newtonian damping (or anti-damping since the sign of ${}^3K_{(1)}$ is not fixed) effect on the motion of particles. This is a inertial effect (hidden in the lapse function) not existing in Newton theory where the Euclidean 3-space is absolute. In the 2-body case we get that for Keplerian circular orbits of radius r the modulus of the relative 3-velocity can be written in the form $\sqrt{\frac{G\,(m+\triangle\,m(r))}{r}}$ with $\triangle\,m(r)$ function only of ${}^3K_{(1)}$. Now the rotation curves of galaxies (see Ref.[23] for a review) imply that this quantity goes to constant for large r (instead of vanishing): as a consequence $\triangle\,m(r)$ is interpreted as a *dark matter halo* around the galaxy. With our approach this dark matter would be a *relativistic inertial effect* consequence of the non-Euclidean nature of 3-space. This option would differ: 1) from the non-relativistic MOND approach [24] (where one modifies Newton equations); 2) from modified gravity theories like the $f(R)$ ones (see for instance Refs.[25]; here one gets a modification of the Newton potential); 3) from postulating the existence of WIMP particles [26].

Since, as already said, at the experimental level *the description of baryon matter is intrinsically coordinate-dependent*, namely is connected with the conventions used by physicists, engineers and astronomers for the modeling of space-time. As a consequence of the dependence on coordinates of the description of matter, our proposal for solving the gauge problem in our Hamiltonian framework with non-Euclidean 3-spaces is to choose a gauge (i.e. a 4-coordinate system) in non-modified Einstein gravity which is in agreement with the observational conventions in astronomy. Since ICRS [5] has diagonal 3-metric, our 3-orthogonal gauges are a good choice. We are left with the inertial gauge variable ${}^3\mathcal{K}_{(1)} = \frac{1}{\triangle}\,{}^3K_{(1)}$ not existing in Newtonian gravity. As already said the suggestion is to try to fix ${}^3\mathcal{K}_{(1)}$ in such a way to eliminate dark matter as much as possible, by reinterpreting it as a relativistic inertial effect induced by the shift from Euclidean 3-spaces to non-Euclidean ones (independently from cosmological assumptions). As a consequence, ICRS should be reformulated not as a *quasi-inertial* reference frame in Galilei space-time, but as a reference frame in a PM space-time with ${}^3K$ (i.e. the clock synchronization convention) deduced from the data connected to dark matter. Then automatically BCRS would be its quasi-Minkowskian approximation for the Solar System. This point of view could also be useful for the ESA GAIA mission (cartography of the Milky Way) [27].

In conclusion the Hamiltonian formulation of Einstein theory done in a form which takes into account the problem of 3-space (i.e. of clock synchronization) and the coordinate-dependent description of matter (i.e. metrology) opens a new scenario for dark matter. Besides looking for other experimental signatures of the York time, we also have the possibility to explore its role in the back-reaction approach [28] to dark energy, according to which dark energy is a byproduct of the non-linearities of general relativity when one considers spatial mean values on large scales to get a cosmological description of the universe taking into account the inhomogeneity of the observed universe. In the York canonical basis all the relevant quantities are 3-scalars and it is possible to study the mean value of nearly all the Hamilton equations.

---

[1] D.Alba and L.Lusanna, *Charged Particles and the Electro-Magnetic Field in Non-Inertial Frames: I. Admissible 3+1 Splittings of Minkowski Spacetime and the Non-Inertial Rest Frames*, Int.J.Geom.Methods in Physics **7**, 33 (2010) (0908.0213) and *II. Applications: Rotating Frames, Sagnac Effect, Faraday Rotation, Wrap-up Effect (0908.0215)*, Int.J.Geom.Methods in Physics, **7**, 185 (2010).

[2] T.D.Moyer, *Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation* (John Wiley, New York, 2003).

[3] *IERS Conventions (2003)*, eds. D.D.McCarthy and G.Petit, IERS TN 32 (2004), Verlag des BKG. G.H.Kaplan, *The IAU Resolutions on Astronomical Reference Systems, Time Scales and Earth Rotation Models*, U.S.Naval Observatory circular No. 179 (2005) (astro-ph/0602086).

[4] M.Soffel, S.A.Klioner, G.Petit, P.Wolf, S.M.Kopeikin, P.Bretagnon, V.A.Brumberg, N.Capitaine, T.Damour, T.Fukushima, B.Guinot, T.Huang, L.Lindegren, C.Ma, K.Nordtvedt, J.Ries, P.K.Seidelmann, D.Vokroulicky', C.Will and Ch.Xu, *The IAU 2000 Resolutions for Astrometry, Celestial Mechanics and Metrology in the Relativistic Framework: Explanatory Supplement* Astron.J., **126**, pp.2687-2706, (2003) (astro-ph/0303376).

[5] J.Kovalevski, I.I.Mueller and B.Kolaczek, *Reference Frames in Astronomy and Geophysics* (Kluwer, Dordrecht, 1989). O.J.Sovers and J.L.Fanselow, *Astrometry and Geodesy with Radio Interferometry: Experiments, Models, Results*, Rev.Mod.Phs. **70**, 1393 (1998).

[6] M.Bartelmann, *The Dark Universe*, (0906.5036).

[7] L. Lusanna, *The N- and 1-Time Classical Descriptions of N-Body Relativistic Kinematics and the Electromagnetic Interaction*, Int. J. Mod. Phys. **A12**, 645 (1997); *The Chrono-Geometrical Structure of Special and General Relativity: A Re-Visitation of Canonical Geometrodynamics*, lectures at 42nd Karpacz Winter School of Theoretical Physics: Current Mathematical Topics in Gravitation and Cosmology, Ladek, Poland, 6-11 Feb 2006, Int.J.Geom.Methods in Mod.Phys. **4**, 79 (2007) (gr-qc/0604120).

[8] D.Alba and L.Lusanna, *Generalized Radar 4-Coordinates and Equal-Time Cauchy Surfaces for Arbitrary Accelerated Observers*, Int.J.Mod.Phys. **D16**, 1149 (2007) (gr-qc/0501090).

[9] L.Cacciapuoti and C.Salomon, *ACES: Mission Concept and Scientific Objective*, 28/03/2007, ESA document, Estec (ACES_Science_v1_printout.doc). L.Blanchet, C.Salomon, P.Teyssandier and P.Wolf, *Relativistic Theory for Time and Frequency Transfer to Order* $1/c^3$, Astron.Astrophys. **370**, 320 (2000).

[10] L.Lusanna and M.Pauri, *The Physical Role of Gravitational and Gauge Degrees of Freedom in General Relativity. I: Dynamical Synchronization and Generalized Inertial Effects; II: Dirac versus Bergmann Observables and the Objectivity of Space-Time*, Gen.Rel.Grav. **38**, 187 and 229 (2006) (gr-qc/0403081 and 0407007); *Dynamical Emergence of Instantaneous 3-Spaces in a Class of Models of General Relativity*, in *Relativity and the Dimensionality of the World*, ed. V.Petkov (Springer Series Fundamental Theories of Physics, Berlin, 2007) (gr-qc/0611045).

[11] H.Crater and L.Lusanna, *The Rest-Frame Darwin Potential from the Lienard-Wiechert Solution in the Radiation Gauge*, Ann.Phys.(N.Y.) **289**, 87 (2001)(hep-th/0001046). D.Alba, H.Crater and L.Lusanna, *The Semiclassical Relativistic Darwin Potential for Spinning Particles in the Rest-Frame Instant Form: Two-Body Bound States with Spin 1/2 Constituents*, Int.J.Mod.Phys. **A16**, 3365 (2001) (hep-th/0103109); *Hamiltonian Relativistic Two-*

Body Problem: Center of Mass and Orbit Reconstruction, J.Phys. **A40**, 9585 (2007) (gr-qc/0610200).

[12] D.Alba, H.W.Crater and L.Lusanna, *The Rest-Frame Instant Form and Dirac Observables for the Open Nambu String*, arXiv 1005.3653; *Massless Particles plus Matter in the Rest-Frame Instant Form of Dynamics*, J.Phys. **A43**, 405203 (2010) (arXiv 1005.5521).

[13] D.Alba, L.Lusanna and M.Pauri, *New Directions in Non-Relativistic and Relativistic Rotational and Multipole Kinematics for N-Body and Continuous Systems* (2005), in *Atomic and Molecular Clusters: New Research*, ed.Y.L.Ping (Nova Science, New York, 2006) (hep-th/0505005); *Centers of Mass and Rotational Kinematics for the Relativistic N-Body Problem in the Rest-Frame Instant Form*, J.Math.Phys. **43**, 1677-1727 (2002) (hep-th/0102087); *Multipolar Expansions for Closed and Open Systems of Relativistic Particles* , J. Math.Phys. **46**, 062505, 1-36 (2004) (hep-th/0402181).

[14] D.Alba, H.W.Crater and L.Lusanna, *Towards Relativistic Atom Physics. I. The Rest-Frame Instant Form of Dynamics and a Canonical Transformation for a system of Charged Particles plus the Electro-Magnetic Field*, Canad.J.Phys. **88**, 379 (2010) (arXiv: 0806.2383); *Towards Relativistic Atom Physics. II. Collective and Relative Relativistic Variables for a System of Charged Particles plus the Electro-Magnetic Field*, Canad.J.Phys. **88**, 425 (2010) (arXiv: 0811.0715).

[15] D.Alba, H.W.Crater and L.Lusanna, *Relativistic Quantum Mechanics and Relativistic Entanglement in the Rest-Frame Instant Form of Dynamics*, (arXiv 0907.1816).

[16] L.Lusanna, *The Rest-Frame Instant Form of Metric Gravity*, Gen.Rel.Grav. **33**, 1579 (2001)(gr-qc/0101048). L.Lusanna and S.Russo, *A New Parametrization for Tetrad Gravity*, Gen.Rel.Grav. **34**, 189 (2002)(gr-qc/0102074). R.DePietri, L.Lusanna, L.Martucci and S.Russo, *Dirac's Observables for the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge*, Gen.Rel.Grav. **34**, 877 (2002) (gr-qc/0105084).

[17] R.Beig and Ó Murchadha, *The Poincar Group as the Symmetry Group of Canonical General Relativity*, Ann.Phys.(N.Y.) **174**, 463 (1987).

[18] D.Alba and L.Lusanna, *The York Map as a Shanmugadhasan Canonical Transformationn in Tetrad Gravity and the Role of Non-Inertial Frames in the Geometrical View of the Gravitational Field*, Gen.Rel.Grav. **39**, 2149 (2007) (gr-qc/0604086, v2).

[19] D.Christodoulou and S.Klainerman, *The Global Nonlinear Stability of the Minkowski Space* (Princeton, Princeton, 1993).

[20] J.Isenberg and J.E.Marsden, *The York Map is a Canonical Transformation*, J.Geom.Phys. **1**, 85 (1984).

[21] I.Ciufolini and J.A.Wheeler, *Gravitation and Inertia* (Princeton Univ.Press, Princeton, 1995).

[22] D.Alba and L.Lusanna, *The Einstein-Maxwell-Particle System in the York Canonical Basis of ADM Tetrad Gravity: I) The Equations of Motion in Arbitrary Schwinger Time Gauges.*, 2009 (arXiv 0907.4087); *II) The Weak Field Approximation in the 3-Orthogonal Gauges and Hamiltonian Post-Minkowskian Gravity: the N-Body Problem and Gravitational Waves with Asymptotic Background.*, (1003.5143); *III) The Post-Minkowskian N-Body Problem, its Post-Newtonian Limit in Non-Harmonic 3-Orthogonal Gauges and Dark Matter as an Inertial Effect.* (1009.1794).

[23] E.Battaner and E.Florido, *The Rotation Curve of Spiral Galaxies and its Cosmological Implications*, Fund.Cosmic Phys. **21**, 1 (2000). D.G.Banhatti, *Disk Galaxy Rotation Curves and Dark Matter Distribution*, Current Science **94**, 986 (2008). W.J.G. de Blok and A.Bosma, *High Resolution Rotation Curves of Low Surface Brightness Galaxies*, Astron.Astrophys. **385**, 816

(2002) (astro-ph/0201276).

[24] M.Milgrom, *New Physics at Low Accelerations (MOND): An Alternative to Dark Matter*, (0912.2678).

[25] S.Capozziello, V.F.Cardone and A.Troisi, *Low Surface Brightness Galaxy Rotation Curves in the Low Energy Limit of R^n Gravity: No Need for Dark Matter?*, Mon.Not.R.Astron.Soc. **375**, 1423 (2007) (astro-ph/0603522).

[26] J.L.Feng, *Dark Matter Candidates from Particle Physics and Methods of Detection* , (arXiv 1003.0904).

[27] S.Jordan, *The GAIA Project: Technique, Performance and Status*, Astron.Nachr. **329**, 875 (2008) (DOI 10.1002/asna.200811065).

[28] R.J.van den Hoogen, *Averaging Spacetime: Where do we go from here ?*, (1003.4020). T.Buchert, *Dark Energy from Structure: a Status Report*, Gen.Rel.Grav. **40**, 467 (2008) (0707.2153).