Phase transition to the state with nonzero average helicity in dense neutron matter

A. A. Isayev*

Kharkov Institute of Physics and Technology, Academicheskaya Street 1, Kharkov, 61108, Ukraine

J. Yang†

Department of Physics and the Institute for the Early Universe,
Ewha Womans University, Seoul 120-750, Korea

The possibility of the appearance of the states with a nonzero average helicity in neutron matter is studied in the model with the Skyrme effective interaction. By providing the analysis of the self-consistent equations at zero temperature, it is shown that neutron matter with the Skyrme BSk18 effective force undergoes at high densities a phase transition to the state in which the degeneracy with respect to helicity of neutrons is spontaneously removed.

PACS numbers: 21.65.Cd, 21.10.Hw, 26.60.-c, 21.30.Fe

The issue of spontaneous appearance of spin polarized states in nuclear matter is a topic of a great current interest due to relevance in astrophysics. In particular, the scenarios of supernova explosion and cooling of neutron stars are essentially different, depending on whether nuclear matter is spin polarized or not. On the one hand, the models with the Skyrme effective nucleon-nucleon (NN) interaction predict the occurrence of spontaneous spin instability in nuclear matter at densities in the range from ϱ_0 to $4\varrho_0$ for different parametrizations of the NN potential [1]-[9] ($\varrho_0 \simeq 0.16\,\mathrm{fm}^{-3}$ is the nuclear saturation density). On the other hand, for the models with the realistic NN interaction, no sign of spontaneous spin instability has been found so far at any isospin asymmetry up to densities well above ϱ_0 [10]-[16]. In order to reconcile two different approaches, based on the effective and realistic NN interactions, recently a new parametrization of the Skyrme interaction, BSk18, has been proposed [17], aimed to avoid the spin instability of nuclear matter at densities beyond the nuclear saturation density. This is achieved by adding new density-dependent terms to the standard Skyrme force. The advantage of the BSk18 parametrization is that it also preserves the high-quality fits to the mass data obtained with the conventional Skyrme force as well as it satisfactorily reproduces the results of microscopic neutron matter calculations (equation of state [18], $^1S_0$ pairing gap [19]). Hence, this Skyrme parametrization has a good potentiality in the studies of various neutron star properties [20].

In terms of Landau Fermi liquid parameters, the ferromagnetic instability of neutron matter is prevented by the requirement $G_0 > -1$, where G_0 is the zeroth coefficient in the expansion of the dimensionless spin-spin interaction amplitude on the Legendre polynomials, $G = \sum_l G_l P_l(\cos\theta)$ [21]. Although this condition can hold true for all relevant densities, nevertheless, the first coefficient G_1, with increasing density, can become large and negative, so that the condition $G_1 < -3$ could be reached in the high-density region of neutron matter. These conditions, considered together, mean quite strong attractive interaction between neutron spins in the triplet spin state and repulsive, or weakly attractive interaction in the singlet spin state. As a result, neutron matter becomes unstable at high densities, and a new state characterized by a nonzero average helicity of neutrons, $\langle \boldsymbol{\sigma}\boldsymbol{p}^0\rangle \neq 0$, is formed. Such a possibility was first studied with respect to an electron liquid in metals in Ref. [22] and, later, in the framework of a microscopic model, in Ref. [23]. Our primary goal here is to develop the proper formalism for the description of the states with a nonzero average helicity in neutron matter and to provide the corresponding analysis of the self-consistent equations for the BSk18 Skyrme force.

The nonsuperfluid states of neutron matter are described by the normal distribution function of neutrons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\,\varrho a^+_{\kappa_2} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma)$, $\mathbf{p}$ is momentum, σ is the projection of spin on the third axis, and ϱ is the density matrix of the system [8, 9]. The self-consistent matrix equation for determining the distribution function f follows from the minimum condition of the thermodynamic potential [24, 25] and is

$$f = \left\{\exp(Y_0\varepsilon + Y_4) + 1\right\}^{-1} \equiv \left\{\exp(Y_0\xi) + 1\right\}^{-1}. \quad (1)$$

Here the single particle energy ε and quantity Y_4 are the matrices in the space of κ variables, with $Y_{4\kappa_1\kappa_2} = Y_4\delta_{\kappa_1\kappa_2}$, $Y_0 = 1/T$, and $Y_4 = -\mu_0/T$ being the Lagrange multipliers, μ_0 being the chemical potential of neutrons, and T being the temperature.

For the BSk18 interaction, the ferromagnetic instability is avoided by adding to the standard Skyrme force the additional density-dependent terms. Nevertheless, although spontaneous spin polarization is excluded at all densities relevant for neutron stars, there is still the possibility, related to the spontaneous appearance of the state with a nonzero average helicity

$$\lambda \equiv \langle \boldsymbol{\sigma}\mathbf{p}^0\rangle,\ \mathbf{p}^0 = \mathbf{p}/p, \quad (2)$$

where $\langle\ldots\rangle \equiv \mathrm{tr}\, f\ldots/\mathrm{tr}\, f$, tr ... being the trace in the space of κ variables. For this state, the single particle energy

* isayev@kipt.kharkov.ua
† jyang@ewha.ac.kr

of neutrons reads

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p})\sigma_0 - \Delta(\boldsymbol{p})\boldsymbol{\sigma}\boldsymbol{p}^0, \tag{3}$$

where σ_i are the Pauli matrices in the spin space and $2\Delta(\boldsymbol{p})$ is the energy splitting between the neutron spectra with different helicities (positive and negative). It follows from Eqs. (1) and (3), that the distribution function of neutrons has the structure

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_{||}(\mathbf{p})\boldsymbol{\sigma}\boldsymbol{p}^0, \tag{4}$$

where

$$f_0 = \frac{1}{2}\{n(\omega_+) + n(\omega_-)\},\ f_{||} = \frac{1}{2}\{n(\omega_+) - n(\omega_-)\}. \tag{5}$$

Here $n(\omega) = \{\exp(Y_0\omega) + 1\}^{-1}$ and

$$\omega_\pm = \varepsilon_0 - \mu_0 \mp \Delta. \tag{6}$$

The quantity $\omega_\pm$, being the exponent in the Fermi distribution function n, entering Eqs. (5), plays the role of the quasiparticle spectrum. There are two branches of the quasiparticle spectrum, corresponding to neutrons with definite helicity, $\boldsymbol{\sigma}\boldsymbol{p}^0 = \pm 1$.

Note that the distribution function f_0 should satisfy the normalization condition

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \tag{7}$$

where ϱ is the total density of neutron matter. The average helicity λ plays the role of the order parameter of a phase transition to the state, in which the majority of neutron spins are oriented along, or opposite to their momenta. By calculating the traces in Eq. (2), it is easy to find that

$$\lambda = \frac{\sum_{\mathbf{p}} f_{||}(\mathbf{p})}{\sum_{\mathbf{p}} f_0(\mathbf{p})}. \tag{8}$$

In order to get the self–consistent equations for the components of the single particle energy, one has to set the energy functional of the system, which reads [9, 24]

$$E(f) = E_0(f) + E_{int}(f), \tag{9}$$

$$E_0(f) = 2\sum_{\mathbf{p}} \underline{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}),\ \underline{\varepsilon}_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0},$$

$$E_{int}(f) = \sum_{\mathbf{p}}\{\tilde\varepsilon_0(\mathbf{p}) f_0(\mathbf{p}) + \tilde\varepsilon_i(\mathbf{p}) f_i(\mathbf{p})\},$$

where

$$\tilde\varepsilon_0(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}),\ \mathbf{k} = \frac{\mathbf{p}-\mathbf{q}}{2}, \tag{10}$$

$$\tilde\varepsilon_i(\mathbf{p}) = \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_i(\mathbf{q}),\ f_i(\mathbf{q}) = f_{||}(\mathbf{q}) q_i^0. \tag{11}$$

Here $\underline{\varepsilon}_0(\mathbf{p})$ is the free single particle spectrum, m_0 is the bare mass of a neutron, $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$ are the normal Fermi liquid (FL) amplitudes, and $\tilde\varepsilon_0, \tilde\varepsilon_i$ are the FL corrections to the free single particle spectrum. Using equation $\delta E = \mathrm{tr}\,\varepsilon(f)\delta f$ [21], we get the self-consistent equations in the form

$$\xi_0(\mathbf{p}) = \underline{\varepsilon}_0(\mathbf{p}) + \tilde\varepsilon_0(\mathbf{p}) - \mu_0, \tag{12}$$

$$\xi_i(\mathbf{p}) \equiv -\Delta(\mathbf{p}) p_i{}^0 = \tilde\varepsilon_i(\mathbf{p}). \tag{13}$$

To obtain numerical results, we utilize the BSk18 parametrization of the Skyrme interaction, developed in Ref. [17] and generalizing the conventional Skyrme parametrizations. In the conventional case, the amplitude of NN interaction reads [26]

$$\hat v(\mathbf{p},\mathbf{q}) = t_0(1 + x_0 P_\sigma) + \frac{1}{6} t_3 (1 + x_3 P_\sigma)\varrho^\alpha + \frac{1}{2\hbar^2} t_1 (1 + x_1 P_\sigma)(\mathbf{p}^2 + \mathbf{q}^2) + \frac{t_2}{\hbar^2}(1 + x_2 P_\sigma)\mathbf{p}\mathbf{q}, \tag{14}$$

where $P_\sigma = (1 + \boldsymbol{\sigma_1}\boldsymbol{\sigma_2})/2$ is the spin exchange operator, t_i, x_i and α are some phenomenological parameters specifying a given parametrization of the Skyrme interaction. The Skyrme interaction suggested in Ref. [17] has the form

$$\hat v'(\mathbf{p},\mathbf{q}) = \hat v(\mathbf{p},\mathbf{q}) + \frac{\varrho^\beta}{2\hbar^2} t_4 (1 + x_4 P_\sigma)(\mathbf{p}^2 + \mathbf{q}^2) + \frac{\varrho^\gamma}{\hbar^2} t_5 (1 + x_5 P_\sigma)\mathbf{p}\mathbf{q}. \tag{15}$$

In Eq. (15), two additional terms are the density-dependent generalizations of the t_1 and t_2 terms of the usual form.

The normal FL amplitudes U_0, U_1 can be expressed in terms of the Skyrme force parameters. For conventional Skyrme force parametrizations, their explicit expressions are given in Refs. [24, 25]. As follows from Eqs. (14) and (15), in order to obtain the corresponding expressions for the generalized Skyrme interaction (15), one should use the substitutions

$$t_1 \to t_1 + t_4\varrho^\beta,\ t_1x_1 \to t_1x_1 + t_4x_4\varrho^\beta, \tag{16}$$

$$t_2 \to t_2 + t_5\varrho^\gamma,\ t_2x_2 \to t_2x_2 + t_5x_5\varrho^\gamma. \tag{17}$$

Therefore, the FL amplitudes are related to the parameters of the Skyrme interaction (15) by formulas

$$U_0^n(\mathbf{k}) = 2t_0(1 - x_0) + \frac{t_3}{3}\varrho^\alpha(1 - x_3) + \frac{2}{\hbar^2}[t_1(1 - x_1) + t_4(1 - x_4)\varrho^\beta + 3t_2(1 + x_2) + 3t_5(1 + x_5)\varrho^\gamma]\mathbf{k}^2, \tag{18}$$

$$U_1^n(\mathbf{k}) = -2t_0(1 - x_0) - \frac{t_3}{3}\varrho^\alpha(1 - x_3) + \frac{2}{\hbar^2}[t_2(1 + x_2) + t_5(1 + x_5)\varrho^\gamma - t_1(1 - x_1) - t_4(1 - x_4)\varrho^\beta]\mathbf{k}^2 \equiv a_n + b_n\mathbf{k}^2. \tag{19}$$

It follows from Eqs. (12) and (18) that

$$\xi_0 = \frac{p^2}{2m_n} - \mu, \tag{20}$$

where the effective neutron mass m_n is defined by the formula

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{8}[t_1(1-x_1) + t_4(1-x_4)\varrho^\beta + 3t_2(1+x_2) + 3t_5(1+x_5)\varrho^\gamma], \tag{21}$$

and the renormalized chemical potential μ should be determined from Eq. (7).

Taking into account the explicit form of the FL amplitude U_1 in Eq. (19), solution of Eq. (13) for the energy gap Δ should be sought in the form

$$\Delta(p) = \frac{b_n}{4}\nu p, \tag{22}$$

where ν is some unknown quantity satisfying the equation

$$\nu = \int_0^\infty \frac{q^3}{6\pi^2\hbar^3} f_{||}(q) d\,q. \tag{23}$$

This equation can be obtained from Eqs. (11),(13) after passing from summation to integration, $\frac{1}{V}\sum \ldots \to \int \frac{d^3q}{(2\pi\hbar)^3}\ldots$, and performing then the angle integration.

Thus, with account of Eqs. (5) for the distribution functions f, we obtain the self–consistent equations (7) and (23) for the renormalized chemical potential μ and the unknown ν, determining the splitting Δ in the energy spectrum (3) of neutrons with different helicities. Note that the energy spectrum (3) is invariant under the time reversion but not under the parity transformation. Hence, the state with $\Delta \neq 0$ is characterized by a spontaneously broken P-symmetry.

Now we present the solutions of the self-consistent equations at zero temperature for BSk18 Skyrme force [17]. Note that the self-consistent equations have always the trivial solution $\Delta = 0$ (or $\nu = 0$), corresponding to the normal neutron matter. Besides, the self-consistent equations are invariant with respect to the change $\Delta \to -\Delta$, and, hence, nontrivial solutions for Δ enter in a pair with the same magnitude and opposite sign. The majority of neutrons will have positive helicity, if $\Delta > 0$, and negative helicity, if $\Delta < 0$. After solving the self-consistent equations, the average helicity λ, playing the role of the order parameter of a phase transition, can be obtained from Eq. (8). According to Eq. (8), both signs of the helicity of the given magnitude are possible because of the two possible signs of the energy splitting Δ. Note that in order to preserve the realistic EoS of neutron matter obtained in Ref. [18], the following constraints on the additional parameters of the BSk18 parametrization were set

$$\beta = \gamma,\ t_4(1-x_4) = -3t_5(1+x_5). \tag{24}$$

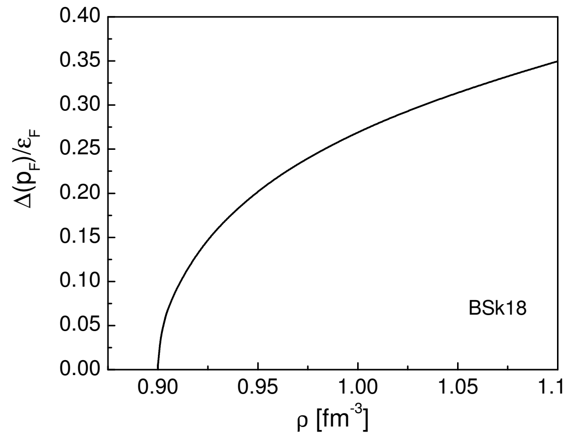

FIG. 1. The energy splitting $\Delta(p_F)$ between the neutron spectra with different helicities normalized to the neutron Fermi energy as a function of density at zero temperature for BSk18 interaction.

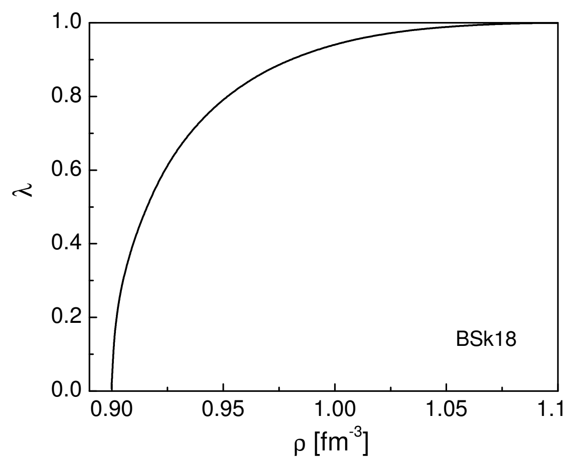

FIG. 2. Same as in Fig. 1 but for the average helicity of neutron matter.

Because of these constraints, the t_4 and t_5 terms cancel completely in the FL amplitude U_0 and in the effective neutron mass m_n, and only the FL amplitude U_1 is affected by the new terms.

Fig. 1 shows the energy splitting $\Delta(p = \hbar k_F)$ between the branches of the neutron spectra with different helicities normalized to the Fermi energy $\varepsilon_F = \frac{\hbar^2 k_F^2}{2m_n}$ of the normal neutron matter as a function of density. A spontaneous phase transition to the state with a nonzero helicity occurs at the critical density $\varrho \approx 5.67\,\varrho_0$ ($\varrho_0 = 0.1586\,\mathrm{fm}^{-3}$ for BSk18 force). The energy splitting continuously increases with the density and becomes comparable with the neutron Fermi energy ε_F. Note that only the branch with the positive energy gap is shown in Fig. 1 while the symmetric branch ($\Delta \to -\Delta$) with the negative energy gap is not presented there.

Fig. 2 shows the average helicity of neutron matter as a function of density obtained with the BSk18 Skyrme

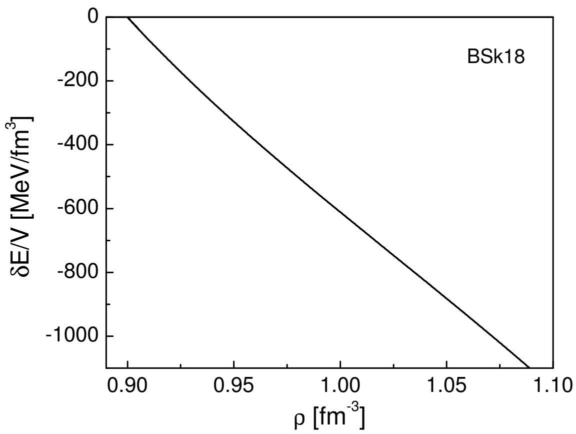

FIG. 3. Same as in Fig. 1 but for the difference between the energy densities of the state with a nonzero average helicity and the normal state ($\lambda = 0$) of neutron matter.

interaction. The average helicity monotonously increases from zero till it is saturated and reaches the value $\lambda = 1$ at $\varrho \approx 6.93\,\varrho_0$. Beginning from that density, all neutron spins will be aligned along their momenta (or opposite to them for the branch with the negative helicity, not shown in Fig. 2).

In order to clarify whether the state with a nonzero average helicity is thermodynamically preferable over the normal state of neutron matter, we should compare the corresponding energies (at zero temperature). Fig. 3 shows the difference between the energy densities of the state with a nonzero average helicity and the normal state of neutron matter. It is seen that for all densities where nontrivial solutions (with $\Delta \neq 0$) exist, this difference is negative and, hence, the state with the majority of neutron spins directed along (or opposite) to their momenta is preferable at that density range.

In summary, we have considered the states with a spontaneous nonzero average helicity in neutron matter with the BSk18 Skyrme NN interaction, which are characterized by broken parity. The self-consistent equations for the parameter, determining the energy splitting between the neutron spectra with different helicities, and the effective chemical potential of neutrons have been obtained and analyzed at zero temperature. It has been shown that the self-consistent equations have solutions corresponding to a nonzero average helicity beginning from the critical density $\varrho \approx 5.67\,\varrho_0$. Under increasing density, the magnitude of the average helicity increases and is saturated at the density $\varrho \approx 6.93\,\varrho_0$, when all neutron spins are aligned along ($\lambda = 1$), or opposite ($\lambda = -1$) to their momenta. The comparison of the respective energies at zero temperature shows that the state with a nonzero average helicity is preferable over the normal state at the densities beyond the critical one. The possible existence of the state with a nonzero average helicity in the dense core of a neutron star will affect the neutrino opacities, and, hence, may be of importance for the adequate description of the thermal evolution of pulsars.

[1] M.J. Rice, Phys. Lett. **A29**, 637 (1969).
[2] S.D. Silverstein, Phys. Rev. Lett. **23**, 139 (1969).
[3] E. Østgaard, Nucl. Phys. **A154**, 202 (1970).
[4] A. Viduarre, J. Navarro, and J. Bernabeu, Astron. Astrophys. **135**, 361 (1984).
[5] S. Reddy, M. Prakash, J.M. Lattimer, and J.A. Pons, Phys. Rev. C **59**, 2888 (1999).
[6] A.I. Akhiezer, N.V. Laskin, and S.V. Peletminsky, Phys. Lett. **B383**, 444 (1996); JETP **82**, 1066 (1996).
[7] M. Kutschera, and W. Wojcik, Phys. Lett. **325B**, 271 (1994).
[8] A.A. Isayev, JETP Letters **77**, 251 (2003).
[9] A.A. Isayev, and J. Yang, Phys. Rev. C **69**, 025801 (2004); A.A. Isayev, Phys. Rev. C **74**, 057301 (2006).
[10] V.R. Pandharipande, V.K. Garde, and J.K. Srivastava, Phys. Lett. **B38**, 485 (1972).
[11] S.O. Bäckmann and C.G. Källman, Phys. Lett. **B43**, 263 (1973).
[12] P. Haensel, Phys. Rev. C **11**, 1822 (1975).
[13] I. Vidaña, A. Polls, and A. Ramos, Phys. Rev. C **65**, 035804 (2002).
[14] S. Fantoni, A. Sarsa, and E. Schmidt, Phys. Rev. Lett. **87**, 181101 (2001).
[15] F. Sammarruca, and P. G. Krastev, Phys. Rev. C **75**, 034315 (2007).
[16] G.H. Bordbar, and M. Bigdeli, Phys. Rev. C **75**, 045804 (2007).
[17] N. Chamel, S. Goriely, and J. M. Pearson, Phys. Rev. C **80**, 065804 (2009).
[18] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).
[19] L. G. Cao, U. Lombardo, and P. Schuck, Phys. Rev. C **74**, 064301 (2006).
[20] J. Rikovska Stone, J. C. Miller, R. Koncewicz, P. D. Stevenson, and M. R. Strayer, Phys. Rev. C **68**, 034324 (2003).
[21] L. D. Landau and E. M. Lifshitz, Statistical Physics, Part 2 (Pergamon, Oxford, 1980).
[22] I. A. Akhiezer, and E. M. Chudnovskii, Phys. Lett. **65A**, 433 (1978).
[23] E. M. Chudnovsky, and A. Vilenkin, Phys. Rev. B **25**, 4301 (1982).
[24] A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, A. P. Rekalo, and A. A. Yatsenko, JETP **85**, 1 (1997).
[25] A.A. Isayev, and J. Yang, in *Progress in Ferromagnetism Research*, edited by V.N. Murray (Nova Science Publishers, New York, 2006), p. 325.
[26] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).