Environmental effects on the x^4 model in Tsallis statistics

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Abstract

The author studied the effects of the environment described by Tsallis statistics in quantum mechanics, when the deviation from Boltzmann-Gibbs (BG) statistics is small. The x^4 model was used and the squeeze angle caused by the difference between Tsallis and BG statistics was calculated perturbatively in the mean field approximation as a function of the dimensionless parameters: the inverse temperature β_p and the coupling strength λ_p. The author found that the effect of the deviation from BG statistics is relatively large at high temperature. The squeeze angle as a function of β_p has a dip structure, and the dip is deeper with the increase of λ_p. The angle as a function of β_p changes the sign. These facts indicate that the frequency is modulated by the difference between these statistics.

Keywords: Non-extensive statistics; Squeezing; x^4 model

1. Introduction

An extended equilibrium statistics, so-called Tsallis statistics [1], was introduced and have been investigated in the past few decades. This statistics shows a power-law distribution and a non-additive entropy. A weighted integral of Gauss distribution (superstatistics) is equal to a Tsallis distribution [2]. It was shown that a certain Langevin equation leads to a Tsallis distribution [3]. These facts imply that the non-extensivity should appear generally. Some physical origins of Tsallis distribution were suggested [4]. For an example, long-range forces may cause power-law distributions. The distribution of such a system whose ingredients interact with each other through long-range forces, may be different from that in the Boltzmann-Gibbs (BG) statistics. Then some environments should be described by Tsallis statistics.

This non-extensive statics has been widely applied to various phenomena and methods, such as particle distribution at high energies [4], non-extensive network [5], generalized simulated annealing algorithm[6] and so on. This statistics has two parameters: one is the inverse of the temperature β and the other is the parameter q which indicates the difference between Tsallis statistics and BG statistics. The parameter q obtained by

fitting the particle distribution [4] is close to the value $q = 1$ at which the distribution corresponds to BG distribution.

The effects of the environment described by Tsallis statistics in Quantum Field Theory were also studied. An example is the calculation of gluon propagator in the environment[7]. It is well-known that the propagator in the thermal environment described by BG statistics is modified, and then the propagator in the environment described by Tsallis statistics is also modified. Similarly, it is expected that the expectation values of other physical quantities are affected by the environment.

The study of the modification of a physical quantity by the environment described by Tsallis statistics is important to clarify the effects of the Tsallis distribution, because power-like distributions appear in many systems. The absolute value $|1 - q|$ (the index of the deviation from BG statistics) is small in many cases, as shown in other studies. Therefore I attempt to calculate the physical quantities, such as the expectation value of the square of the coordinate, under the influence of the environment described by Tsallis statistics, when the deviation from the BG statistics is small enough. I use the x^4 model to study the effects of the environment through the anharmonic potential. I apply a mean field approximation and Bogoliubov transformation. The energy depends on the dimensionless parameters, the inverse temperature β_p and the coupling strength λ_p, and is calculated in the self-consistent manner. The effects of the deviation from BG statistics are given numerically by estimating the squeeze angle. I found that the squeeze angle as a function of β_p has a dip structure and the angle changes the sign. The frequency is modulated by the difference between these statistics, because the coefficient of the x^2 term is modified. These results provide the insight in the study of the phenomena described by the model with self-coupling, such as the ϕ^4 theory.

2. Squeeze angle

2.1. Squeeze angle in a mean field approximation

I deal with the x^4 model in the present paper. The Hamiltonian is

$$H = \frac{1}{2m} \left(p^2 + m^2 \omega^2 x^2 \right) + \frac{1}{4!} m \omega^2 \lambda x^4, \tag{1}$$

where x is the coordinate, p is the momentum, m is the mass, ω is the frequency and λ is the coupling strength. The mean field approximation [8, 9] is applied:

$$x^{2n} \rightarrow \frac{(2n)!}{2^n (n-1)!} \left(\langle x^2 \rangle_q \right)^{n-1} x^2 - \frac{(n-1)[(2n)!]}{2^n n!} \left(\langle x^2 \rangle_q \right)^n, \tag{2a}$$

$$x^{2n+1} \rightarrow \frac{(2n+1)!}{2^n n!} \left(\langle x^2 \rangle_q \right)^n x, \tag{2b}$$

where $\langle O \rangle_q$ is the statistical average of a physical quantity O in Tsallis statistics. The mean field Hamiltonian is given by

$$H^{\mathrm{MF}} = \frac{1}{2m} \left(p^2 + M^2(q, \beta) \omega^2 x^2 \right) - \frac{1}{8} m \omega^2 \lambda \left(\langle x^2 \rangle_q \right)^2, \tag{3}$$

where β is the inverse of the temperature and $M^2(q, \beta)$ is defined as $m^2 \left[1 + \frac{\lambda}{2} \langle x^2 \rangle_q \right]$. The coefficient of the x^2 term is influenced by the environment [10] through $\langle x^2 \rangle_q$. I

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introduce an arbitrary mass M and define creation and annihilation operators, $a^\dagger$ and a, which are related to x and p:

$$x = \sqrt{\frac{\hbar}{2M\omega}} \left(a^\dagger + a\right), \qquad p = i\sqrt{\frac{M\omega\hbar}{2}} \left(a^\dagger - a\right).$$ (4)

With eqs.(4), the mean field Hamiltonian is given by

$$H^{\mathrm{MF}} = \frac{M}{m}\frac{\hbar\omega}{2} \left\{ \left(1 + \frac{M^2(q,\beta) - M^2}{2M^2}\right) \left(aa^\dagger + a^\dagger a\right) \right.$$
$$\left. + \left(1 + \frac{M^2(q,\beta) - M^2}{2M^2}\right) \left(a^2 + \left(a^\dagger\right)^2\right) \right\} - \frac{1}{8}m\omega^2\lambda \left(\left\langle x^2\right\rangle_q\right)^2.$$ (5)

The Bogoliubov transformation [11] is applied to diagonalize H^{MF}:

$$a = \cosh\theta \cdot a_q + \sinh\theta \cdot a_q^\dagger, \qquad a^\dagger = \cosh\theta \cdot a_q^\dagger + \sinh\theta \cdot a_q,$$ (6)

where the parameter θ is the squeeze angle. The quantum state $|0, q; \beta\rangle$ is defined by the equation, $a_q|0, q; \beta\rangle = 0$. Here, the function $g(q, \beta)$ is defined by

$$g(q, \beta) := \frac{M^2(q, \beta) - M^2}{2M^2}.$$ (7)

The mean field Hamiltonian is diagonalized, if the following equation is satisfied:

$$\tanh(2\theta(q, \beta)) = -\frac{g(q, \beta)}{1 + g(q, \beta)}.$$ (8)

The mean field Hamiltonian is given by

$$H^{\mathrm{MF}} = \frac{\Omega(q, \beta)}{2} \left(a_q a_q^\dagger + a_q^\dagger a_q\right) - \frac{1}{8}m\omega^2\lambda \left(\left\langle x^2\right\rangle_q\right)^2,$$ (9)

where $\Omega(q, \beta)$ is defined by

$$\Omega(q, \beta) := \left(\frac{M}{m}\right) \hbar\omega \left(1 + \frac{g(q, \beta)}{1 + g(q, \beta)}\right) \cosh(2\theta).$$ (10)

I show another way to obtain eq. (8). Similarly to eq. (4), the operators, x and p, are related to a_q and $a_q^\dagger$:

$$x = \sqrt{\frac{\hbar}{2M(q, \beta)\omega}} \left(a_q^\dagger + a_q\right), \qquad p = i\sqrt{\frac{M(q, \beta)\omega\hbar}{2}} \left(a_q^\dagger - a_q\right).$$ (11)

From eqs. (4), (6), and (11), I obtain the following equation:

$$\exp(\theta) = \sqrt{\frac{M}{M(q, \beta)}}.$$ (12)

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Apparently, this equation satisfies eq. (8). Then I define θ_{ij} as follows:

$$\exp\left(\theta_{ij}\right) := \sqrt{\frac{M_i}{M_j}}. \tag{13}$$

Equation (13) indicates that

$$\theta_{13} = \theta_{12} + \theta_{23}. \tag{14}$$

This relation is useful to find the value of the parameter θ.

The Hamiltonian is reduced to be harmonic in the mean field approximation. The expressions of $\left\langle x^2 \right\rangle_q$ and $\left\langle p^2 \right\rangle_q$ for the Hamiltonian are given by

$$\left\langle x^2 \right\rangle_q = \left(\frac{\hbar}{2M\omega}\right) e^{2\theta} \left(2 \left\langle a_q^\dagger a_q \right\rangle_q + 1\right), \qquad \left\langle p^2 \right\rangle_q = \left(\frac{M\hbar\omega}{2}\right) e^{-2\theta} \left(2 \left\langle a_q^\dagger a_q \right\rangle_q + 1\right), \tag{15}$$

where the equation $\left\langle a_q \right\rangle_q = 0$ is satisfied. Here I note that the mass M is not specified in the previous discussion.

2.2. The squeeze angle caused by the difference between Tsallis and Boltzmann-Gibbs statistics

In Tsallis statistics, the statistical average of a physical quantity O is given by

$$\langle O \rangle_q = \frac{\sum_i \langle i | O \left\{ \left[1 - (1-q)\frac{\beta}{c_q}(H - \langle H \rangle_q) \right]^{\frac{q}{1-q}} \right\} | i \rangle}{\sum_j \langle j | \left[1 - (1-q)\frac{\beta}{c_q}(H - \langle H \rangle_q) \right]^{\frac{q}{1-q}} | j \rangle}, \tag{16}$$

where $|i\rangle$ is a quantum state labelled i, c_q is the normalization factor and $\langle H \rangle_q$ is the expectation value of the Hamiltonian.

I use the approximated Hamiltonian H^{MF} instead of H in eq. (16). To simplify, I introduce the Hamiltonian $\tilde{H}$ shifted as $\tilde{H} = H^{\mathrm{MF}} - \langle 0, q; \beta | H^{\mathrm{MF}} | 0, q; \beta \rangle$. When the quantity O is the Hamiltonian $\tilde{H}$, I obtain the self-consistent equation for $\tilde{H}$ by using the number state $|n, q; \beta\rangle$ which is the n particle state constructed with $a_q^\dagger$ on the vacuum $|0, q; \beta\rangle$:

$$\langle \tilde{H} \rangle_q = \frac{\sum_{n=0}^{\infty} \left\{ \Omega(q,\beta)\, n \left[1 - (1-q)\frac{\beta}{c_q}(\Omega(q,\beta)\, n - \langle \tilde{H} \rangle_q) \right]^{\frac{q}{1-q}} \right\}}{\sum_{n=0}^{\infty} \left[1 - (1-q)\frac{\beta}{c_q}(\Omega(q,\beta)\, n - \langle \tilde{H} \rangle_q) \right]^{\frac{q}{1-q}}}. \tag{17}$$

In this paper, I attempt to find the quantum state (namely squeeze angle θ) in Tsallis statistics with small deviation from $q = 1$. Conventionally, the deviation ϵ from $q = 1$ is defined by $\epsilon := 1 - q$. I use this parameter ϵ instead of q in the following calculations. I

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assume that the lowest contribution of θ for small ϵ is proportional to ϵ. The functions, $\Omega(q,\beta)$, c_q and $\theta(q,\beta)$, are expanded as series of ϵ:

$$\Omega(q,\beta) \equiv \Omega(1-\epsilon,\beta) = \Omega_0 - \epsilon\Omega_1 + O\left(\epsilon^2\right), \tag{18a}$$

$$c_q \equiv c_{1-\epsilon} = c_0 - \epsilon c_1 + O\left(\epsilon^2\right), \tag{18b}$$

$$\theta(q,\beta) \equiv \theta(1-\epsilon,\beta) = \theta_0 - \epsilon\,\Theta + O\left(\epsilon^2\right). \tag{18c}$$

I specify the mass M explicitly to calculate the parameter θ. I choose $M(q=1,\beta)$ as M. To avoid the confusion, I define the symbol M_β by $M_\beta = M(q=1,\beta)$. In such the case, the angle θ satisfying eq. (8) reaches zero as $\epsilon \to 0$. Therefore, the term θ_0 in eq. (18c) is equal to zero in the case that M is equal to M_β.

The expectation value $\langle\tilde{H}\rangle_q$ is also a function of ϵ. I expand this expectation as follows:

$$\langle\tilde{H}\rangle_q \equiv \langle\tilde{H}\rangle_{1-\epsilon} = E_0 - \epsilon E_1 + O(\epsilon^2) \tag{19}$$

The expressions of c_0 and c_1 can be obtained by using the relation between c_q and the partition function Z_q:

$$c_q = (Z_q)^{1-q}. \tag{20}$$

In the similar way, Z_q is expanded as $Z_{1-\epsilon} = Z_0 - \epsilon Z_1 + O(\epsilon^2)$, where Z_0 is the partition function at $q=1$. From eq. (20), these quantities are given by

$$c_0 = 1, \qquad c_1 = -\ln Z_0. \tag{21}$$

The quantities, E_0 and E_1, are obtained by substituting eqs. (18a), (18b), (19) into eq. (17). The quantity E_0 is given by

$$E_0 = \Omega_0 \left(\frac{S_1}{S_0}\right), \tag{22}$$

where S_p is defined by

$$S_p := \sum_{n=0}^{\infty} n^p \exp\left[-\beta\left(\frac{\Omega_0}{c_0}\right)n\right]. \tag{23}$$

The quantity Ω_0 is apparently given by

$$\Omega_0 = \left(\frac{M_\beta}{m}\right)\hbar\omega. \tag{24}$$

I obtain easily the expression of $\left\langle x^2\right\rangle_{q=1-\epsilon}$ by the perturbation with respect to ϵ:

$$\begin{aligned}
\left\langle x^2\right\rangle_{q=1-\epsilon} =&\frac{\hbar}{2\omega M_\beta}R_1 \\
&+ \frac{\epsilon\hbar}{\omega M_\beta}\left\{-\Theta R_1 + \left[\beta\left(1-\frac{c_1}{c_0}\right)\left(\frac{\Omega_0}{c_0}\right) + \beta\left(\frac{\Omega_1}{c_0}\right) + \beta^2\left(\frac{\Omega_0}{c_0}\right)\left(\frac{E_0}{c_0}\right)\right]R_2 \right. \\
&\left. + \frac{\beta^2}{2}\left(\frac{\Omega_0}{c_0}\right)^2 R_3\right\} + O\left(\epsilon^2\right).
\end{aligned} \tag{25}$$

The functions R_1, R_2 and R_3 are defined by

$$R_1 := 1 + 2\left(\frac{S_1}{S_0}\right), \quad R_2 := \left(\frac{S_1}{S_0}\right)\left[\left(\frac{S_2}{S_1}\right) - \left(\frac{S_1}{S_0}\right)\right], \quad R_3 := \left(\frac{S_1}{S_0}\right)\left[\left(\frac{S_2}{S_0}\right) - \left(\frac{S_3}{S_1}\right)\right].$$
(26)

Apparently, the quantity $\langle x^2 \rangle_q$ - $\langle x^2 \rangle_{q=1}$ is $O(\epsilon)$. Then the order of $g(1 - \epsilon, \beta)$ is $O(\epsilon)$. By using the definition of $g(q, \beta)$ and eq. (25), the expression of $g(1 - \epsilon, \beta)$ for small ϵ is given as follows:

$$g(1 - \epsilon, \beta)$$
$$= \epsilon \frac{\kappa\left\{-R_1\Theta + \left[\beta\left(1 - \frac{c_1}{c_0}\right)\left(\frac{\Omega_0}{c_0}\right) + \beta\left(\frac{\Omega_1}{c_0}\right) + \beta^2\left(\frac{\Omega_0}{c_0}\right)\left(\frac{E_0}{c_0}\right)\right]R_2 + \frac{1}{2}\beta^2\left(\frac{\Omega_0}{c_0}\right)^2 R_3\right\}}{1 + \kappa R_1}$$
$$+ O(\epsilon^2),$$
(27)

where $\kappa = (\lambda \hbar)/(4\omega M_\beta)$. Therefore, eq. (10) gives the equation between Ω_1 and $g(1 - \epsilon, \beta)$ to the order ϵ by ignoring $O(\epsilon^2)$ terms:

$$\epsilon\Omega_1 = -\Omega_0 g(1 - \epsilon, \beta).$$
(28)

Similarly, eq. (8) gives us the equation to the order ϵ:

$$2\epsilon\Theta = g(1 - \epsilon, \beta).$$
(29)

I finally obtain the expression of Θ with eqs. (27) (28) and (29):

$$\Theta = \frac{\kappa\left\{\left[\beta\left(1 - \frac{c_1}{c_0}\right)\left(\frac{\Omega_0}{c_0}\right) + \beta^2\left(\frac{\Omega_0}{c_0}\right)\left(\frac{E_0}{c_0}\right)\right]R_2 + \frac{1}{2}\beta^2\left(\frac{\Omega_0}{c_0}\right)^2 R_3\right\}}{2 + 3\kappa R_1 + 2\kappa\beta\left(\frac{\Omega_0}{c_0}\right)R_2},$$
(30)

where c_1 is given by

$$c_1 = -\ln Z_0 = -\ln S_0 - \beta\left(\frac{\Omega_0}{c_0}\right)\left(\frac{S_1}{S_0}\right).$$
(31)

The function S_p is simplified with the functions $f(x)$ and $\tilde{f}(x)$ which are defined by

$$f(x) := \frac{e^x}{e^x - 1}, \qquad \tilde{f}(x) := \frac{1}{e^x - 1}.$$
(32)

For an example, the quantity S_1/S_0 is equal to $\tilde{f}(\beta\Omega_0/c_0)$.

The squeezing also occurs due to the difference between M_β and m. The corresponding squeeze angle is given by

$$\exp(\varphi_m(\beta)) = \left(\frac{m}{M_\beta}\right)^{1/2}.$$
(33)

The quantity $M(q = 1, \infty)$ is not equal to m when λ is not zero. Therefore $\varphi_m(\infty)$ is not zero generally. From eq. (14), the total squeeze angle φ_{tot} of the order ϵ is given by

$$\varphi_{\text{tot}} = \varphi_m(\beta) + \theta(1 - \epsilon, \beta) = \varphi_m(\beta) - \epsilon\Theta + O\left(\epsilon^2\right).$$
(34)

The angle φ_{tot} is calculated by solving the temperature dependence of M_β and using eq. (30).

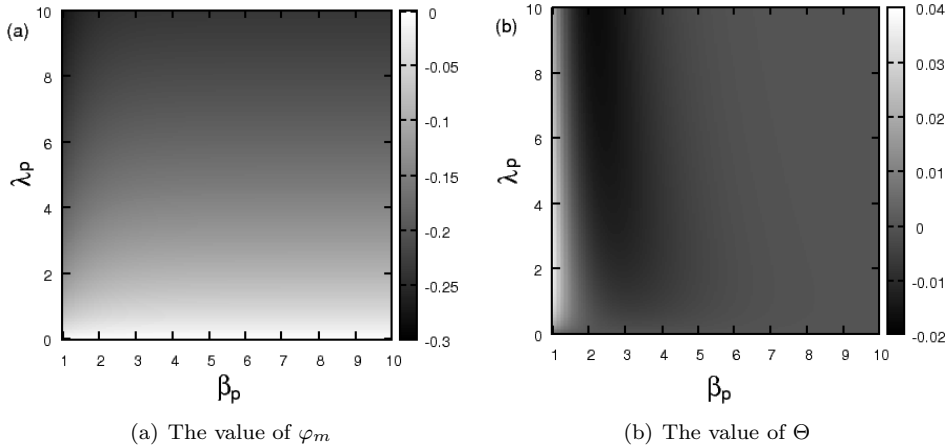(a) The value of φ_m
(b) The value of Θ

Figure 1: The squeeze angle in the ranges of $1.0 \leq \beta_p \leq 10$ and $0 \leq \lambda_p \leq 10$.

3. Numerical Estimation of the Squeeze Angle

In this section, I calculate numerically the angle Θ given by eq. (30) and the angle $\varphi_m(\beta)$ given by eq. (33). I introduce the dimensionless parameters, β_p and λ_p, by

$$\beta_p = \beta\hbar\omega, \qquad \lambda_p = \lambda\frac{\hbar}{m\omega}. \qquad (35)$$

The parameters in these numerical calculations, $\hbar$, ω and m, are set to 1.

I show the values of φ_m and Θ at various values of β_p and λ_p. Figure 1(a) is the map of φ_m in the ranges of $1 \leq \beta_p \leq 10$ and $0 \leq \lambda_p \leq 10$. As is well-known, the temperature-dependent mass M_β increases generally with the temperature, because of the existence of the term $m^2\lambda\langle x^2 \rangle_{q=1}/2$. Then φ_m is negative at finite temperature and decreases with the temperature β_p^{-1} and the coupling λ_p, as shown in Fig. 1(a). Figure 1(b) shows the map of Θ in the same range. The value of Θ goes to zero from negative value as β_p goes to infinity, as seen in this figure. This behavior is reasonable because the parameter β_p corresponds to the inverse of the temperature. As shown in this figure, Θ has a local minimum as a function of β_p at a fixed λ_p and the sign of Θ changes in the vicinity of $\beta_p = 2 \sim 3$, while φ_m decreases monotonically and the sign of φ_m is negative.

The sign of Θ depends on the values of β_p and λ_p, and this fact indicates that the sign of $\theta = -\epsilon\Theta$ does not depend on only the sign of the parameter ϵ. In other words, the sign of θ cannot be determined by only the property of statistics, subextensive ($\epsilon < 0$) or superextensive ($\epsilon > 0$). From eq. (12) and (18c), I have

$$M(q = 1 - \epsilon, \beta) = M_\beta \exp(2\epsilon\Theta). \qquad (36)$$

Then $M(q, \beta)$ is larger than M_β for $\epsilon\Theta > 0$. Figure 1 indicates that $M(q, \beta)$ at $q \neq 1$ is smaller than M_β for small and positive ϵ in the large area of the parameters, where Θ is negative. This tendency is probable intuitively. The cutoff exists in Tsallis statistics of $q < 1$ ($\epsilon > 0$), then the fluctuation should be smaller than that in BG statistics.

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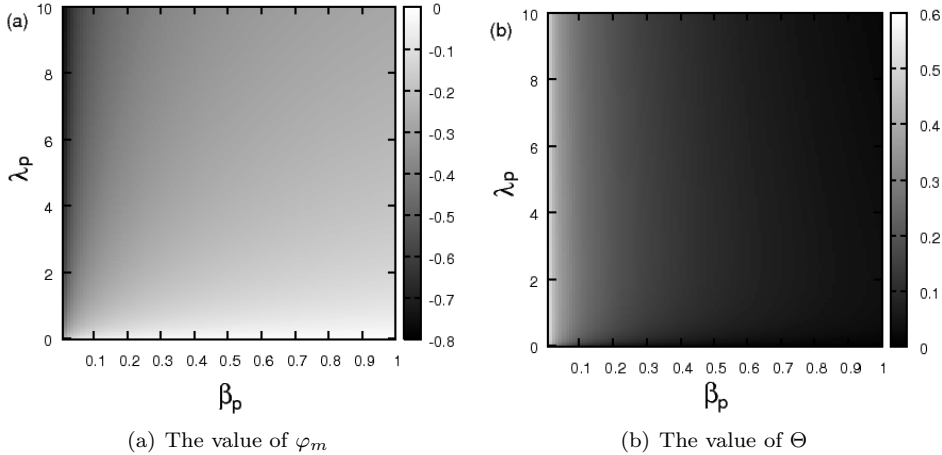(a) The value of φ_m (b) The value of Θ

Figure 2: The squeeze angle in the ranges of $0.01 \leq \beta_p \leq 1$ and $0 \leq \lambda_p \leq 10$.

Therefore $\langle x^2 \rangle$ in Tsallis statistics of $q < 1$ should be smaller than that in BG statistics. Then $M(q < 1, \beta)$ should be smaller than M_β. However Θ is positive for small β_p, and then $M(q < 1, \beta)$ is larger than M_β even when ϵ is positive. In addition, the modification of φ_{tot} due to the deviation from $q = 1$ is at most $O\left(|\epsilon \varphi_m|\right)$ in the large area of this figure, because the absolute value of Θ is less than that of φ_m.

The absolute value of Θ becomes large as β_p goes to zero in Fig. 1(b). Therefore I show the map of Θ at small β_p. Figure 2(a) is the map of φ_m in the ranges of $0.01 < \beta_p < 1$ and $0 < \lambda_p < 10$. Figure 2(b) is the map of Θ in the same ranges. The absolute value of Θ is close to that of φ_m in this region. Therefore the difference between BG statistics and Tsallis statistics will be important. The modification due to $q \neq 1$ is given as $-\epsilon \Theta$, and then the ratio of modification is $O\left(|\epsilon|\right)$ in the present case. The quantity $M(q, \beta)$ is larger than M_β for $\epsilon > 0$, because the sign of Θ is positive at small β_p.

From Fig. 1 and Fig. 2, the effects of the discrepancy between $q \neq 1$ and $q = 1$ is relatively large at small λ_p (weak coupling) and small β_p (high temperature). Therefore the experiment to measure the deviation from the Boltzmann-Gibbs statistics should be performed at weak coupling and high temperature in the system described by the x^4 potential.

4. Discussion and Conclusion

I studied the effects of the environments described by Tsallis statistics in quantum mechanics, when the deviation from Boltzmann-Gibbs (BG) statistics is small. I calculated the squeeze angle (the angle in Bogoliubov transformation) in the mean approximation. I used the x^4 model, and the effects of $q \neq 1$ in Tsallis statistics were taken into the coefficient of the x^2 term through the expectation value $\langle x^2 \rangle_q$. The squeeze angles were displayed for various sets of the dimensionless parameters, the inverse of the temperature β_p and the coupling strength λ_p.

In the numerical calculations, I showed that the sign of the squeeze angle Θ changes as a function of β_p and λ_p. Then the sign of $\theta(q, \beta) = -\epsilon\Theta(\beta)$ is not determined by only the sign of the parameter ϵ which is defined by $1 - q$, where this parameter indicates the property of the statistics, subextensive or superextensive. Intuitively, it seems that the expectation value $\langle x^2 \rangle_{q=1-\epsilon}$ is large in the superextensive case. However, the fact that Θ changes the sign due to the values, β_p and λ_p, indicates that this conjecture is not always correct.

The squeeze angle as a function of β_p has a dip structure, and this structure is easily seen at large λ_p. This structure is characteristic, because the angle due to the thermal effect is a monotone function. Considering the ratio of the squeeze angle caused by the difference between BG statistics and Tsallis statistics to the squeeze angle caused by BG static (thermal contribution), I found that the absolute value of this ratio at high temperature and weak coupling is relatively large in the ranges of the numerical calculations. From these facts, the effects caused by the difference between Tsallis statistics and BG statistics will be observed at high temperature.

The effect of the environment was included in the quantity $M(q, \beta)$. The x^2 term in eq. (3) can be rewritten as follows:

$$\frac{1}{2m}M^2(q, \beta)\,\omega^2 x^2 = \frac{1}{2m}m^2\left(\frac{M^2(q, \beta)}{m^2}\,\omega^2\right)x^2 \tag{37}$$

The right-hand side of eq. (37) is interpreted as the modulation of the frequency of a particle with mass m. This effect may be found experimentally in a quantum mechanical system.

I studied the effects of the environment described by Tsallis statistics in this paper. I hope that this work is useful in the future studies of the non-extensive statistic.

Acknowledgements

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