Distribution function of persistent current

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We introduce a variant of the replica trick within the nonlinear sigma model that allows calculating the distribution function of the persistent current. In the diffusive regime, a Gaussian distribution is derived. This result holds in the presence of local interactions as well. Breakdown of the Gaussian statistics is predicted for the tails of the distribution function at large deviations.

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Introduction. - A striking manifestation of quantum mechanics in the mesoscopic physics of electrons is that an equilibrium persistent current (PC) can flow in normal-metallic rings threaded by a magnetic flux [1]. In the diffusive regime, this property arises from a flux-dependent interference contribution to the electron density of states [2] that survives in presence of the impurity-induced static potential disorder. The amplitude of PC is quite small and varies strongly from sample to sample: The theory for non-interacting electrons [3, 4] predicts that the ensemble average vanishes, while the typical amplitude is Φ_0-periodic ($\Phi_0 = h/e$ is the flux quantum) and scales as $I_{\rm typ} \sim e/\tau_D$, where τ_D is the diffusion time along the ring. Thus, $I_{\rm typ} \sim 1$nA in micrometer-size rings made of conventional metals. Electron-electron interactions were predicted to induce a $\Phi_0/2$-periodic average current of order $I_{\rm av} \sim \lambda_{\rm eff} e/\tau_D$, where $\lambda_{\rm eff}$ is an effective coupling constant [5]. In superconducting rings, this would yield a diamagnetic average current due to superconducting fluctuations well above the superconducting critical temperature [6].

Early experiments measured the PC by detecting the small magnetic field that they produce with superconducting quantum interference devices (SQUIDs). Both the $\Phi_0/2$-periodic average current in large ensembles of rings [7, 8] and Φ_0-periodic current in single rings and small ensembles of rings [9–12] were recorded; a low-flux diamagnetic response was observed in [8, 11]. However, it was not always possible to reconcile their amplitude with the theories described above. This could be due to subtle effects related to canonical vs.grand canonical averaging in isolated rings [13], magnetic impurities [14], or the great sensitivity of the PC to its electromagnetic environment [15]. A recent experiment on several rings addressed separately with a scanning SQUID microscope showed however a good agreement with the non-interacting theory for the typical current and no sign of average current [16].

An experimental breakthrough was made recently by measuring PCs with high-precision cantilever torque magnetometry [17]. Notably, the technique allows better sensitivity and works under large magnetic fields (compared with the SQUID technique). The PC could thus be recorded over a huge number of flux periods. The amplitude of the typical PC was found to be in good agreement with the prediction for non-interacting electrons and no average current was detected.

The aim of the present work is to address the distribution function of PC that seems within reach of the new experimental technique. We demonstrate that in the diffusive regime the statistics is Gaussian. It justifies characterizing the PC with its first two cumulants only. The Gaussian distribution carries on in the presence of local interactions as well. To derive the result, we make use of a replica trick in the nonlinear sigmal model that allows obtaining the distribution function at once. It provides an alternative to the evaluation of all cumulants order by order [18, 19]. It could serve as a useful starting point to address questions such as canonical vs. grand canonical averaging, localization and cross-over to non-Gaussian statistics.

Gaussian distribution for the persistent current of non-interacting electrons. - The PC flowing in a metallic ring pierced by a magnetic flux Φ,

$$I(\Phi) = -\frac{\partial F}{\partial \Phi}, \tag{1}$$

is related to the free energy $F(\Phi) = -kT \ln Z_\Phi$, where Z_Φ is the partition function. The PC distribution function at given Φ is the probability density for $I(\Phi)$ to be equal to $\mathcal{I}$,

$$P(\mathcal{I}) = \langle \delta\left(\mathcal{I} - I(\Phi)\right)\rangle, \tag{2}$$

where the brackets denote an ensemble average over different disorder configurations in the ring. Using the identity $\delta(s) = \int (dx/2\pi) e^{ixs}$ and the definition of the derivative, we express Eq. (2) as

$$P(\mathcal{I}) = \lim_{\Phi' \to \Phi} \int \frac{dx}{2\pi} e^{i\mathcal{I}x} \langle Z_\Phi^n Z_{\Phi'}^{n'} \rangle, \tag{3}$$

where $n = -n' = -ixkT/(\Phi - \Phi')$.

The disorder-averaging for free fermions in Eq. (3) can be performed with a variant of the replica trick [20] in the fermionic non-linear sigma model [21, 22]. For this, we consider that the system is formed of n replicas with flux Φ and n' replicas with flux Φ' (n and n' integers) and we evaluate

$$\langle Z_\Phi^n Z_{\Phi'}^{n'}\rangle = \int \mathcal{D}Q e^{-S[Q]}. \tag{4}$$

Here, Q is a matrix field acting in the direct product of the replica space of dimension $n+n'$, infinite Matsubara energy space, two-dimensional Gorkov-Nambu space (Pauli matrices τ_α), and two-dimensional spin space (Pauli matrices σ_k). The Q matrix obeys the nonlinear constraint $Q^2 = 1$ and the charge conjugation symmetry $Q = \overline{Q} \equiv \tau_1\sigma_2 Q^T \sigma_2\tau_1$, where Q^T stands for the full matrix transposition. The action of the model is [22, 23]

$$S[Q] = \frac{\pi\nu}{8}\int d\boldsymbol{r}\mathrm{Tr}\left[D(\boldsymbol{\partial}Q)^2 - 4\epsilon\tau_3 Q\right]. \tag{5}$$

Here, ν is the single-particle density of states per spin, D is the diffusion coefficient in the metal, $\boldsymbol{\partial} = \boldsymbol{\nabla} + (ie/\hbar)[\boldsymbol{A}\tau_3, .]$ includes the effect of a vector potential associated with the magnetic field $\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$ (different in each replica), ϵ is a fermionic Matsubara energy, and the trace 'Tr' is taken over all spaces of the Q matrix.

Let us consider a quasi one-dimensional circular ring with length $L = 2\pi R$ (R is the radius of the ring). The circular gauge $\boldsymbol{A} = (\Phi/L)\boldsymbol{u}$, where $\boldsymbol{u}$ is a unitary vector normal to the ring, is used. We introduce a coordinate y along the ring measured in units of L, a flux vector $\hat{\phi} = \{\phi_a\}_a$ in replica space measured in units of Φ_0, with components $\phi_a = \phi \equiv \Phi/\Phi_0$ ($\phi_a = \phi' \equiv \Phi'/\Phi_0$) for $1 \le a \le n$ ($n < a \le n+n'$), and $\varepsilon = \epsilon/E_c$, where $E_c = \hbar D/R^2$ is the Thouless energy related to diffusion time ($E_c = 4\pi^2\hbar/\tau_D$). Then, Eq. (5) reads

$$S[Q] = \frac{E_c}{32\pi\delta}\int_0^1 dy\mathrm{Tr}\left[(\partial_y Q)^2 - 16\pi^2\varepsilon\tau_3 Q\right], \tag{6}$$

where δ is the mean level spacing in the ring and $\partial_y = \nabla_y + 2\pi i[\hat{\phi}\tau_3, .]$. The single-valuedness of the Q field fixes the boundary conditions: $Q(y=0) = Q(y=1)$ and $\nabla_y Q(y=0) = \nabla_y Q(y=1)$.

In the metallic regime, the ring's conductance measured in units of the conductance quantum, $g = E_c/(2\pi\delta)$, is large. Thus, we can evaluate Eq. (4) within the saddle-point approximation. The Q_0 field that minimizes the action (6) is proportional to unity in spin and replica spaces, and diagonal in Matsubara space with value $Q_0(\varepsilon) = \tau_3\mathrm{sign}(\varepsilon)$. The action at the saddle point is $(n+n')S_0$ where $S_0 = -(2\pi E_c/\delta)\sum_\varepsilon|\varepsilon|$; it does not depend on the flux.

In order to study fluctuations near this saddle point we write matrices close to Q_0 as

$$Q = Q_0(1 + W + W^2/2 + \ldots). \tag{7}$$

The constraints on the Q field imply that $\{Q_0, W\} = 0$ and $W = -\overline{W}$; the requirement of convergency of the sigma model on the perturbative level implies that $W^\dagger = -W$. Then, we decompose the W field into its elements in Nambu and spin spaces, and its Fourier components:

$$W_{\varepsilon a,\varepsilon' b}(y) = \sum_{p=-\infty}^{\infty}\sum_{k=0}^{3}\begin{pmatrix} W^{k(p)}_{\varepsilon a,\varepsilon' b} & \tilde{W}^{k(p)}_{\varepsilon a,\varepsilon' b} \\ -\tilde{W}^{k(-p)*}_{\varepsilon' b,\varepsilon a} & \varsigma_k W^{k(p)}_{-\varepsilon' b,-\varepsilon a}\end{pmatrix}_{\mathrm{Nambu}} \otimes \sigma_k e^{2\pi i p y}, \tag{8}$$

where $\varsigma_k = \pm$ for $k = 1,2,3$ and $k = 0$, respectively, and the components $W^{k(p)}_{\varepsilon a,\varepsilon' b}$ ($\tilde{W}^{k(p)}_{\varepsilon a,\varepsilon' b}$) only exist at $\varepsilon\varepsilon' < 0$ ($\varepsilon\varepsilon' > 0$). An independent set of complex integration variables is then obtained with $W^{k(p)}_{\varepsilon a,\varepsilon' b}$ and $\tilde{W}^{k(p)}_{\varepsilon a,\varepsilon' b}$ at $a > b$, and $W^{k(p)}_{\varepsilon a,\varepsilon' a}$ and $\tilde{W}^{k(p)}_{\varepsilon a,\varepsilon' a}$ at $\varepsilon > 0$.

Expanding the action (6) near the saddle point up to quadratic terms in W, one finds:

$$S^{(2)}[W] = \frac{\pi E_c}{2\delta}\sum_{\varepsilon\varepsilon'}\sum_{ab}\sum_k\sum_p\left\{\left[|\varepsilon| + |\varepsilon'| + (p + \phi_a - \phi_b)^2\right]|W^{k(p)}_{\varepsilon a,\varepsilon' b}|^2 + \left[|\varepsilon| + |\varepsilon'| + (p + \phi_a + \phi_b)^2\right]|\tilde{W}^{k(p)}_{\varepsilon a,\varepsilon' b}|^2\right\} \tag{9}$$

The Gaussian integration over the W field is then straightforward and yields:

$$\langle Z_\Phi^n Z_{\Phi'}^{n'}\rangle = e^{-n^2\Xi(\phi,\phi) - n'^2\Xi(\phi',\phi') - 2nn'\Xi(\phi,\phi')}, \tag{10}$$

where we have omitted a factor which is equal to 1 in the replica limit $n' = -n$, and

$$\Xi(\phi,\phi') = 4\sum_p\sum_{\varepsilon,\varepsilon'>0}\sum_{s=\pm}\ln\left[\varepsilon + \varepsilon' + (p + \phi - s\phi')^2\right]. \tag{11}$$

The replica trick now consists in assuming that Eq. (10) can be analytically continued to pure imaginary variables n and n', with $n' = -n$. Then, inserting Eq. (10) into (3), we find

$$P(\mathcal{I}) = \int \frac{dx}{2\pi} e^{i\mathcal{I}x} e^{-x^2 I_{\rm typ}^2/2} = e^{-\mathcal{I}^2/(2I_{\rm typ}^2)}/\sqrt{2\pi I_{\rm typ}}, \tag{12}$$

where $I_{\rm typ}^2 = -2(kT/\Phi_0)^2 \partial^2 \Xi(\phi,\phi')/(\partial\phi\partial\phi')|_{\phi'=\phi}$. That is, the distribution function of the PC is Gaussian with a zero mean value and typical value $I_{\rm typ}$. Using the Poisson summation rule, we can convert the sum over p in Ξ into an integral and finally obtain, at zero temperature,

$$I_{\rm typ}^2(\Phi) = \frac{24E_c^2}{\pi^2\Phi_0^2} \sum_{q=1}^{\infty} \frac{1}{q^3} \sin^2(2\pi q\Phi/\Phi_0). \tag{13}$$

Thus, the PC has typical amplitude $E_c/\Phi_0 \sim e/\tau_D$. The distribution function (12) is in agreement with the known results [4] for the average current $\langle I(\Phi)\rangle$ and its cumulant $\langle\langle I(\Phi)I(\Phi')\rangle\rangle \equiv \langle I(\Phi)I(\Phi')\rangle - \langle I(\Phi)\rangle\langle I(\Phi')\rangle$ in non-interacting diffusive rings, with $I_{\rm typ} = \langle\langle I(\Phi)^2\rangle\rangle^{1/2}$ [24].

This section contains the main result of this article, Eq. (12). In the following we illustrate several directions where it can be extended.

Distribution function for the harmonics. - It may be more convenient experimentally to characterize the flux-current relation by its harmonic content. In this section, we show that the distribution function for the harmonics in the diffusive regime is also Gaussian.

Due to time-reversal symmetry and flux-periodicity, the current-flux relation,

$$I(\Phi) = \sum_{q=1}^{\infty} I_q \sin 2\pi q\phi, \tag{14}$$

is fully characterized by its harmonics I_q. The distribution function for the harmonics,

$$P_q(\mathcal{J}) = \langle\delta(\mathcal{J} - I_q)\rangle, \tag{15}$$

can also be determined with a replica trick. Indeed, using Eqs. (1), (14), and integrating by parts, we first note that $I_q = (8\pi q/\Phi_0)\int_0^{1/2} d\phi \cos(2\pi q\phi) F(\phi)$. By definition of the integration, it also reads:

$$I_q = \lim_{N\to\infty} (4\pi q/N\Phi_0) \sum_{\ell=1}^{N} \cos(2\pi q\phi_\ell) F(\phi_\ell), \tag{16}$$

where $\phi_\ell = \ell/(2N)$. Now, inserting the representation of the delta-function and Eq. (16) into (15), we find

$$P_q(\mathcal{J}) = \lim_{N\to\infty} \int \frac{dx}{2\pi} e^{i\mathcal{J}x} \langle\prod_{\ell=1}^{N} Z_{\phi_\ell}^{n_\ell}\rangle, \tag{17}$$

where $n_\ell = i4\pi xqkT\cos(2\pi q\phi_\ell)/(\Phi_0 N)$.

The average over the disorder can also be performed within the fermionic sigma model by considering that the system is formed of n_ℓ replicas (n_ℓ integer) with flux ϕ_ℓ ($1 \le \ell \le N$). In the saddle point approximation, one would find as a generalization of Eq. (10):

$$\langle\prod_\ell Z_{\phi_\ell}^{n_\ell}\rangle = \exp\left[-\sum_{\ell\ell'} n_\ell n_{\ell'} \Xi(\phi_\ell, \phi_{\ell'})\right]. \tag{18}$$

Taking the replica limit, one again obtains a Gaussian distribution $P_q(\mathcal{J}) \sim \exp(-\mathcal{J}^2/2\langle\langle I_q^2\rangle\rangle)$ for the harmonics, with zero average value and variance

$$\begin{aligned}\langle\langle I_q^2\rangle\rangle &= -\frac{32k^2T^2}{\Phi_0^2}\int_0^{1/2} d\phi d\phi' \sin(2\pi q\phi)\sin(2\pi q\phi') \\ &\quad \times \frac{\partial^2\Xi(\phi,\phi')}{\partial\phi\partial\phi'}.\end{aligned} \tag{19}$$

In particular, $\langle\langle I_q^2\rangle\rangle = 96E_c^2/(\pi^2\Phi_0^2 q^3)$ at $T = 0$, in agreement with Eq. (13).

Interactions. - The effect of electron-electron interactions can also be taken into account. To be specific, we consider the case of attractive, local pairing between electrons with opposite spins that was theoretically debated after the early experiments on PC. Then, the action (5) should be supplemented with an interaction term [22],

$$\begin{aligned}S_{\rm int}[Q] &= -\frac{\nu\lambda\pi^2k^2T^2}{16}\sum_a \int d\boldsymbol{r}d\tau \left[({\rm tr}\tau_1 Q_{\tau a,\tau a})^2\right. \\ &\quad \left. + ({\rm tr}\tau_2 Q_{\tau a,\tau a})^2\right],\end{aligned} \tag{20}$$

where λ is the Bardeen-Cooper-Schrieffer coupling constant, τ is imaginary time, and the trace 'tr' is taken over spin and Nambu spaces.

Above the superconducting critical temperature, the action can be evaluated in the Gaussian approximation near the metallic saddle point Q_0. For the one-dimensional ring, Eq. (20) results in an interacting contribution adding to (9):

$$S_{\rm int}^{(2)}[W] = -\frac{4\lambda\pi^2T}{\delta}\sum_{p,\omega,a}\sum_{\varepsilon,\varepsilon'>0} \tilde{W}^{0(p)}_{\varepsilon a,\varepsilon+\omega a}\tilde{W}^{0(p)*}_{\varepsilon' a,\varepsilon'+\omega a}. \tag{21}$$

where ω is a bosonic Matsubara energy (also measured in units of E_c). Gaussian integration over the W field including Eqs. (9), (21) can be performed; it yields

$$\langle Z_\Phi^n Z_{\Phi'}^{n'}\rangle_\lambda = \langle Z_\Phi^n Z_{\Phi'}^{n'}\rangle_{\lambda=0} e^{-n\Xi_{\rm int}(\phi) - n'\Xi_{\rm int}(\phi')}, \tag{22}$$

where

$$\Xi_{\rm int}(\phi) = \sum_{p\omega} \ln\left[1 - \sum_{|\omega|<\varepsilon<\frac{\Omega}{E_c}} \frac{(4\lambda\pi kT/E_c)}{2\varepsilon - |\omega| + (p+2\phi)^2}\right]. \tag{23}$$

Here, Ω fixes the energy bandwidth around the Fermi level over which pairing is effective. By introducing the critical temperature $T_c \simeq (1.14\Omega/k)e^{-1/\lambda}$, one gets

$$\Xi_{\rm int}(\phi) = \sum_{p\omega} \ln \left[\ln \frac{T}{T_c} + \psi\left(\frac{1}{2} + \frac{|\omega| + (p+2\phi)^2}{4\pi kT/E_c}\right) - \psi\left(\frac{1}{2}\right)\right] . \quad (24)$$

where ψ is the digamma function. Inserting Eq. (22) into (3), one again finds that the distribution function for PC is Gaussian, with average value $\langle I(\Phi)\rangle = -(kT/\Phi_0)\partial\Xi_{\rm int}/\partial\phi$ and the same variance as in the non-interacting case. The average current was discussed in Ref. [6], it is $\Phi_0/2$-periodic with amplitude $I_{\rm av} \sim \lambda_{\rm eff} e/\tau_D$ where $\lambda_{\rm eff} \sim \ln^{-1}(E_c/kT_c)$ at $T_c < T \ll E_c$.

Discussion. - We first note that spin and orbital effects, such as the penetration of the magnetic field within rings with finite thickness [25] are important for a quantitative comparison with the experiment [17]. Taking these effects into account within our formalism can be done easily; it would not change the prediction of a Gaussian distribution in the diffusive regime.

On the other hand, the Gaussian statistics clearly fails in the insulating regime, at $g < 1$. Actually, its alteration is expected already at large, but finite g, in relation with the Anderson localization phenomenon. A similar question on the statistics of the – dissipative – conductance of diffusive wires was addressed [26]. Log-normal tails in the probability distribution were predicted at large deviations from the average conductance. However, the present case differs by the fact that PC is a thermodynamic quantity.

To estimate the range of validity of the Gaussian statistics for PC, we expand the action (5) in vicinity of the metallic saddle point Q_0 up to fourth order terms in the field W. Then, we evaluate the generated terms perturbatively with the Gaussian action. As a result, we found that the leading correction to the integrand in Eq. (12) arises in order $\propto x^3 I_{\rm typ}^3/g$, consistent with the recently derived third order cumulant [28]. The same way, we also obtain that n-th order cumulants scale as $I_{\rm typ}^n/g^{n-2}$ at $n \geq 3$. Subsequently, this implies that the Gaussian distribution is not reliable at large deviations, when $|\mathcal{I}| \gtrsim g^{1/3} I_{\rm typ} \gg I_{\rm typ}$. A more detailed investigation of the behaviour of $P(\mathcal{I})$ at large deviations is left for future study.

In the absence of interactions, the average current vanishes. However, this result is an artefact of the grand canonical averaging tacitly performed here. When (canonical) averaging is done with keeping the number of electrons constant in the ring, a small, but finite, average current $I_{\rm av} \sim I_{\rm typ}/g$ is obtained [27]. Including this effect in the framework of this article remains an open question.

Conclusion.- The persistent current has been mostly characterized by its first two cumulants. Here, we proposed a replica trick allowing to calculate at once all the cumulants or, equivalently, the complete distribution function. We mostly applied this trick to the diffusive regime, when the statistics is Gaussian and higher order cumulants are negligible. We believe that the trick could be extended to regimes where the Gaussian statistics breaks down.

The replica trick introduced in this paper can be applied to the evaluation of the probability distribution of other thermodynamic quantities. For instance, the non-linear sigma-model was used to calculate the mesoscopic fluctuations of the supercurrent in metallic Josephson junctions [29]. We would easily find that the statistics of the supercurrent is also Gaussian in the diffusive regime.

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