Observation of subdiffusion of a disordered interacting system

E. Lucioni,* B. Deissler, L. Tanzi, G. Roati, M. Modugno,† M. Zaccanti,‡ M. Inguscio, and G. Modugno§
LENS and Dipartimento di Fisica e Astronomia, Università di Firenze, and CNR-INO
Via Nello Carrara 1, 50019 Sesto Fiorentino, Italy
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We study the transport dynamics of matter-waves in the presence of disorder and nonlinearity. An atomic Bose-Einstein condensate with tunable atom-atom interaction is let free to expand in a quasiperiodic lattice. While the non-interacting system is localized, the presence of a repulsive interaction between the atoms allows a slow expansion with a subdiffusive behavior. The dependence of the diffusion exponent on the system parameters allows us to get an insight on the microscopic dynamics of the expansion process and raises a question about the role of the spatial correlations of the disordered potential.

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The combination of disorder and nonlinearities determines the transport properties of many physical systems, including normal conductors and superconductors [1], graphene [2] and DNA [3], or light in disordered nonlinear media [4–8]. While a full understanding of the interplay of disorder and nonlinearities has long been sought, the incomplete control over experimental parameters in these systems makes systematic investigations of this interplay difficult. There are still several open questions, such as the fate of a wavepacket expanding in a disordered potential in presence of a nonlinearity. There have been various predictions in theory and debate over the results of numerical experiments over the last 20 years about this subject [9–16]. Most authors agree that the nonlinearity should destroy localization and the wavepacket should expand in a way that is slower than normal diffusion. However, it has not been possible so far to observe such subdiffusive behavior in experiments.

Bose-Einstein condensates with tunable interatomic interaction have recently appeared as a very promising system in this respect: for instance, the delocalizing effect of a repulsive interaction was observed by studying the momentum distribution in equilibrium [17–19]. We now employ a Bose-Einstein condensate to study the expansion dynamics of an initially confined system along a one-dimensional quasiperiodic lattice. Despite its large spatial correlations, this kind of potential is known [20] to feature exponentially localized states that are equivalent to those appearing in a lattice with uncorrelated random disorder that is described by the Anderson model [21]. In absence of interaction, a condensate is localized with exponential tails in such potential, and no transport is observed [22]. Adding a controlled interatomic repulsion in the system, we observe a change of shape and a slow increase of the width σ of the sample that follows a subdiffusive law: $\sigma(t) \propto t^{\alpha}$, with $\alpha = 0.2 - 0.4$. We find that the exponent increases with the interaction energy and the localization length, in agreement with both numerical simulations and the predictions of a heuristic model we develop. The observed exponent is however larger than the one calculated for uncorrelated disordered potentials [9–16], suggesting a role of the spatial correlations of the disorder.

The one-dimensional quasiperiodic potential is created by perturbing a primary optical lattice with a weaker secondary lattice of incommensurate periodicity [23]: $V(x) = V_1 \cos^2(k_1 x) + V_2 \cos^2(k_2 x + \phi)$. Here $k_i = 2\pi/\lambda_i$ are the wavevectors of the lattices, which are chosen such that their ratio $\beta = \lambda_1/\lambda_2$ is far from a simple fraction (in our experiment $\lambda_1 = 1064.4$ nm and $\lambda_2 = 859.6$ nm). This potential is characterized by the spacing $d = \lambda_1/2$ and the tunneling energy J of the primary lattice, and by the disorder strength Δ, which scales linearly with V_2 [24]. In the case of non-interacting particles this system constitutes an experimental realization of the Harper or Aubry-André model [20] which shows a transition between extended and localized state for a finite value of the disorder $\Delta/J = 2$ [22].

We employ a Bose-Einstein condensate of ^{39}K atoms in their lowest internal state, whose s-wave scattering length a can be tuned by means of a Feshbach resonance [25, 26]. The condensate is produced in an optical trap at a scattering length $a = 280a_0$, and contains about 3×10^4 atoms. To study the dynamics along the quasiperiodic lattice we first load the interacting condensate into the quasiperiodic lattice with a constant $\Delta \approx 3J$. The trap harmonic potential has a radial (axial) frequency of $2\pi \times 50(70)$ Hz, while the lattice beams give an additional radial potential with frequency $\omega_r = 2\pi \times 50$ Hz. At a given time $t = 0$ the optical trap is suddenly switched off, giving the possibility to the condensate to expand along the lattice; at the same time, the disorder strength and the scattering length are tuned to their final values within 10 ms, and kept there for the rest of the evolution. The subsequent change of the spatial distribution $n(x)$ of the sample is then monitored by in-situ absorption imaging for increasing times, up to $t = 10$ s. The width of the distribution is measured as the square root of its second moment: $\sigma = \sqrt{\int x^2 n(x) dx}$. The initial interaction energy per particle is estimated

as $E_{int} = (2\pi\hbar^2 a/m)\, N \int \varphi^4 d^3x$, where m is the atomic mass, φ is the calculated single-site wavefunction and N is the average atom number per site. We estimate that the initial distribution occupies on average 20±7 sites of the lattice; the uncertainty arises from an incomplete knowledge of the distribution after the preparation procedure, and translates into a 35% uncertainty on E_{int}.

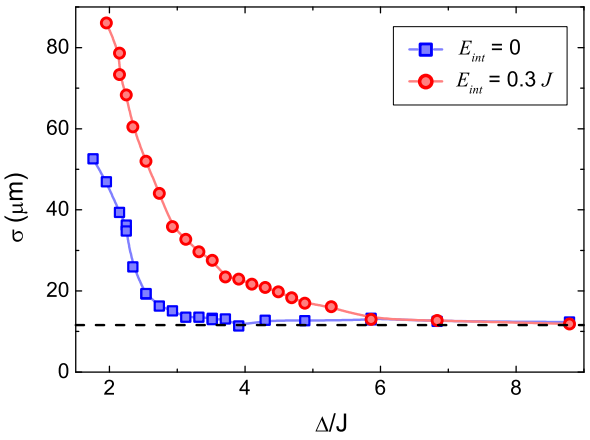

FIG. 1. (color online). Width of the sample after 10 seconds of evolution in the quasiperiodic lattice without interaction (blue squares) and with interaction (red circles). The dashed line indicates the initial size of the sample. The continuous lines are a guide to the eye. The horizontal scale has a 10% uncertainty due to the experimental calibration.

In a first measurement we studied σ after 10 s of evolution, for different values of Δ, see Fig. 1. For negligible interaction we observe that the system becomes almost completely localized at $\Delta/J \approx 3$, as in Ref.[22]. If we add some repulsive interaction, the system becomes localized only at a larger $\Delta/J \approx 6$, indicating that a localized noninteracting system is at least partially delocalized by the interaction.

To understand the nature of the interacting system above the localization threshold, we have then performed a systematic study of the temporal evolution of the width for a wide range of the system's parameters, as shown for instance in Fig. 2. These measurements confirm that the repulsive interaction allows the system to expand, although the expansion is not ballistic but its velocity decreases as the width increases. Note that the change in size is very slow on the timescale given by the tunnelling rate of the lattice, which is $h/J \approx 55$ ms for the data in Fig. 2. To model the expansion dynamics we fit the measured evolution data with

$$\sigma(t) = \sigma_0(1 + t/t_0)^{\alpha}\,, \qquad (1)$$

where σ_0 is the initial width, t_0 is an "activation time" and α is the diffusion exponent. Here, a ballistic expansion corresponds to $\alpha = 1$, while normal diffusion gives $\alpha = 0.5$. For the data in Fig. 2 we measure instead

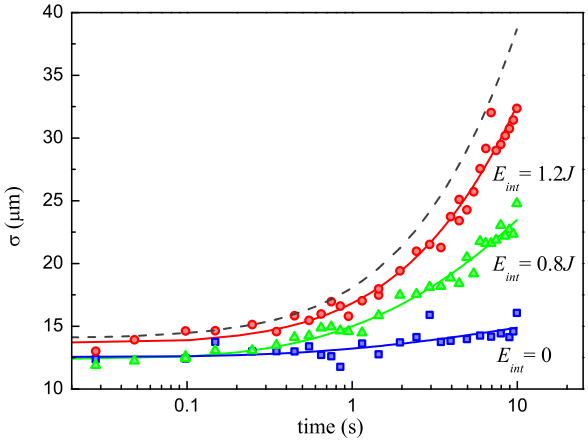

FIG. 2. (color online). Time-evolution of the width σ in the quasiperiodic lattice for different initial interaction energies. The dashed line shows how the cloud would evolve in case of normal diffusion. The continuous lines are the fit with eq.1. The very slow expansion of the noninteracting sample is presumably due to noise. The lattice parameters are $J/h =$ 180 Hz, $\Delta/J = 4.9$.

$\alpha \approx 0.2-0.4$, indicating a subdiffusive behavior. The expansion becomes faster when increasing the interaction energy, suggesting an increased ability of the interaction to delocalize the system. This behavior is confirmed by the systematic study reported in Fig. 3a, where one sees a clear increase of α with the interaction energy E_{int}, followed by a saturation approximately at $\alpha = 0.4$. The related measurement performed at fixed interaction energy in Fig. 3b shows instead that α decreases slightly when increasing Δ, confirming the trend of Fig. 1. We have repeated the experiments for two different depths of the primary lattice, hence for two different J, to check that α is independent of J alone, as expected.

The observed behavior gives hints on the microscopic mechanism of the expansion for $\Delta/J > 3$. In this regime, the single-particle eigenstates of the quasiperiodic lattice are all exponentially localized, with an almost constant localization length $\xi \approx d/\ln(\Delta/2J)$. The interaction breaks the orthogonality of these states, and makes possible the transfer of population from occupied states to initially empty states that are within a few ξ. Since this microscopic transfer happens at a rate Γ that scales with the interaction energy, one can expect that the velocity of expansion decreases as the sample expands and becomes less dense, as we observe in the experiment. Various authors have built heuristic models for the expansion that relate Γ to the variation of the width σ in a variety of random models, such as the kicked rotor [9], the Anderson model and Klein-Gordon oscillators chains [11, 12, 15, 27]. It is possible to derive from basic principles that the rate has a dependence on the number of atoms per state $\Gamma \propto N^3$, where N scales as $1/\sigma$. The idea [12, 15] is then that in a random system $\Gamma\sigma$ is essentially

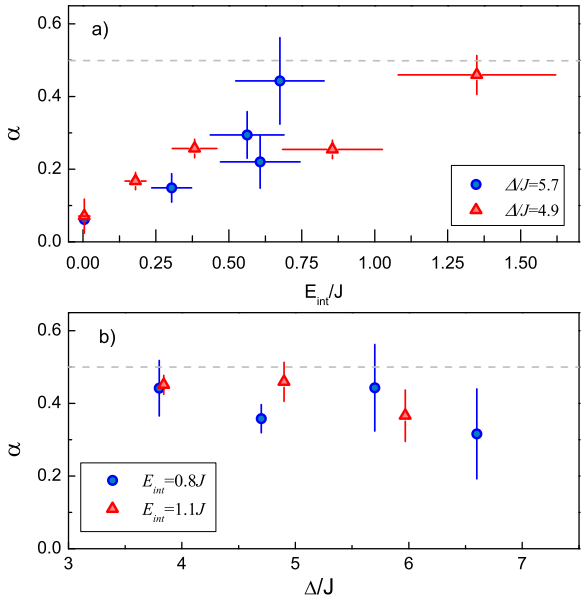

FIG. 3. (color online). Dependence of the diffusion exponent α on: (a) the initial interaction energy E_{int}; (b) the disorder strength Δ, for two different values of the tunneling energy: $J/h = 180$ Hz (triangles) and $J/h = 300$ Hz (circles). The vertical bars are the fitting error of eq.1 to the data, while the horizontal bars in (a) are the statistical error. In (b) there is a 10% uncertainty on the horizontal scale.

the instantaneous diffusion constant D, that is related to σ by $d\sigma^2/dt = D$. The resulting equation $d\sigma^2/dt \propto \sigma^{-2}$ can be easily integrated to get $\sigma(t) \propto t^{\alpha}$, with $\alpha = 1/4$. This result applies to the regime of strong interaction, where many final states can be coupled to the initial ones, otherwise the expected exponent has been derived to be $\alpha = 1/6$ [15].

The exponent α we observe is about twice these values, suggesting that the large correlations of the quasiperiodic lattice might play a role. Actually, all the states of our system have almost the same shape and their energies are arranged in a quasiperiodic way. This might allow the expansion process to proceed through almost coherent hopping processes, in contrast to the incoherent processes expected for uncorrelated random systems [9, 12]. Heuristically, one could then relate the quantity $\Gamma\sigma$ to the instantaneous velocity, i.e. $d\sigma/dt \propto \Gamma\sigma$. With this assumption the model gives an exponent $\alpha = 1/3$ for the regime of strong interaction. This regime is reached when the interaction energy is larger than the standard deviation of the single-particle energies δE. With the aid of numerical calculations we find that for the opposite regime $E_{int} < \delta E$ there is an additional scaling of Γ approximately as E_{int} due to the reduced capability of the interaction to provide an efficient coupling of the states. Since also E_{int} scales as $1/\sigma$, in this regime one gets a smaller exponent around $\alpha = 1/4$, which also decreases with the localization length. This heuristic model is in qualitative agreement with the measured exponents and could explain the increasing trend of Fig.3a. Our observations are also in agreement with numerical simulations performed on a discrete model [13], which give exponents ranging from 0.2 to 0.35.

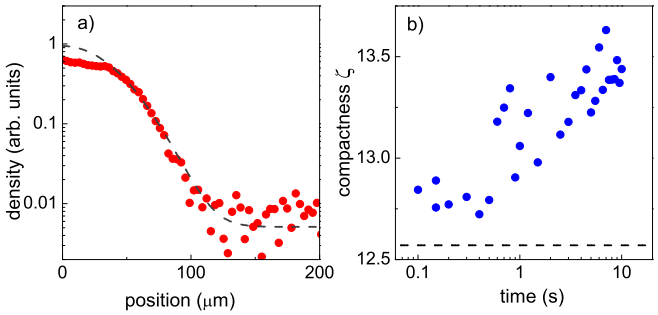

FIG. 4. (color online). a) Shape of an interacting sample after 10 s of evolution; the dashed curve is a gaussian fit of the tails. b) time-evolution of the compactness index ζ; the dashed line is the expected compactness for a gaussian distribution, $\zeta = 4\pi$. For (b) the interaction energy is $E_{int} = 1.2J$, while the lattice parameters are the same of Fig.1.

Since the coupling between localized states is larger at the center of the sample, where E_{int} is large, one might expect a faster depletion at the center than in the tails [9]. This behaviour is clearly visible in the shape of the clouds at a long evolution time shown in Fig. 4a, which features an extended flat top and rapidly decaying tails. Note that the samples initially have an approximately gaussian profile, while the flat-top shape gradually appears during the expansion. This can be seen in the evolution of the so-called compactness [27], which is the ratio of the squared participation number to the second moment, $\zeta = (\int n^2(x)dx)^{-2}/\sigma^2$. The reference value for the compactness is $\zeta = 4\pi$ of a gaussian distribution, while it increases for a distribution with more rapidly decaying tails. As expected, the measured ζ in Fig. 4b shows an increase during the 10 s evolution time.

From a study of the radial momentum distribution we detect a non negligible radial heating during the 10 s evolution time, with a rate of the order of 5-10 nK/s. We attribute it to mechanical noise on the lattice laser beams and also to three-body losses at the largest E_{int}. A question is then about the role of the radial excitation in the axial dynamics. An obvious consequence of the finite radial temperature T is that the on-site radial density of particles decreases, and with that also the interaction energy, as $E_{int}(T) \propto 1/T$. One might then guess that an increase of the temperature rapidly drives the system in the weak interaction regime, where the number of coupled states is vanishingly small. However, it is important to note that the harmonic oscillator energy $\hbar\omega_r$ is considerably smaller than δE (in our case, $\delta E \approx 3J$ and $\hbar\omega_r \approx J/5$). For finite T, the axial dynamics can happen also through excited radial states, which therefore pro-

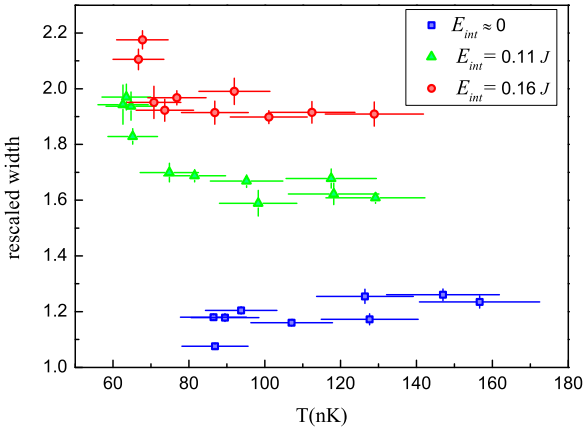

FIG. 5. Evolution of the width after 10 s, rescaled by the initial width, for an increasing radial temperature of the sample, for three different values of the scattering length. The datasets are labeled with E_{int} at the lowest temperature. The typical decrease of E_{int} for the two interacting datasets from the lowest to the highest T is a factor of 3. The error bars are the statistical deviation of about 10 independent measurements. The lattice parameters are $J/h = 290$ Hz, $\Delta/J = 3.9$.

vide a smaller energy scale to be compensated by the interaction energy. This implies that also axial states with $\delta E > E_{int}$, which would not be efficiently coupled in absence of the radial degree of freedom, can participate in the axial diffusion process. Qualitatively, we would then expect that the characteristic velocity does not decrease significantly for increasing temperature.

We have checked this expectation in the experiment, by studying the expansion of samples that were intentionally heated in the radial directions by parametric excitation. Fig. 5 shows that, while an increasing T does not affect the expansion of a non-interacting sample, as expected, it causes a decrease of the expansion velocity of interacting samples. In this case we typically observe an initial decrease of $\sigma(t{=}10$ s), followed by a plateau up to the highest temperatures we can reach. The presence of this plateau is not surprising, since in our case $\delta E/k_B \approx 45$ nK and for temperatures sufficiently above this characteristic value most of the excited radial states can participate to the axial motion. A more detailed modeling of the finite temperature dynamics is left to a future work.

In conclusion, our study gives evidence of the subdiffusive character of the expansion of a disordered interacting system and elucidates the microscopic mechanism of such expansion. This is a further evidence of the delocalizing role of a repulsive interaction in a disordered system [17]. It would be now interesting to extend our study to less correlated disorder to quantify the effect of spatial correlations of the disorder in the dynamics. Also, we could drive our system into the quasi-1D regime by increasing the radial confinement to freeze out the radial degrees of freedom. There one could study the predicted temperature-induced metal-insulator transition [28], or investigate the role of quantum fluctuations in the dynamics of a disordered system, both in the weakly and strongly interacting regimes [29, 30].

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* lucioni@lens.unifi.it
† Permanent addresses: Department of Theoretical Physics and History of Science, UPV-EHU, 48080 Bilbao, Spain; IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain
‡ Present address: Institut für Quantenoptik und Quanteninformation, Innsbruck, Austria
§ modugno@lens.unifi.it

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