Fractionalization in Three-Components Fermionic Atomic Gases in a One-Dimensional Optical Lattice

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We study a three-components fermionic gas loaded in a one-dimensional optical trap at half-filling. We find that the system is fully gapped and may order into 8 possible phases: four $2k_F$ atomic density wave and spin-Peierls phases with all possible relative π phases shifts between the three species. We find that trionic excitations are unstable toward the decay into pairs of kinks carrying a fractional number, Q=3/2, of atoms. These sesquions eventually condense upon small doping and are described by a Luttinger liquid. We finally discuss the phase diagram of a three component mixture made of three hyperfine level of $^6\mathrm{Li}$ as a function of magnetic field.

PACS numbers: 03.75.Mm, 71.10.Pm, 71.10.Fd,

A problem analogue to quark confinement in particle physics has been recently addressed in systems with multi-components fermionic atoms loaded in an optical lattice¹⁻¹⁰. These studies strongly support the formation of "baryonic" states made of bound states of n > 2atoms. In one dimension, for example, trionic states, made of bound state of three atoms, were predicted to be stable at generic densities and sufficiently low temperatures, typically of the order $\sim 30-100 \text{nK}^{11}$. This result opens the exciting possibility to probe in a new context and in future experiments a simplified version of the "quark" confinement phenomenon in quantum chromodynamics. In all these previous studies the attention has been drawn on the formation of baryonic (or molecular) states that contains an integer number of atoms. Here we shall focus on the intriguing situation where the lowenergy elementary excitations carry a fractional number of atoms. Although it may appears counter-intuitive, fractionalization of quantum numbers is a well established phenomenon in condensed matter physics. Celebrated examples are fractionally charged excitations in the quantum Hall state¹² and in quasi-one-dimensional polymers¹³. In this work we shall present strong arguments that fractionalization can also occur in ultra-cold atomic physics. We shall give evidences that at densities close to half-filling a three-components fermionic mixture loaded in a one-dimensional optical trap may support low-energy excitations carrying a fractional number, Q=3/2, of atoms, the sesquions.

When loaded in a one-dimensional optical lattice of wavelength λ , a three-components mixture is well described, away from resonance, by a Hubbard-type hamiltonian of the form¹⁴:

$$\mathcal{H} = -t \sum_{i,a} \left[c_{i,a}^{\dagger} c_{i+1,a} + \text{H.C.} \right] + \sum_{i,a < b} U_{ab} \rho_{i,a} \rho_{i,b} \quad (1)$$

where $c_{i,a}^{\dagger}$ is the creation operator for a fermionic atom of species a=(1,2,3), at site i, and $\rho_{i,a}=c_{i,a}^{\dagger}c_{i,a}$ is the local density of the atomic species a. The parameters t and the couplings U_{ab} can be expressed in term

of the recoil energy, the laser intensity and wavelength as well as the s-wave scattering lengths $s_{ab}(B)$ between the species. For generic external magnetic fields B, the s_{ab} are in general different and so are the couplings U_{ab} . Thus the physical symmetry group of (1) is $U(1)^3$ corresponding to the conservation of the number of atoms of each species. Such a small symmetry, which is an essential feature of atomic mixtures, make the elucidation of the physics associated with (1) a difficult task. However, as we shall see, much can be said in the weak coupling, low-energy, limit. The physics described by (1) strongly depends on the density of atoms $\bar{\rho} = \bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}_3$. Away from half-filling, i.e. when $\bar{\rho} \neq 1/2$, it was shown in Ref. 11 that for generic couplings the dominant fluctuations consists into massless $2k_F$ Atomic Density Waves (ADW) and massless trionic excitations carrying total atomic number Q=3. At half filling, when $\bar{\rho}=1/2$, the physics is, as we shall see, radically different.

Effective Low Energy Hamiltonian. The low energy effective theory associated with the Hubbard Hamiltonian (1) can be derived, as usual, from the linearization at the two Fermi points $\pm \mathbf{k}_F$ of the dispersion relation of free three-component fermions:

$$c_{i,a} \sim \Psi_{aR} e^{ik_F x} + \Psi_{aL} e^{-ik_F x} \quad a = (1, 2, 3), \quad (2)$$

where $x=ia_0$, $a_0=\lambda/2$ is the lattice spacing, λ the laser wavelength, and $k_F=\pi\bar{\rho}/a_0$ is the Fermi wave-vector. Finally $\bar{\rho}=1/2$ is the density per species. In the weak coupling limit $|U_{ab}|/t << 1$ the effective hamiltonian associated with (1) is found to be:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{I}} \tag{3}$$

with

$$\mathcal{H}_0 = -iv_F \sum_a (\Psi_{aR}^{\dagger} \partial_x \Psi_{aR} - \Psi_{aL}^{\dagger} \partial_x \Psi_{aL}) \tag{4}$$

and

$$\mathcal{H}_{I} = \sum_{a < b} \left(\mu_{ab} \ h_{aR} h_{bL} + \lambda_{ab}^{-} I_{abR}^{\dagger} I_{abL} + \lambda_{ab}^{+} J_{abR}^{\dagger} J_{abL} \right)$$

$$+ (R \leftrightarrow L)$$

$$(5)$$

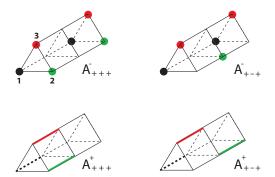


FIG. 1: Generalized atomic density wave and spin-Peierls phases. Atomic species are labelled 1,2 and 3. Among the 8 possible phases A_{+++}^{\pm} , A_{++-}^{\pm} , A_{+-+}^{\pm} , A_{-++}^{\pm} four are represented. In the A_{+-+}^{\pm} phase the species 1 and 3 are in phase and out of phase with the species 2.

where

$$h_{aR(L)} = \Psi_{aR(L)}^{\dagger} \Psi_{aR(L)}, I_{abR(L)}^{\dagger} = \Psi_{aR(L)}^{\dagger} \Psi_{bR(L)}, \quad (6)$$

and

$$J_{abR(L)}^{\dagger}=\Psi_{aR(L)}^{\dagger}\Psi_{bR(L)}^{\dagger}. \tag{7}$$

The fermi velocity is given by $v_F = 2t$, $\mu_{ab} = U_{ab}$, $\lambda_{ab}^{\pm} = \pm U_{ab}$ and we have omitted a term $\sum_{a \leq b} (h_{aR} h_{bR} +$ $h_{aL}h_{bL}$) that account for a non-uniform velocity renormalization. In the absence of the λ^+ -term we recover the hamiltonian studied in Ref. 11 in the $\bar{\rho} \neq 1/2$ case which involves the 9 $U(3)|_{R(L)}$ currents $\mathcal{J}_{\parallel R(L)} \equiv$ $(h_{aR(L)}, I_{abR(L)}^{\dagger}, I_{abR(L)})$. The λ^+ -term is a $4k_F$ contribution of the density-density interaction among the species and is present only at half-filling. It involves the 6 currents $\mathcal{J}_{\perp R(L)} \equiv (J_{abR(L)}^{\dagger}, J_{abR(L)})$ that create or destroy pairs of atoms. Together with the $U(3)|_{R(L)}$ currents they generate the 15 $SO(6)|_{R(L)}$ currents $\mathcal{J}_{R(L)} =$ $\mathcal{J}_{\parallel R(L)} \oplus \mathcal{J}_{\perp R(L)}$. The non interacting part of (3), \mathcal{H}_0 , is that of relativistic free fermions and has the maximally available symmetry $SO(6)_R \otimes SO(6)_L$ generated by the $\mathcal{J}_{R(L)}$'s. The interaction hamiltonian breaks the later symmetry down to $U(1)_R^3 \otimes U(1)_L^3|_{\text{diag}}$ corresponding to the conservation of the number of atoms of each species a. Though the lattice hamiltonian (1) depends on three couplings U_{ab} the hamiltonian (3) is the most general hamiltonian for the three species problem with an $U(1)^3$ symmetry and one has to consider the role of the 15 couplings $(\lambda_{ab}^{\pm}, \mu_{ab})$ that encode all possible competing orders. Which one is likely to be stabilized in the low energy limit depends on the asymptotic behavior of the Renormalization Group (RG) flow.

Renormalization Group Analysis. We have obtained the one-loop RG equations associated with (3). They will be given elsewhere²¹ and we shall only present in the following our results. Due of the lack of symmetry in the problem, and consequently of the relatively large number of independent couplings $(\lambda_{ab}^{\pm}, \mu_{ab})$, it may

seems an akward task to draw any general conclusions on the phase diagram associated with (3). Fortunately, it has been recognized that due to the importance of strong quantum fluctuations in one dimensional systems the symmetry at level of the lattice spacing a_0 is likely to be enlarged at low energies which thus considerably simplify the problem. Phrased in the RG language such a Dynamically Symmetry Enlargement (DSE) correspond to a situation where the hamiltonian (3) is attracted under the RG flow toward an effective hamiltonian H^* with a higher symmetry. As shown in Ref. 17 the possible DSE fixed points H^* depend only on the symmetry breaking pattern described by (3). In the present case we find that for generic initial conditions of the RG flow, $(\lambda_{ab}^{\pm}, \mu_{ab})$, the low-energy physics associated with (3) is described by one of the fixed points hamiltonians:

$$\mathcal{H}_{\epsilon_{1}\epsilon_{2}\epsilon_{3}}^{\pm} = -iv \sum_{a} (\Psi_{aR}^{\dagger} \partial_{x} \Psi_{aR} - \Psi_{aL}^{\dagger} \partial_{x} \Psi_{aL})$$

$$\pm G \left(\sum_{a} \epsilon_{a} (\Psi_{aR}^{\dagger} \Psi_{aL} \mp \Psi_{aL}^{\dagger} \Psi_{aR}) \right)^{2}$$
(8)

where v is a renormalized velocity, $\epsilon_a = \pm 1$, and G is some positive coupling. As (8) is invariant under the simultaneous change $\epsilon_a \to -\epsilon_a$, there are 8 independent fixed points, with $(\epsilon_1\epsilon_2\epsilon_3) = (+++), (-++), (+-+)$ and (++-), that describe phases, labelled $A^{\pm}_{\epsilon_1\epsilon_2\epsilon_3}$, with qualitativally different physical properties. The phases $A^+_{\epsilon_1\epsilon_2\epsilon_3}$ and $A^-_{\epsilon_1\epsilon_2\epsilon_3}$, are generalized $2k_F$, Spin-Peierls (SP) and Atomic Density Wave (ADW) phases. The ϵ_a account for all possible relative π -phase shifts between the species a. A pictorial representation of the ground states is presented in Fig.1. The corresponding lattice order parameters can be readily obtained from the structure of the interacting part of (8) and are given by:

$$\mathcal{O}_{\epsilon_{1}\epsilon_{2}\epsilon_{3}}^{+} = \sum_{a} \frac{(-1)^{i}}{2} \epsilon_{a} (c_{ai}^{\dagger} c_{ai+1} + \text{h.c})$$
 (9)
$$\mathcal{O}_{\epsilon_{1}\epsilon_{2}\epsilon_{3}}^{-} = \sum_{a} \frac{(-1)^{i}}{2} \epsilon_{a} (c_{ai}^{\dagger} c_{ai} + \text{h.c}).$$

In each phase the ground state is doubly degenerated and when $\langle \mathcal{O}_{\epsilon_1\epsilon_2\epsilon_3}^{\pm} \rangle \neq 0$ there is spontaneous symmetry breaking of translational invariance by one lattice site. As a consequence we expect kinks (or solitonic) excitations that interpolate between the two ground states to be present in the spectrum. As we shall see these have fractional quantum numbers.

Spectrum and Fractionalization. Remarkably enough, the different hamiltonians (8) can be brought to the same form by mean of duality transformations¹⁷:

$$\mathcal{H}_{\epsilon_1 \epsilon_2 \epsilon_3}^{\pm}(G, \Psi) = \mathcal{H}_{+++}^{+}(G, \omega_{\epsilon_1 \epsilon_2 \epsilon_3}^{\pm}(\Psi)), \tag{10}$$

where the duality transformations ω act only on the right-moving fermions as:

$$\omega_{\epsilon_1 \epsilon_2 \epsilon_3}^{\pm}(\Psi_{aR}) = \mathrm{e}^{i\frac{\pi}{4}(1\mp 1)} \; \epsilon_a \Psi_{aR}. \tag{11}$$

We therefore find that the elucidation of the lowenergy physics described by the fixed points hamiltonians $\mathcal{H}_{\epsilon_1 \epsilon_2 \epsilon_3}^{\pm}$ stem from the knowledge of those of H_{+++}^{+} . The latter hamiltonian has an enlarged SO(6) symmetry generated by the $\mathcal{J}^A = \int dx (\mathcal{J}_R^A + \mathcal{J}_L^A), A = (1,...,15),$ and is that of the SO(6) Gross-Neveu (GN) model. The other fixed points hamiltonians (8) has a dual extended symmetry $\tilde{S}O(6)$ generated by the dual currents $\tilde{\mathcal{J}}^A = \int dx (\omega_{\epsilon_1 \epsilon_2 \epsilon_3}^{\pm}(\mathcal{J}_R^A) + \mathcal{J}_L^A)$. Fortunately, the SO(6) GN model is integrable¹⁵ so that its spectrum is exactly known and by duality the one in the other phases $A^{\pm}_{\epsilon_1\epsilon_2\epsilon_3}$. In all cases it consists into two set of four kinks, $S^{\pm}_{\alpha=0,1,2,3}$, of mass $m_S \sim t \mathrm{e}^{-t/U}$ (*U* being a characteristic energy scale), belonging to the two spinorial representations of SO(6) (or $\tilde{S}O(6)$) and six real (Majorana) fermions $\xi_{\beta=1,\dots,6}$ of mass $m_F=\sqrt{2}~m_S$ transforming according to the vectorial representation of SO(6) (or SO(6)). Though their wave functions are different in all the phases $A_{\epsilon_1\epsilon_2\epsilon_3}^{\pm}$, these particles are described by the same quantum numbers since the duality transformations (11) do not affect the 3 conserved charges (or Cartan generators):

$$q_a = \int dx \; (\Psi_{aR}^{\dagger} \Psi_{aR} + \Psi_{aL}^{\dagger} \Psi_{aL}), \; a = (1, 2, 3).$$
 (12)

These are nothing but the total number of atoms of a given species a and the particles of the spectrum are labelled by the set of quantum numbers (q_1,q_2,q_3) . The kinks quantum numbers are $fractional: S_0^{\pm} = \pm 1/2(1,1,1), S_1^{\pm} = \pm 1/2(1,-1,-1), S_2^{\pm} = \pm 1/2(-1,1,-1)$ and $S_3^{\pm} = \pm 1/2(-1,-1,1)$. The fermions are bound states of two kinks and have the same quantum numbers as the original lattice fermions: $\xi_{\beta} = (\pm 1,0,0), (0,\pm 1,0)$ and $(0,0,\pm 1)$. There are no other stable particles. In particular there are no stable trions in contrast with what happens at incommensurate fillings. Trionic excitations, $T^{\dagger} = c_1^{\dagger}c_2^{\dagger}c_3^{\dagger}$, have total atomic number Q=3, where

$$Q = q_1 + q_2 + q_3, (13)$$

and are unstable toward the decay into elementary kinks or fermions. A trion has quantum numbers (1,1,1) and the most energetically favorable process is $T^{\dagger} \to S_0^+ S_0^+$ so that one may think of the kink S_0^+ as "half" a trion. As it has total atomic number Q=3/2 one may call it a sesquion. The existence of these fractional kinks as the lowest energy excitations in generic three-components ultra-cold atomic systems is an unexpected and nontrivial finding and constitute one of the main results of the present work. It is therefore worth discussing the stability of the above excitations. Indeed, as noticed in both Refs. (16,17), the DSE is only approximate and we expect residual symmetry breaking operators to survive even in the low-energy limit. As the SO(6) GN particles are labeled by the conserved quantum numbers (12) associated with the $U(1)^3$ symmetry of the problem, small

residual anisotropy will result only into a small splitting of the particle spectrum. For large enough anisotropy and/or strong couplings it may eventually happens that the above description of the spectrum breaks down. We expect however that the DSE description of the hamiltonian (1) (and hence the stability of the sesquions) holds in a relatively large portion of the phase diagram. Indeed the accuracy of the DSE description has been checked numerically in the different context of the SU(4) Hubbard model at half-filling where the adiabatic continuity of the SO(8) GN spectrum in this case has been explicitly observed for small enough interactions ¹⁸.

Doping. To model small doping we consider adding a chemical potential term $H_Q = -\mu Q$ (we consider hole doping with $\mu > 0$). As Q is invariant under the duality transformation (11) it is sufficient to consider doping the SO(6) Gross-Neveu model. The chemical potential term breaks the SO(6) symmetry but since $[H_{+++}^+, Q] = 0$ doping does not spoil integrability and the following picture emerges. At non zero μ , the particle spectrum is splitted according to the values of Q. A particle with mass m and atomic number Q will lower its energy to $m-\mu Q$. When its energy becomes negative the ground state start to fill with these particles. When $\mu > 2m_S/3$ the first particle that start filling the ground state is the sesquion S_0^+ of mass m_S and Q=3/2. As μ is increased further other particles would like to enter the ground state like other members of the kinks multiplets with Q = 1/2 or the fermions with Q = 1. However the increase of the chemical potential is counteracted by the repulsion felt by the kinks and the fermions to the sesquions^{19,20}. As a result for $\mu > 2m_S/3$ the ground state is only filled by sesquions which become massless excitations. The effective theory describing these massless fractional Q = 3/2 excitations is a Luttinger liquid with a stiffness K. At these dopings the kinks $S_{\alpha=1,2,3}^{\pm}$ and the fermions ξ_{β} remain massive, with renormalized masses. Both the renormalized masses and the stiffness K could be in principle computed from the Bethe ansatz solution in a similar way as done in Ref. 19. At large doping, i.e. when $\mu >> m_S$, the $4k_F$ term λ^+ in (5) decouples and one recovers the physic described above for the generic filling case with massless Q=3 trionic excitations. We may therefore expect that below some critical value of the density $\bar{\rho} < \bar{\rho}_c$ sesquions get confined into trions. The elucidation of the nature as well as the location of such a confinement/deconfinement transition goes beyond the scope of this work and will studied elsewhere 21 .

Three-Species Problem and Experiments. The phase diagram in the three-dimensional space $(U_{12}/t, U_{23}/t, U_{31}/t)$ is rich and complex, revealing the delicate balance between the different competing orders. We find that among the 8 possible phases only 5 are stabilized: the four ADW phases $A_{\epsilon_1\epsilon_2\epsilon_3}^-$ and the uniform SP phase A_{+++}^+ . We show in Fig.2. two projections of the phase diagram in the $(U_{23}/t, U_{31}/t)$ plane for typical value of $U_{12}/t = \pm 0.5$. Though it is

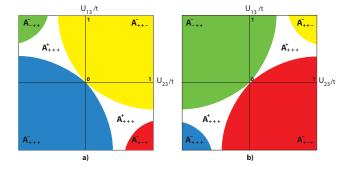


FIG. 2: 2D projections of the weak coupling phase diagram of the anisotropic Hubbard model for fixed values of U_{12}/t : a) $U_{12}/t = -0.5$ and b) $U_{12}/t = 0.5$. The SP phase A_{+++}^+ occurs in the frustrated region (i.e. when $(\Pi_{a < b} \ U_{ab}) > 0$) around the isotropic points $|U_{ab}| = U$.

difficult to draw any general quantitative picture of the phase diagram we observe that: i) in the unfrustrated regions, where $(\Pi_{a < b} U_{ab}) < 0$, the ADW phases that are stabilized are the one that minimize the density-density potential in (1) and ii) that in the frustrated regions, when $(\Pi_{a < b} U_{ab}) > 0$, the kinetic energy term play an important role when all couplings are of the same order of magnitude. In particular in the vicinity of the isotropic rays $|U_{ab}| = U$ a uniform SP phase A_{+++}^+ is likely to be stabilized. Eventually the above SP phase is destabilized in favor of various ADW phases for sufficiently large anisotropies. In experiments once the optical lattice parameters and the density are fixed the only control parameter is the external magnetic The phase diagram as a function of the field B. magnetic field B is a line in the three-dimensional space $(U_{12}(B)/t, U_{23}(B)/t, U_{31}(B)/t)$ which dependence on B essentially depends on the mixture through the s-wave scattering lengths. Taking for example 10 a mixture made of a balanced population of three hyperfine states of

⁶Li atoms, $|F, m_F\rangle = |1\rangle = |1/2, 1/2\rangle, |2\rangle = |1/2, -1/2\rangle,$ and $|3\rangle = |3/2, -3/2\rangle$, with typical optical lattice parameters¹¹ and a laser wavelength $\lambda = 1 \mu m$, we find a weak coupling regime where a non trivial SP phase may be observed. At half-filling using our one loop RG equations the following phase diagram as a function of the magnetic field emerges. For small fields $B < B_{c1}$, a uniform ADW phase A_{+++}^- is stabilized while at larger fields $B > B_{c2}$ an ADW phase A_{-++}^- , where the species labeled 1 is in phase opposition with species 2 and 3, shows off. This match the result of Ref. 11 for $\bar{\rho} \neq 1/2$ where at these filings ADW phases of the same type with quasi-long range order were predicted. The essential difference with the above case is that when $\bar{\rho} = 1/2$ an intermediate uniform SP phase $A_{\perp\perp\perp}^+$ is locked in the region $B_{c2} < B < B_{c1}$. Within the one loop accuracy we find $B_{c2} \sim 560 \text{G}$ and $B_{c1} \sim 540 \text{G}$. This is an interesting result from the experimental point of view since in both A_{+++}^+ and A_{-++}^- phases we expect that effect of the three-body losses¹¹ will be considerably reduced. The knowledge of actual values of the binding energies of the sesquions and hence of the temperature scale below which these phases could be stabilized call for an alternative approach such like numerical calculations²¹. To summarize we have shown that in the vicinity of half-filling fractional excitations carrying Q = 3/2 atoms, the sesquions, are the relevant low energy excitations in a generic three-components Fermi mixture. These sesquions are likely to get confined into Q = 3 trionic excitations when one moves sufficiently far away from half-filling. We therefore expect that both the confined (trionic) and unconfined (sesquionic) phases could be probed in future experiments.

We thank E. Boulat, S. Capponi, V. Dubois, P. Lecheminant and M. Najac for discussions and encouragements.

P. Lecheminant, E. Boulat, and P. Azaria, Phys. Rev. Lett. 95, 240402 (2005).

² C. J. Wu, Phys. Rev. Lett. **95**, 266404 (2005).

³ H. Kamei and K. Miyake, J. Phys. Soc. Jpn. **74**, 1911 (2005).

⁴ A. Rapp *et al.*, Phys. Rev. Lett. **98**, 160405 (2007); A. Rapp, W. Hofstetter, and G. Zaránd, Phys. Rev. B **77**, 144520 (2008).

⁵ S. Capponi *et al.*, Phys. Rev. A **77**, 013624 (2008).

⁶ X.-J. Liu, H. Hu, and P. D. Drummond, Phys. Rev. A 77, 013622 (2008).

⁷ X. W. Guan *et al.*, Phys. Rev. Lett. **100**, 200401 (2008).

⁸ G. Roux et al., Eur. Phys. J. 68, 293 (2009)

 $^{^9\,}$ R. A. Molina et al. arXiv: 0807.1886.

¹⁰ T. B. Ottenstein *et al.*, Phys. Rev. Lett. **101**, 203202 (2008).

¹¹ P. Azaria, S. Capponi and P. Lecheminant, Phys. Rev. A 80, 041604(R) (2009).

¹² R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1982).

¹³ W.P. Su, J.R. Schrieffer, and A.J. Heeger, Phys. Rev. Lett. 42, 1698 (1979), R. Jackiw and C. Rebby, Phys. Rev. D 13, 3398 (1976)

¹⁴ D. Jaksch and P. Zoller, Ann. Phys. (N.Y.) **315**, 52 (2005).

¹⁵ N. Andrei and J. H. Lowenstein, Phys. Lett. B **90**, 106 (1980).

¹⁶ H.H. Lin, L. Balents and M. Fisher, Phys. Rev. B 58, 1794 (1998).

¹⁷ E. Boulat, P. Azaria and P. Lecheminant, Nucl. Phys. B 822, 367 (2009).

¹⁸ R. Assaraf et al, Phys. Rev. Lett. 93, 016407 (2004)

¹⁹ R. Konik *et al*, Phys. Rev. Lett. 42, 1698 (1998)

²⁰ J. Evans and T. Hollowood, Nucl. Phys. Proc. Suppl. 45A, 130 (1996).

²¹ P. Azaria, S. Capponi and H. Nonne, in preparation.