# Comments on Degrees of freedom region for $K$-user interference channel with $M$ antennas

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**Abstract**

For a $K$-user interference channel with $M$ antenna at each transmitter and each receiver, the maximum total DoF of this channel has been previously determined to be $\max \sum_{k=1}^{K} d_k = MK/2$. However, the DoF region remains to be unknown. In this short note, through a simple time-sharing argument, we obtain the degrees of freedom (DoF) region of this channel.

**Index Terms**

Interference alignment, Degrees of freedom region,interference channel

Consider a $K$-user interference channel with $M$ antennas at each transmitter and each receiver, the same as in [1]. Let $d_k$ denote the degrees of freedom (DoF) of user $k$, $k = 1, \ldots, K$. The maximum total DoF $\max \sum_{k=1}^{K}$ has been found in [1] to be $MK/2$. We have the following result regarding the DoF region.

*Theorem 1:* The degrees of freedom region of the $K$-user interference channel with $M$-antennas at both receivers and transmitters is characterized as follows :

$$\mathcal{D} = \{(d_1, d_2, \cdots, d_K) : \quad d_i + d_j \leq M, \forall 1 \leq i, j \leq K, i \neq j\}. \tag{1}$$

*Proof:* The converse argument is the same as the converse argument in [1, Theorem 1]. We show the achievability as follows.

Without loss generality, suppose $d_1^* \geq d_2^* \geq d_k^*, k = 3, \cdots, K$, and $d_i^* + d_j^* \leq d_1^* + d_2^* \leq M$, $\forall i, j \in [1, K]$. We would like to show that $(d_1, d_2, \ldots, d_M) = (d_1^*, d_2^*, \ldots, d_M^*)$ is achievable.

It is obvious that

$$(d_1, d_2, \ldots, d_K) = (M, 0, \ldots, 0)$$

can be achieved by single user transmission. It is also known from [1] that the point

$$(d_1, d_2, \ldots, d_K) = (M/2, M/2, \ldots, M/2)$$

is achievable. Trivially, the point

$$(d_1, d_2, \dots, d_K) = (0, 0, \dots, 0)$$

is achievable.

By time sharing, with weights $(d_1 - d_2)/M$, $2d_2/M$ and $1 - d_1/M - d_2/M$ among the three points, in that order, it follows that the point

$$(d_1, d_2, \dots, d_K) = (d_1^*, d_2^*, d_2^*, \dots, d_2^*)$$

is achievable. This is already no smaller than the DoF we would like to have.

■

## REFERENCES

[1] V. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the $k$-user interference channel," *IEEE Trans. On Information Theroy*, , no. 5, pp. 3425–3440, 2008.