Non-Abelian Vortex in Graphenelike Systems?

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Abstract

The Abelian Higgs model is an appropriate field theoretic framework for the description of superconductivity and its topological excitations(Nielson-Olesen-Abrikosov vortex). Recently, C.-Y. Hou etl [Phys. Rev. lett.**98** 186809 (2007)]. show that such excitations exist in graphene structures. In this paper, we show that a pseudo non-Abelian Higgs field can also be generated in graphenelike systems. Introducing a non-Abelian gauge field, we extend the chiral gauge theory for graphene constructed by R. Jackiw and S.-Y. Pi [Phys. Rev. Lett. **98**, 266402 (2007)] to the non-Abelian regime.

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I. INTRODUCTION

Graphene, despite its short history, has attracted considerable interest due to its chiral Dirac-like electronic excitations, which leads to many unusual phenomena as the half-integer quantum Hall effect even at room temperature[1]. Accordingly, it is possible to observe exotic quantum electrodynamics (QED) effects, such as Klein tunneling in graphene. On the other hand, for its 2 dimensional configuration, the behavior of the excitations differs in interesting ways from the standard behavior we are used to in classical and QED.

Recently, It was showed that fractional charge bound to a vortice can be generated in graphene with a Kekulé distortion.[2] Furthermore, R.Jackiw and S.-Y.Pi proposed identifying this kind of vortex with the Nielsen-Olesen-Abrikosov (NOA) vortex, which is a topological excitation in Abelian Higgs model.[3] A relevant gauge potential, coupling to the vortex, was also introduced to describe the vortex dynamics. Thereby the whole system becomes invariant under chiral gauge transformation.

By analogy with high energy physics, this kind of physics can be described by massless fermions interacting with the Abelian Higgs particles. An immediate question will be whether it is possible to extend to an non-Abelian regime. While it seems plausible to explore such an analog in condensed matter system, but as optics techniques' development, synthesizing a non-Abelian regime by means of laser-assisted ultracold atoms, tunneling in optical lattices, has been explored.[4][5]

In this paper we construct a non-Abelian chiral gauge theory for an unusual honeycomb lattice system. By analog with the non-abelian Higgs model frequently discussed in high energy physics, we show that topological excitation could also be generated in this novel system.

The tight-binding Hamiltonian, for fermions with a uniform hopping constant t, is given by

$$H = -t \sum_{\mathbf{r}\in\Lambda_A} \sum_{j=1,2,3} a_\tau^\dagger(\mathbf{r}) b_\tau(\mathbf{r}+\mathbf{u}_j) + H.c. \tag{1}$$

where $a_\tau(a_\tau^\dagger)$ annihilates (creates) with spin τ on sublattice $\mathbf{A}$ (an equivalent consideration is for $b_\tau(b_\tau^\dagger)$) and the spin index is not written out explicitly. This Hamiltonian can be diagonalized in momentum space. After linearization around the Dirac points $\mathbf{K}_\pm$, we would obtained the familiar Dirac type Hamiltonian.

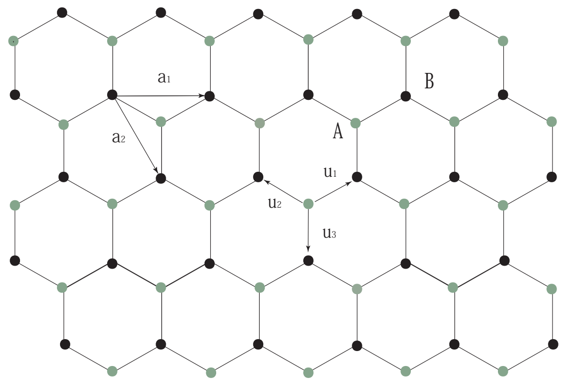

Figure 1: Honeycomb lattice is composed of two interpenetrating triangular lattices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{a}_1$ and $\mathbf{a}_2$ are their unit basis vectors; $\mathbf{u}_j$ are the nearest neighbor vectors.

When we take into account a small perturbation in the hopping constant, such a extra term results from lattice distortion or the laser-induced cold atom tunneling. Hamiltonian then becomes

$$H = -\sum_{\mathbf{r}\in\Lambda_A}\sum_{j=1,2,3}(t\delta_{\tau'\tau} + \delta t_{\mathbf{r},j,\tau',\tau})a^{\dagger}_{\tau'}(\mathbf{r})b_{\tau}(\mathbf{r}+\mathbf{u}_j) + H.c. \tag{2}$$

By similar consideration as Ref.[2][5], we propose a spin dependent Kekulé mechanism, both the chiral degree and spin degree are coupled in this mechanism. The perturbation hopping matrix is

$$\delta t_{\mathbf{r},j,\tau',\tau} = \Delta_{\tau'\tau}(\mathbf{r})e^{iK_+\cdot\mathbf{u}_j}e^{i\mathbf{G}\cdot\mathbf{r}}/3 + \bar{\Delta}_{\tau'\tau}(\mathbf{r})e^{iK_-\cdot\mathbf{u}_j}e^{-i\mathbf{G}\cdot\mathbf{r}}/3 \tag{3}$$

with coupled wave vector $\mathbf{G} := \mathbf{K}_+ - \mathbf{K}_-$. To this end, we argue that $\Delta_{\tau'\tau}(\mathbf{r})$ is a spatial modulation of hopping combined with local transformation in fermion spin space. The Hamiltonian subjected to spin dependent Kekule hopping is

$$H = \int d^2\mathbf{r}\psi^{\dagger}(\mathbf{r})\mathcal{K}_D\psi(\mathbf{r}) \tag{4}$$

$$\mathcal{K}_D = \begin{pmatrix} 0 & -2i\partial_z & \Delta(\mathbf{r}) & 0 \\ -2i\partial_{\bar{z}} & 0 & 0 & \Delta(\mathbf{r}) \\ \bar{\Delta}(\mathbf{r}) & 0 & 0 & 2i\partial_z \\ 0 & \bar{\Delta}(\mathbf{r}) & 2i\partial_{\bar{z}} & 0 \end{pmatrix} \tag{5}$$

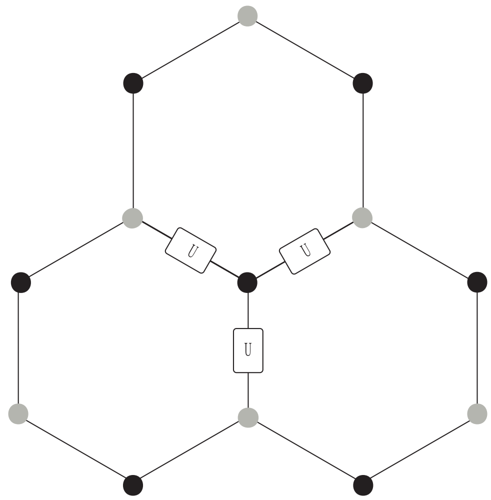

Figure 2: Non-Abelian honeycomb lattice, the paths between nearest sites have been dressed by a $U(k)$ or $SU(k)$ transformation.

with

$$\Delta(\mathbf{r}) = \begin{pmatrix} \Delta_{11}(\mathbf{r}) & \Delta_{12}(\mathbf{r}) \\ \Delta_{21}(\mathbf{r}) & \Delta_{22}(\mathbf{r}) \end{pmatrix}$$

If we set $\Delta(\mathbf{r}) = \Delta_0 I$, which means hopping is spin independent, this simply gives a mass gerneration. However, if we could control the hopping matrix in terms of $SU(2)$ generators$\Delta(\mathbf{r}) = \phi = gT^a\phi^a$,$T^a = \frac{1}{2}\sigma^a$, something interesting would happen. We can identify this term with a coupling term for fermions mediating tough a (pseudo) non-Abelian Higgs field. Particularly, when $\Delta(\mathbf{r}) = \Delta_0 T^3$, this term could be described as a spin orbital interaction as in Ref.[6][7] discussing spin Hall effects in graphene and cold atom system. We note that the left- and right-handed fermions are coupled to each other by the Higgs fields. Finally, the Hamiltonian density can be expressed as

$$\mathcal{H} = \psi^\dagger[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(\phi(\frac{1-\gamma^5}{2}) + \phi^\dagger(\frac{1+\gamma^5}{2}))]\psi \tag{6}$$

$$\mathcal{L} \; = \; \bar{\psi}[i\gamma^\mu\partial_\mu - \phi(\frac{1-\gamma^5}{2}) - \phi^\dagger(\frac{1+\gamma^5}{2})]\psi \tag{7}$$

The Dirac matrices are defined below

$$\boldsymbol{\alpha} = (\alpha^1, \alpha^2) = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\boldsymbol{\gamma} = \beta\boldsymbol{\alpha}, \qquad \gamma^0 = \beta, \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\alpha^1\alpha^2\alpha^3$$

The usual commutation and anti-commutation between Dirac matrices and gamma matrices are fulfilled i.e. α^i anti-commutes with each other and β, while commutes with γ^5.

II. CHIRAL GAUGE THEORY AND NON-ABELIAN VORTEX

To make Lagrangian density invariant under chiral $SU(2)$ gauge transformations, a relevant gauge potential is introduced by similar arguments with Ref.[3]. While the difference is that to obtain a vortex solution in non-Abelian regime, it needs at least two Higgs fields. However, this condition can be maintained by introducing an additional hopping superposing on the original one. The Lagrangian density becomes

$$\mathcal{L} \;=\; \bar{\psi}[i\gamma^\mu(\partial_\mu - i\gamma^5 A_\mu - (\phi+\chi)(\frac{1+\gamma^5}{2}) - (\phi^\dagger + \chi^\dagger)(\frac{1-\gamma^5}{2}))]\psi \tag{8}$$

This Lagrangian density is obviously invariant against a local chiral non-Abelian gauge transformation. The vortex sector is defined by the Lagrangian density given below

$$\mathcal{L}_v = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu\phi^a)^2 + \frac{1}{2}(D_\mu\chi^a)^2 + V(\phi^a,\chi^a) \tag{9}$$

(10)

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon^{abc}A^b_\mu A^c_\nu$$
$$D_\mu\phi^a = \partial_\mu\phi^a - \epsilon^{abc}A^b_\mu\phi^c$$

The vortex solutions can be obtained by the ansatz

$$\bar{\phi}(\mathbf{r}) = f(r)\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}, \qquad \bar{\chi}(\mathbf{r}) = g(r)\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix} \tag{11}$$

$$\bar{\mathbf{A}}^a(\mathbf{r}) = \hat{e}_\varphi\frac{1-H(r)}{r}\delta^{a3}, \qquad \bar{A}^0 = 0 \tag{12}$$

the boundary conditions are

$$f(\infty) = \phi_0, \qquad g(\infty) = \chi_0, \qquad H(\infty) = 0, \qquad f(0) = g(0) = 0, \qquad H(0) = 1 \tag{13}$$

where ϕ_0 and χ_0 are constants. And just as its Abelian counterpart, the non-Abelian one also preserves the parity ($\mathcal{P}$) , charge conjugation ($\mathcal{C}$) and time reversal ($\mathcal{T}$) invariance.

A Z_2 vortex may be also possible

$$\bar{\phi}(\mathbf{r}) = f(r)\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}, \qquad \bar{\chi}(\mathbf{r}) = g(r)\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}, \qquad \bar{\mathbf{A}}^a(\mathbf{r}) = \hat{e}_\varphi \alpha(r)\delta^{a3}, \qquad \bar{A}^0 = 0 \quad (14)$$

As for NOA vortex or the t'Hooft-Polyakov monopole, the topological stability of this vortex is maintained by the non-vanishing Higgs fields, enforced by the self-interactions and the asymptotic alignment of the gauge and Higgs fields. And the quantization of the magnetic flux

$$m = \int_{S^2} \mathbf{B}\cdot d\mathbf{a} = -2\pi \quad (15)$$

Finally, We mention that non-Abelian vortex-Fermion systems have been discussed extensively, but most discussions focus on high energy physics. So the rise of graphene or optical lattice seem to give a new sight of these subjects. Atomic physics, condensed matter physics and high energy physics seem to meet in this novel system.

III. POSSIBLE EXPERIMENT REALIZATION

First, we note that a experiment scheme has been proposed to simulate Dirac fermion excitations in grapene in honeycomb optical lattice[8]. The lattice was formed by interference of three laser beams, where cold neutral atoms suc as ^6Li, ^6K are loaded. Next, possible Kekulé pattern could be formed by considering the nearest-neighbor repulsive interaction between dipolar fermionic atoms loaded in an optical honeycomb lattice[2].

For generating the non-Abelian gauge potential in optical lattice, which interacts with the fermion in a spin dependent way, several possibilities have been explored such as Ref.[4], It was showed that the application of laser assisted nonuniform and state dependent tunneling, and coherent transfer between atom multi internal states would generate "artificial non-Abelian $U(k)$ or $SU(k)$ magnetic fields". We mention that combining Kekulé distortion and the scheme for generating non-Abelian gauge potential, possible pseudo Higgs fields should be possible. However, To obtain a stable non-Abelian vortex, the real experimental challenge comes from controlling the gauge fields in a space dependent way. It remains to be determined whether a viable experiment scheme which leads to such non-Abelian votex exists. But with the development of cold atom techniques, such obstacles should be overcome gradually.

IV. CONCLUSION

In conclusion, we construct a non-Abelian chiral gauge theory for describing a novel non-Abelian honeycomb lattice system, where pseudo Higgs fields can be generated via a spin dependent Kekulé scheme. This mechanism is based on effective low-energy hamiltonian of the Dirac excitations in graphenelike system and can be viewed as an low-energy analogue to its high energy counterpart.

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