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Effects of Dielectric Properties on Temperature Distributions in Food Model during Microwave Heating

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To investigate heat transfer phenomena in cylindrical foods of different dielectric properties with microwave heating, the dielectric constant, loss factor and the temperature distributions of samples were measured. The temperature distributions changed with their dielectric properties (dielectric constant, loss factor and penetration depth). As the penetration depth increased, the region of high temperature moved from the surroundings of the cylinder to the center. To describe these phenomena theoretically, the temperature distributions in the samples were calculated under the same conditions as those in the experiments using the mathematical model. The calculated results agreed closely with the experimental values.

Keywords: temperature distribution, microwave heating, dielectric properties, heat transfer analysis, mathematical model

Microwave heating has a tendency to heat foods unevenly causing problems with both sensory and microbiological quality; therefore, control of heating uniformity is essential. The prediction of temperature distributions in dielectric materials heated by microwave radiation has been reported. G.W. Padua developed a mathematical model for predicting the temperature profiles of cylindrical samples of agar gels containing sucrose heated by microwaves (Padua, 1993). Nykvist and Decareau (1978) developed a two-dimensional mathematical model to simulate microwave cooking of cylindrical meat roasts and have demonstrated good agreement with limited experimental data. Ohlsson and Risman (1978) studied the temperature distribution with hot and cold spot in spheres and cylinders using an infrared thermograph technique and using computer simulations in calculations. In those studies, the incident angle of the microwave was considered only towards the center of cylinder, so that the calculated temperature at the center became higher than those in the experiment.

In our previous studies (Cheng *et al.*, 1996), the temperature distributions of a flat cylinder were calculated numerically by fundamental equations with consideration of the incident angle of microwave energy. The calculated temperature distributions that varied with the diameter of the cylinder agreed closely with the measured values.

However, in spite of the fact that the dielectric properties of food affect the internal heat generation, little study has been reported about temperature distributions in cylindrical samples with different dielectric properties. The object of this study is to examine the formation of temperature distributions related to dielectric properties within cylindrical foods. In this study, cylindrical samples were covered by metallic shields on both ends to simulate one-dimensional heating (Padua, 1993; Prosetya & Datta, 1991; Ho & Yam 1992; Mudgett, 1986), as basic research for the study of two-dimensional heating.

Mathematical Model

During microwave heating, the microwave energy was considered to penetrate into the sample and to be converted to heat energy throughout the heated body, as shown in Fig. 1. It was assumed that the thermal properties of the samples were constant and that the samples were thermally insulated during the heating period. A one-dimensional heat transfer equation with a term for internal heat generation, which can be modeled in terms of dielectric properties, is represented as the following,

$$0 \le r \le R_0$$
$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q}{\kappa} \tag{1}$$

where q is the internal heat generation. The details of the method to calculate the heat generation, q, were described in the previous paper (Cheng *et al.*, 1996). The dielectric properties, particularly the penetration depth, notably affect the value of q.

Initial condition and boundary conditions are represented by Eqs. (2)–(5).

$$I.C.: \quad t=0; \quad 0 \le r \le R_0; \quad T=T_i \tag{2}$$
$$B.C.: \quad r=0; \quad \frac{\partial T}{\partial r}=0 \tag{3}$$
$$r=R_0; \quad \frac{\partial T}{\partial r}=0 \tag{4}$$
$$R_i < r \le R_0; \quad q=0 \tag{5}$$

R_i and R_0 are the internal and external radius of the container, respectively. The Eqs. (1)–(5) were solved by finite difference approximation, and the temperature distributions were calculated along the radial direction of the cylindrical sample.

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Materials and Methods

Microwave heating The experimental apparatus used (Cheng *et al.*, 1996) consisted of a magnetron, a power monitor, a matched load, a waveguide and a cavity which had a rating as 2450 MHz, 500 W maximum output. One weight percent agar gel, 10 wt% bentonite paste, 20 wt% mashed potato and synthetic paste were used as samples, and they were contained in cylindrical containers (4.7, 3.8 and 2.0 cm radius and 1 cm height). The synthetic paste was composed of 14% polyvinyl-alcohol and 86% water (Sekisui-Jyusi, Ltd., Osaka). The top and bottom surfaces of the samples were covered with aluminum foil. Therefore, microwaves were reflected from those surfaces and allowed to penetrate into the sample from the curved side surfaces only. A 2.5-cm thick heat insulating material was used to insulate the side, top and bottom from convective heat transfer. The sample, which had been kept at a uniform temperature of 25°C, was positioned at the center of a turntable in the microwave cavity. After heating, the samples were removed from the cavity and, the temperature distributions in the radial direction were measured with thermocouple probes as shown in Fig. 2. The experiments were performed three times and the results showed good reproducibility.

Measurement of dielectric properties During microwave heating, many variables in the food affect the heating performance. One of the most significant factors is the dielectric property, which describes how a material interacts with microwaves, that is, the ability to absorb, transmit and reflect electromagnetic energy.

The dielectric constants and loss factors were measured with a test set (Hewlett-Packard Corp., Santa Rosa, USA) HP85070B combination, and an Open-Ended Coaxial Probe method (David & Charles, 1991) was used. The probe measurements provide dielectric constant ε' and loss factor ε'' over a wide frequency range (200 MHz-13.5 GHz).

Powders of agar (Kokusan Chemical Works, Ltd., Tokyo), bentonite (Wako Pure Chemical Industries, Ltd., Osaka) and mashed potato (Marukyu, Ltd., Omiya) were dried at 70°C under vacuum for 24 h before use. Samples were prepared by mixing with ion-exchanged water. The dielectric properties of the samples were measured in duplicate at each temperature in a constant temperature oven, which was controlled from 10 to 80°C.

Thermal properties The thermal properties of 1 wt% agar gel were assumed to be equal to those of water. The values of 10 wt% bentonite paste, 20 wt% mashed potato and synthetic paste were taken from the literature and are summarized in Table 1.

In order to simplify the calculation, the thermal diffusivity of the container was assumed to be identical with that of the sample.

Energy flux in the surface of sample During microwave heating, the energy flux at the surface of the sample change according to the size and the dielectric properties of the sample. In this study, each energy flux was estimated from the average rate of temperature rise of the sample.

Results and Discussion

In Figs. 3 and 4, the dielectric constant and loss factor of the samples at 2450 MHz are respectively shown as a function of temperature (Sakai *et al.*, 1996). Because water is the main contributor to the dielectric constant of the samples, it was found that the dielectric constant of the samples decreased with a temperature rise. On the other hand, the loss factor decreased with respect to temperature between 10 and 70°C and then increased slowly as the temperature rose. A similar

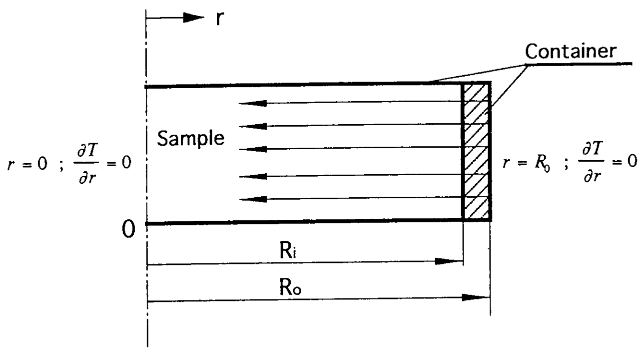

Fig. 1. Penetration model of microwave energy.

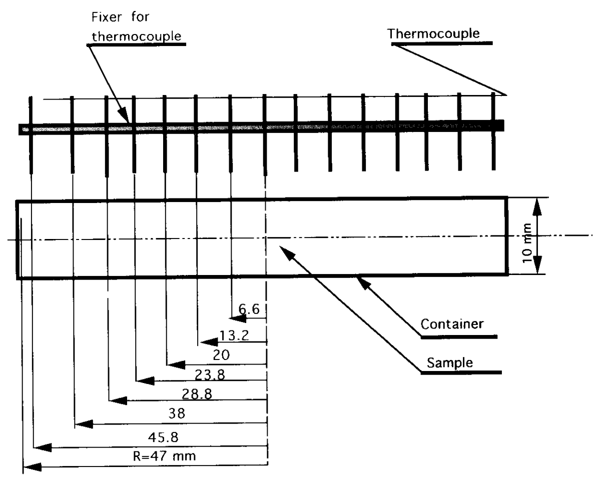

Fig. 2. The position of temperature measurement.

Table 1. Thermal properties of samples.

Samples	ρ [kg/m^3]	C_p [J/g·°C]	κ [W/m·°C]	α [m^2/s]	Ref.
1 wt% Agar gel	1000	4.206	0.558	0.135×10^{-6}	—
10 wt% Bentonite paste	1050	3.85	0.708	0.175×10^{-6}	(Uno & Hayakawa, 1980)
20 wt% Mashed potato	1067	3.542	0.648	0.171×10^{-6}	(Chen *et al.*, 1993)
Synthetic paste	1010	4.52	0.724	0.159×10^{-6}	(Cheng *et al.*, 1996)

phenomenon has also been observed in many food materials containing salt in which the ionic loss is an important part of the increase in the loss factor of the sample and increases as the temperature increases (Bengtsson & Risman, 1971; Roebuck & Goldblith, 1975).

Penetration depth, d, the distance where the microwave power is decreased to $1/e$ of its surface value, is a very important parameter characterizing microwave heating. The penetration depth is calculated (Metaxas & Meredith, 1983) from the dielectric constant and the loss factor using Eqs. (6) and (7).

$$d=\frac{\lambda_0}{2\pi}\sqrt{\frac{1}{2\varepsilon'(\sqrt{1+\tan^2\delta}-1)}} \tag{6}$$

$$\tan\delta=\varepsilon''/\varepsilon' \tag{7}$$

where $\lambda_0=12.237$ cm at 2450 MHz

The penetration depth of the samples as a function of temperature is shown in Fig. 5. The value of the penetration depth decreased in the following order: agar gel, bentonite paste, synthetic paste and mashed potato.

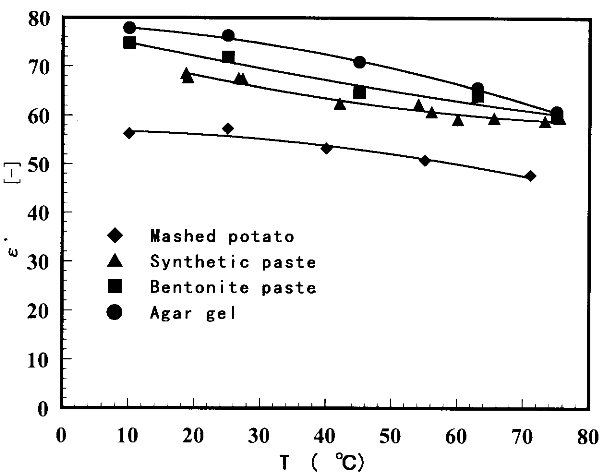

Fig. 3. The dielectric constant of samples as a function of temperature at 2450 MHz.

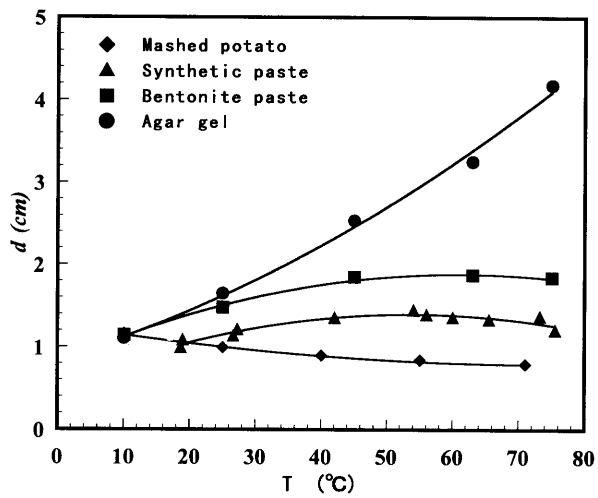

Fig. 5. The penetration depth of samples as a function of temperature at 2450 MHz.

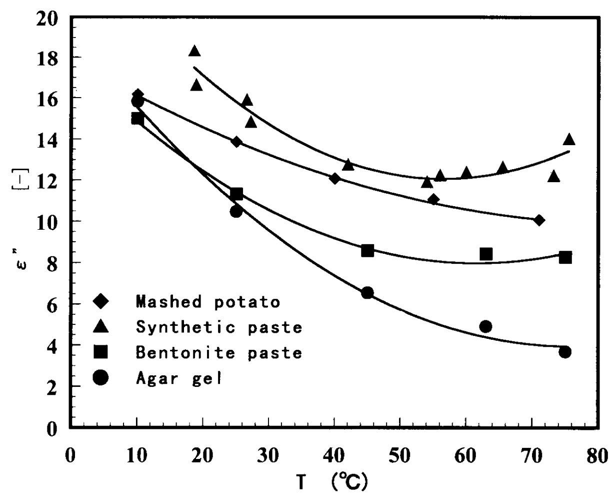

Fig. 4. The loss factor in samples as a function of temperature at 2450 MHz.

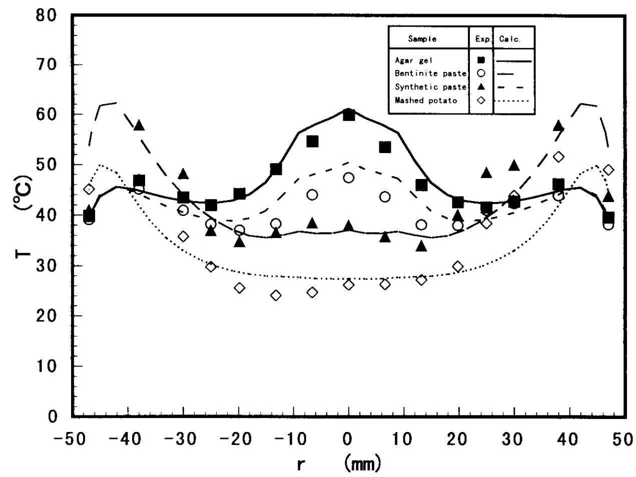

Fig. 6. Comparison between calculated temperature distributions and experimental values for 47-mm radius cylinders.

Table 2. Dielectric properties of samples at 2450 MHz.

Samples	ε' [−]	ε'' [−]	d [cm]
Agar gel	$-2.38\times10^{-3}T^2-6.45\times10^{-2}T+78.84$	$2.68\times10^{-3}T^2-0.406T+19.34$	$2.55\times10^{-4}T^2+2.41\times10^{-2}T+78.84$
Bentonite paste	$8.64\times10^{-4}T^2-0.299T+77.79$	$2.59\times10^{-3}T^2-0.319T+17.78$	$-2.88\times10^{-4}T^2+3.53\times10^{-2}T+0.795$
Synthetic paste	$1.91\times10^{-3}T^2-0.353T+74.5$	$3.77\times10^{-3}T^2-0.426T+24.1$	$-2.67\times10^{-4}T^2+2.88\times10^{-2}T+0.477$
Mashed potato	$-1.89\times10^{-3}T^2-1.4\times10^{-3}T+56.93$	$1.01\times10^{-3}T^2-0.187T+17.86$	$7.99\times10^{-5}T^2-1.17\times10^{-2}T+1.14$

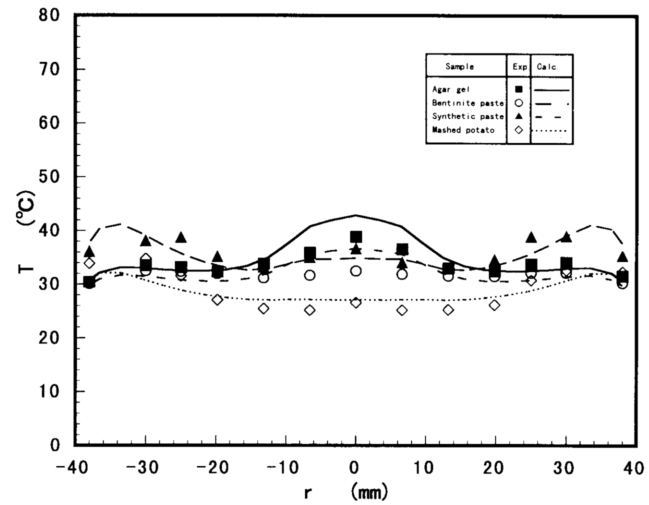

Fig. 7. Comparison between calculated temperature distributions and experimental values for 38-mm radius cylinders.

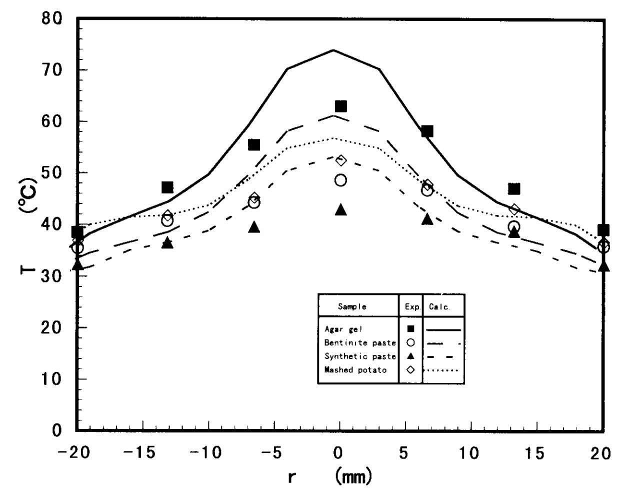

Fig. 8. Comparison between calculated temperature distributions and experimental values for 20-mm radius cylinders.

In order to calculate the internal heat generation in the mathematical model, empirical equations representing the relation between the dielectric properties and temperature were obtained using the least square fit. The results are summarized in Table 2.

The calculated temperatures were compared with the measured temperature profiles along the radial direction of the cylinders after 60 s of microwave heating (Figs. 6–8). As shown in Fig. 6, center heating is observed in the agar cylinder of 4.7 cm radius, because there is focusing of the internal energy. However, surface-heating patterns are observed in the mashed potato and synthetic paste, because the microwave energy was converted to heat energy before focusing of the internal field. On the other hand, both the surface and center heating patterns are obtained in a bentonite sample, and this results in a relatively uniform profile.

Center heating effects depend not only on the penetration depth but also on the diameter of the samples. In a cylinder of 2.0 cm radius (Fig. 8), center effects are also observed for every sample, because focusing of the internal electric field is predominant even in mashed potato. In contrast, the samples of 3.8 cm radius show nearly uniform heating profiles, that is, neither surface nor center-heating patterns were observed (Fig. 7).

In Figs. 6–8, the lines represent the temperature calculated from the mathematical model. Because the calculated results are close to the experimental data, the temperature distributions in the cylindrical container could be successfully predicted by the mathematical model.

The Lambert law, which was used in this calculation, is a simple power formulation that can be used to estimate temperature profiles for sufficiently thick samples. Therefore, when the thickness of the sample in the direction of microwave penetration is not sufficient, the correct heat generation must be computed from Maxwell's equations. The critical thickness L_{crit} above which the Lambert's law limit is valid can be estimated from the following equation (Ayappa *et al.*, 1991).

$$L_{crit}=2.7d-0.08$$

For example, the L_{crit} value was roughly estimated to be 2.35 cm for mashed potato from its average penetration depth of $d=0.9$ cm. In the case of a 2.0 mm radius cylinder, the experimental temperature profiles could not be precisely predicted by the model, because this radius is shorter than its critical thickness.

Nomenclature

C_p	=specific heat of sample	[J/(g•°C)]
d	=penetration depth	[cm]
L_{crit}	=critical thickness	[cm]
q	=internal heat generation	[J/(cm³•s)]
R_i	=internal radius of sample	[cm]
R_o	=external radius of sample	[cm]
r	=radial position of cylinder	[cm]
T	=temperature in samples	[°C]
T_0	=initial temperature of sample	[°C]
t	=heating time	[s]
α	=thermal diffusivity	[m²/s]
ε'	=dielectric constant	[−]
ε''	=loss factor	[−]
$\varkappa$	=thermal conductivity	[W/m•°C]
λ_0	=wave length in vacuum	[cm]
ρ	=density of sample	[g/cm³]

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