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Review

An On-Line Inverse Method for Estimation of Thermal Parameters Which Are Responsible to Predicting Temperature History in Food Heating/Freezing

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Most of the published methods for estimating temperature history during heating/cooling/freezing solid food require data on thermal properties of the product and any relevant heat transfer coefficients. However, there are some difficulties of obtaining thermal data for use in industrial heating/cooling/freezing of food. In this paper the development of a new procedure for estimating the temperature history is briefly reviewed, a procedure which does not require the knowledge of thermal data of the food being heated/frozen. This procedure collects a series of time/temperature data at a point in the food in the early stages of heating/freezing, analyzes these data to predict the thermal parameters which are responsible for heat conduction, and predicts the time/temperature relationship for the remainder of the heating/cooling/freezing phases.

Keywords: temperature history, inverse method, optimum control, thermal processing, freezing

We start with the case of heating for developing the on-line inverse method and then it is extended to freezing process. This is because freezing is more complicated than heating due to the considerably changes of thermal property during food freezing.

Heating a Slab of Food

A number of workers have calculated optimal thermal sterilization conditions to maximize surface or volume average quality retention with a constraint that required inactivation of microorganisms is fulfilled (Silva *et al.*, 1992). Solving optimal sterilization problem may give an optimal temperature history for higher quality products of heat-preserved foods. However, there is a need for a specific control system which may manage the thermal process operation to afford the food with optimal temperature history. This is because thermal properties of food and the surface heat transfer coefficient are not widely available. To determine the precise values for these parameters often requires a separate, lengthy and detailed analysis.

For such situation where system parameters are not clearly known, feed back control is often a good method. Unfortunately, however, it fails to achieve a target-lethality because of a kind of time lag which is considerably enlarged as the increase of food size (Mihori *et al.*, 1991).

In order to get rid of these difficulties, Mihori *et al.* (1991, 1994) proposed an inverse method for on-line control to achieve appropriate sterilization.

In case of heating a slab of food, the temperature at time t and position x may be described as a function of t, x and two parameters Bi and τ:

$$\tau \equiv D^2 \rho Cp/\lambda\ , \tag{1}$$

$$Bi \equiv hD/\lambda\ , \tag{2}$$

$$(T-T_h)/(T_i-T_h)=\eta(t,x,Bi,\tau)\ , \tag{3}$$

which is an analytical solution of the heat conduction equation.

We place a temperature sensing probe in a sample food. The collection of a series of time/temperature data sets at the location of the sensing probe begins when we start a thermal sterilization process.

$$(t_1,S_1),\ (t_2,S_2),\ (t_3,S_3),\ (t_4,S_4), \ldots (t_k,S_k)\ .$$

We attempt to estimate two system parameters τ, Bi and x, the location of sensing probe. To do this we use some initial estimate values for the parameters with which we calculate an approximate values for the temperatures at the true location of the probe.

$$(t_1,T_1),\ (t_2,T_2),\ (t_3,T_3),\ (t_4,T_4), \ldots (t_k,T_k)\ .$$

Using a steepest descent method we can access to true values of τ, Bi and x where the sum of the errors to be minimized:

$$\frac{\partial}{\partial \tau}\sum_j (T_j-S_j)^2=0,\ \frac{\partial}{\partial Bi}\sum_j (T_j-S_j)^2=0,\ \frac{\partial}{\partial x}\sum_j (T_j-S_j)^2=0. \tag{4}$$

Once the parameters are available we can predict the temperature history making use of the parameters. A successful example (Mihori *et al.*, 1991) of heating mannan jelly in a slab container realized to a target lethality is shown in Fig. 1.

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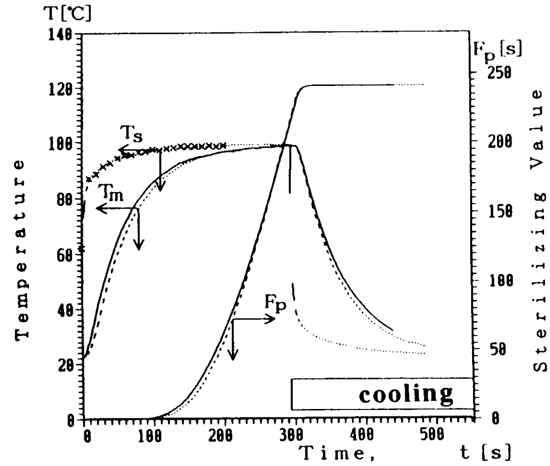

Fig. 1. Time/temperature relationship and lethal rate integral both measured and predicted using a mannan jelly slab. Half-thickness=4.5 mm. T_s=Temperature of sensing probe at outside surface; T_m=temperature at geometric center; F_p=lethal rate integral.

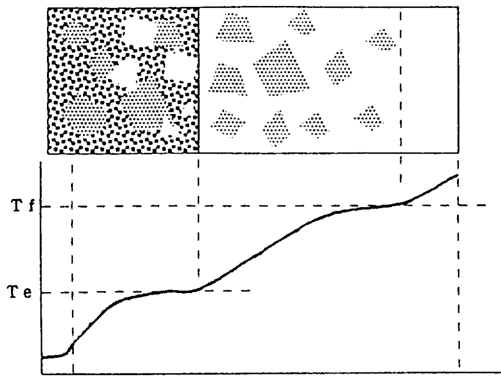

Fig. 2. A disperse system as a model of frozen food at each stages of freezing.

Placing a Sensing Probe on the Outside Surface of the Container

The position of the temperature probe with which the thermal response of food is observed, can be placed anywhere in the food, even on the external surface of the container (Mihori *et al.*, 1994). Locating the sensing probe on the outside surface of the container may have advantages and disadvantages. Advantages are: (1) facilitate setting up the sensing probe, (2) a non-destructive method. The disadvantage is (1) the sensor is able to give little information in case of large Biot number, (2) the surface temperature is sometimes ill-stable. These disadvantage can be overcome by replacing the temperature probe with a heat flux sensor which is placed on the outside surface of the container (Mihori *et al.*, 1994). A variety of film type heat flux sensors are available in the market; so-called "thermopile" composed of different metal foils (thermocouples) sandwiched one another.

A Precise Mathematical Model for Freezing Food

Frozen food as a disperse system In order to extend this on-line inverse method to food freezing, we need a precise mathematical model for freezing food. We assumed a frozen food as a disperse system in which granular pure-ice-crystal is dispersed in concentrated aqueous liquid or in solid solution (Fig. 2). The extent of freezing, R, defined by the mass of ice divided by the mass of food, may be expressed in the form as follows (Mihori & Watanabe, 1994a):

$$R=1-(T_f/T)^n, \tag{5}$$

where T_f is the initial freezing temperature in Celsius. Using R, the volumetric liquid fraction, F, in the food body may be written as:

$$F=\rho_{ice}/(\rho_{ice}+R\rho_m), \tag{6}$$

where ρ_m is the density of unfrozen matrix.

Effective thermal conductivity The effective thermal conductivity λ in a disperse system may depend on the structure of the system and cannot be predicted easily. For the practical use it is often written by a series model or by a parallel model.

$$\lambda=F\lambda_m+(1-F)\lambda_{ice} \quad \text{parallel}, \tag{7}$$

$$\frac{1}{\lambda}=\frac{1}{F\lambda_m}+\frac{1}{(1-F)\lambda_{ice}} \quad \text{series}. \tag{8}$$

Heat conduction equation Considering that the phase change takes place over some extended range of temperature in food freezing, the energy accumulation term in heat conduction equation may be written as:

$$\frac{\partial(\rho C_p T)}{dt}+\rho_f L\frac{\partial F}{\partial t}. \tag{9}$$

Since F depends on T, this may be rewritten and Ψ is defined as:

$$\left\{\frac{d(\rho C_p T)}{dT}+\rho_f L\frac{\partial F}{\partial T}\right\}\frac{\partial T}{\partial t}\equiv\Psi\frac{\partial T}{\partial t}. \tag{10}$$

Thence a general equation for one dimensional heat conduction in food freezing may be written as:

$$\Psi\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left\{\lambda(T)\frac{\partial T}{\partial x}\right\}. \tag{11}$$

When the local temperature at a position in a food body is lowered, extent of freezing at this position may increase and the thermal properties may change. This means that the thermal property may depend on the position in the food body when it has a temperature difference in the food body, which happens in rapid freezing processes. In such cases the heat conduction equation in a solid may be written as follows because λ depends on T which depends on x:

$$\Psi\frac{\partial T}{\partial t}=\lambda(T)\frac{\partial^2 T}{\partial x^2}+\frac{d\lambda(T)}{dT}\left(\frac{\partial T}{\partial x}\right)^2$$

non-linear heat conduction model. (12)

This equation may be referred as a non-linear model.

The second term (non-linear term) in the right-hand side of this equation has been often neglected even though they liked to consider the temperature dependence of λ. In such a case the heat conduction equation may be written as:

$$\Psi\frac{\partial T}{\partial t}=\lambda(T)\frac{\partial^2 T}{\partial x^2} \quad \text{linear heat conduction model.} \tag{13}$$

A rapid freezing experiment to examine the model In an attempt to find which model works for the simulation, we

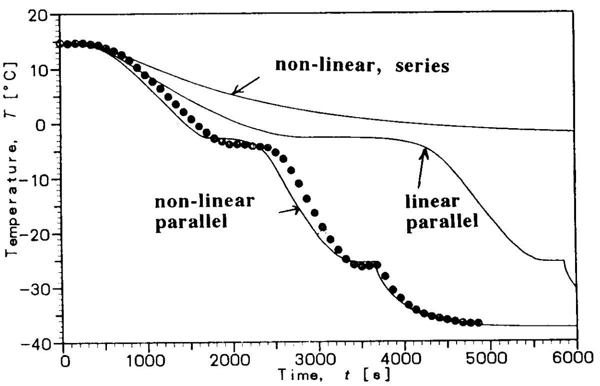

Fig. 3. The time/temperature curve at the center of a slab (5% sodium chloride solution). Closed circle, measured; solid line, calculated.

carried out a rapid freezing experiment on a sodium chloride aqueous solution (Mihori & Watanabe, 1992), because precise thermal data including temperature coefficients and concentration coefficients are available. An experiment of rapid freezing (with large h) is required to observe the freezing time which is reflected by the structure of the food body, eliminating the effects of surface heat transfer coefficient (h) which governs the heat transfer phenomena when h is small. The results of the experiment showed that non-linear heat conduction equation with parallel model heat conductivity works for the simulation of freezing food (Fig. 3).

Analytical Solution Model for Food Freezing

Freezing a slab of food Since the non-linear heat conduction model requires the detailed thermal data such as temperature coefficient and concentration coefficient of heat conductivity and heat capacity (including latent heat), we use a sodium chloride solution as the prototype of model food. When Ψ and λ for a sodium chloride solution can be approximated by (Fig. 4):

$$\Psi=-A/(T+T_c), \qquad \lambda=\Lambda-BT \tag{14}$$

the non-linear heat conduction equation (Eq. (12)) may be written as:

$$-\frac{\partial\theta}{\partial t}=\theta\frac{\partial}{\partial x}\left\{(\beta-\Gamma\theta)\frac{\partial\theta}{\partial x}\right\}, \tag{15}$$

where A, B, T_c and Λ are constants chosen to make a good fit for Ψ and λ; $\beta=(\Lambda+BT_c)/A$, $\Gamma=B/A$ and $\theta=T+T_c$. By using a new variable $\xi=\pm x/\sqrt{2\Gamma t}$, this equation turns into an ordinary non-linear differential equation as:

$$\theta\left\{(\theta-\beta/\Gamma)\frac{d^2\theta}{d\xi^2}+\left(\frac{d\theta}{d\xi}\right)^2\right\}-\xi\frac{d\theta}{d\xi}=0. \tag{16}$$

This equation has an analytical solution: $\theta=\xi$. This means that there exists a linear temperature profile in a food of slab being frozen (Mihori & Watanabe, 1994a). It may be interesting that a linear temperature profile is assumed in Plank's model which is well known to be useful. Another interesting point is that substituting a linear temperature profile into the linear heat conduction equation (Eq. (13)) results in no cooling of the body.

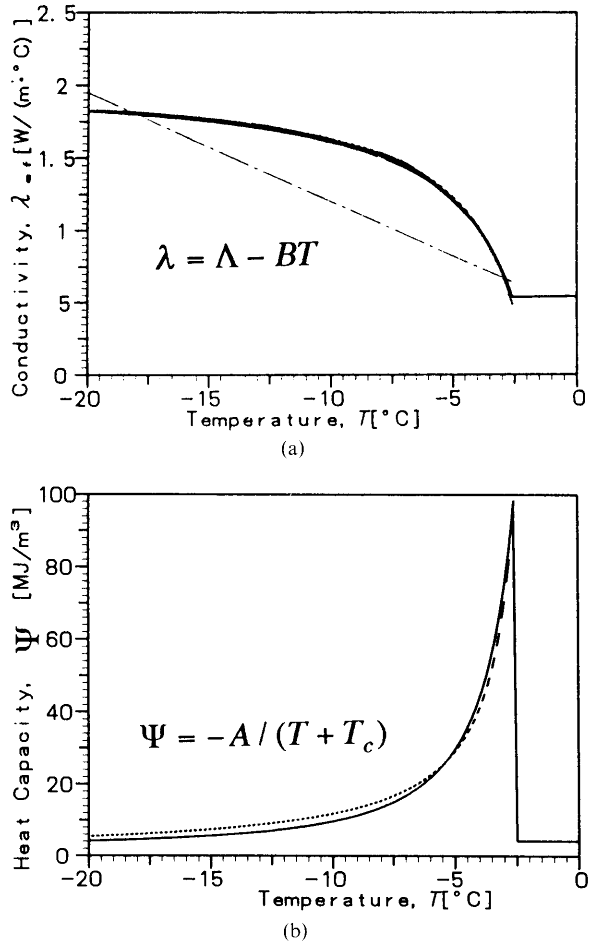

Fig. 4. Conductivity λ and heat capacity Ψ vs. temperature of 5% sodium chloride solution. Solid line, literature value; chain line and dotted line, approximated by Eq. (14).

Using this linear profile as the first stage of freezing followed by a terminating stage profile with constant thermal properties (Mihori & Watanabe, 1994a), the on-line inverse procedure for predicting freezing temperature history was validated to work (Mihori & Watanabe, 1994b). A copy of the monitoring display of an on-line prediction of freezing a slab of Alaska pollack surimi is shown in Fig. 5.

Locating the temperature sensing probe on the outside surface of the food body was found to work also for the case of freezing (Mihori *et al.*, 1997).

Freezing a cylinder and a sphere of food When the shape of the food to be frozen is a cylinder, a linear temperature profile does not exist anymore in the food body during freezing and no analytical solution has been found yet. Fortunately however, a profile method using a logarithmic function was found available to describe the temperature profile with which an approximate equation for predicting freezing time was derived (Hu *et al.*, 1995).

Concerning a sphere of food, however, we could find no function suited to describe the temperature profile during freezing. This difficulty pushed us to step into a numerical model approach.

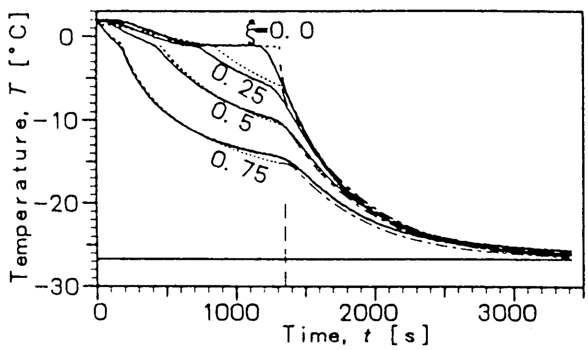

Fig. 5. Temperature history during freezing a slab of Alaska pollack surimi. Solid line, measured; dotted line, predicted. ξ: dimensionless distance from the center.

Table 1. The coefficients used at each step of the iteration in the numerical experiment of freezing lean beef. The simulated measured temperature history included random error of ± 0.2°C.

n	E^2	P_1	P_2	P_3	$P_4 \times 10^8$	P_5
0		1.0	−0.00001	0.1	1.0	1.0
1	725.7	1.0	0.00145	0.009646	0.4865	−0.3519
2	400.1	1.0	0.00041	0.009646	1.021	0.5481
3	82.42	1.0	−0.00738	0.007630	1.927	1.047
15	0.8092	1.0	−0.2795	0.3244	2.898	1.547
16	0.8071	1.0	−0.2856	0.3167	2.873	1.540
52	0.798102	1.0	−0.3452	0.2518	2.616	1.473
53	0.798098	1.0	−0.3454	0.2516	2.614	1.472
True value		1.669	−0.629	0.333	4.68	1.50
error (%)			−8.5	−24.3	−6.8	−1.8

The error for P_3 and P_5 was calculated as $100\times(P_{\mathrm{estimate}}-P_{\mathrm{true}})/P_{\mathrm{true}}$. The error for P and P was calculated as $100\times(P_{\mathrm{estimate}}/P_{1,3\mathrm{estimate}}-P_{\mathrm{true}}/P_{1,\mathrm{true}})/(P_{\mathrm{true}}/P_{1,\mathrm{true}})$.

Numerical Solution Model for Food Freezing

In order to make the on-line inverse method available for a food body of any shape, applying a numerical calculation method may be a good choice. This may be boosted by the recent trends of appearing a rapid personal computer with amazing CPU.

Hu *et al.* (1997) made an attempt to apply a numerical calculation method to develop an inverse procedure for predicting temperature history without the knowledge of thermal properties. They examined the literature data on thermal properties of various kinds of food and found that the temperature dependence of λ and Ψ may be described well in the form as:

$$\lambda(T)=P_1+P_2/(-T)^{P3}, \tag{17}$$

$$\Psi=P_4/(-T)^{P5}. \tag{18}$$

Once the boundary and initial conditions and the parameters $P=\{P_1, P_2, P_3, P_4, P_5\}$ associated with the thermal properties are specified, the direct problem defined by Eqs. (12), (17) and (18) can be solved numerically and the temperature distribution can be calculated as a function of time and position anywhere in the food body.

Now we consider an inverse problem which concerns with the estimation of these five coefficients from the knowledge of transient temperature measurements collected using sensors placed at three different locations in the food body. This may be a problem of finding the unknown parameters P that minimize the following sum of squares function:

$$E^2=\sum_i^{n_t}(T_i(P)-S_i)^2. \tag{19}$$

An important point to be noticed is that the solution of the inverse problem described above does not enable us to access to the true values of the thermal property coefficients P. When Eqs. (17) and (18) are substituted into Eq. (12), one can see that Eq. (12) gives the same solution to any sets of parameters P_1, P_2 and P_4 as long as the ratio of these parameters $P_1 : P_2 : P_4$ is kept constant, while good estimates for the values of P_3 and P_5 can be determined.

The validity and the accuracy of the method of inverse analysis considered here was demonstrated by a numerical experiment where literature data of lean beef (Sweat, 1986) was used. The sensor locations were 1/10, 1/5, 3/10 thick of the slab from the surface. For simplicity the initial freezing temperature was chosen as the initial temperature of the slab. Table 1 shows the coefficients used at each step of the iteration in the numerical experiment of freezing lean beef. The simulated measured temperature included random error of ± 0.2°C. The iteration was successful to converge. P_1 was set to 1. The ratio of $P_1 : P_2 : P_4$ was predicted within 10% error, while the absolute values were in 80% error. Using these parameters, the temperature history during freezing was calculated and compared with the simulated exact temperature history in Fig. 6, which shows that the prediction of the temperature history makes a considerably good agreement.

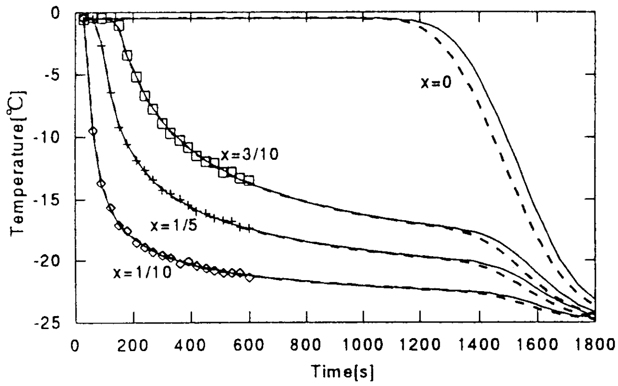

Fig. 6. The comparison between the simulated exact temperature history (solid line) and the predicted temperature history (dash line) for freezing lean beef, when the temperature measurement included ± 0.2°C. The symbols are the temperatures used for the prediction.

Nomenclatures

- Bi Biot number $=D\ h/\lambda\ [-]$
- Cp specific heat capacity [J kg^{-1} °C^{-1}]
- D half-thickness of a slab [m]
- F volumetric liquid fraction [−]
- h surface heat transfer coefficient [W m^{-1} °C^{-1}]
- L latent heat [J kg^{-1}]
- n_t number of temperature recordings [−]

Pi thermal property coefficient
R extent of freezing [−]
T temperature [°C]
T_h heating water temperature [°C]
T_i initial temperature [°C]
t time [s]
x position [m]
λ thermal conductivity [W m^{-1} °C^{-1}]
λ_m λ of saturated NaCl solution [W m^{-1} °C^{-1}]
ρ density [kg m^{-3}]
τ $=\rho CpD^2/\lambda$ [s]
Ψ function defined by Eq. (10).

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