

# NONCOMMUTATIVE POSITIVSTELLENSÄTZE FOR PAIRS REPRESENTATION-VECTOR

JAKA CIMPRİČ

ABSTRACT. We study non-commutative real algebraic geometry for a unital associative  $*$ -algebra  $\mathcal{A}$  viewing the points as pairs  $(\pi, v)$  where  $\pi$  is an unbounded  $*$ -representation of  $\mathcal{A}$  on an inner product space which contains the vector  $v$ . We first consider the  $*$ -algebras of matrices of usual and free real multivariate polynomials with their natural subsets of points. If all points are allowed then we can obtain results for general  $\mathcal{A}$ . Finally, we compare our results with their analogues in the usual (i.e. Schmüdgen's) non-commutative real algebraic geometry where the points are unbounded  $*$ -representation of  $\mathcal{A}$ .

CIMPRİČ JAKOB, UNIVERSITY OF LJUBLJANA, FACULTY OF MATHEMATICS  
AND PHYSICS, DEPARTMENT OF MATHEMATICS, JADRANSKA 19, SI-1000 LJUBL-  
JANA, SLOVENIA, CIMPRIC@FMF.UNI-LJ.SI, HTTP://WWW.FMF.UNI-LJ.SI/~CIMPRIC

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