U(1) × U(1) / Z_2 Kosterlitz-Thouless transition of the Larkin-Ovchinnikov phase in an anisotropic two-dimensional system

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We study Kosterlitz-Thouless (KT) transitions of the Larkin-Ovchinnikov (LO) phase for a two-dimensional system composed of coupled one-dimensional tubes of fermions. The LO phase here is characterized by a stripe structure (periodic in only one direction) in the order parameter. The low energy excitations involve the oscillation of the stripe and the fluctuation of the phase, which can be described by an effective theory composed of two anisotropic XY models. We compute from a microscopic model the coefficients of the XY models from which the KT transition temperatures are determined. We found the $T^{KT} \propto t_\perp$ for small intertube tunneling $t_\perp$. As $t_\perp$ increases the system undergoes a first-order transition to the normal phase at zero temperature. Our method can be used to determine the Goldstone excitations of any stripe order involving charge or spin degrees of freedom.

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The role of topological excitations of striped superconducting states has been intensively studied [1–5] since at finite temperature the proliferation of those defects can lead to possible exotic phases, such as the charge 4 superfluid [5]. A typical striped superconducting state is the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) [6, 7] state which is believed to exist in heavy-fermion superconductor $CeCoIn_5$ [8, 9]. However, since the FFLO order is more likely to occur in the quasi-one-dimensional (1D) system [10], the cold atom system with two imbalanced species of atoms confined in a lattice array of 1D tubes formed by coherent laser beams [11] seems more promising to display the direct evidence. Since the intertube coupling can be tuned relatively with ease in the cold atom system by controlling the intensity of trapping lasers, it is suitable to study the dimensional crossover phenomena [12–15].

Numerous exotic phases have been predicted from effective field theories [3], but the phase diagram of these exotic phases is not established for cold atom experiments yet. In cold atom experiments, the microscopic parameters (like interaction strength) are tunable and measurable, and this motivates our detailed study of the Kosterlitz-Thouless (KT) transitions of the Larkin-Ovchinnikov (LO) phase starting from a microscopic model. In this letter we study a quasi-1D two-dimensional (2D) spinful fermionic system composed of coupled 1D tubes as illustrated in Fig. (1)(a) where at zero temperature the LO order is the ground state. We determine the Kosterlitz-Thouless temperature of LO phase (the FFLO regime in Fig. (1)(b) [10]) as a function of intertube coupling $t_\perp$ from a microscopic model. We found that transition temperature is linear in $t_\perp$ for small $t_\perp$ (Fig. (2)(a)) and at zero temperature the transition from LO to normal phase upon increasing $t_\perp$ is of first order (Fig. (2)(c)). This phase transition is driven by the disappearance of the nesting Fermi surface, which is a key feature for the quasi-1D to 2D dimension crossover.

Before introducing the microscopic model, we first discuss the Goldstone modes of LO phase [3–5] from the symmetry point of view. The LO phase is characterized by an order parameter of stripe configuration

$$\Delta(x,z) \propto \langle c_\downarrow(x,z) c_\uparrow(x,z)\rangle \propto \Delta_0 f(z) \tag{1}$$

where Δ_0 is the amplitude of the order parameter while $f(z)$ describes the stripe satisfying $f(z+L/2) = -f(z)$ and $f(z) = f(-z)$ in our coordinate choice. The LO wave vector Q is defined as $Q \equiv 2\pi/L$ with L the period of the stripe. Since the LO phase breaks both translational and $U(1)$ symmetries, it has two branches of Goldstone modes – the oscillation of the stripe, and the phase fluctuation of the amplitude. Under these fluctuations, the order parameter becomes

$$\Delta(x,z) = \Delta_0 e^{i\theta(x,z)} f(z + u(x,z)) \tag{2}$$

where $u(x,z)$, $\theta(x,z)$ are generalized elastic fields [16] to describe the Goldstone modes. Physically u represents

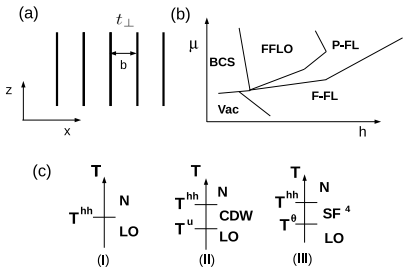

FIG. 1: (a) Configuration of the system: arrays of 1D tubes with intertube distance b and intertube tunneling $t_\perp$. (b) A schematic plot of quasi-1D phase diagram as a function of μ and h. Vac: vacuum state (no particle); P-FL: partially polarized Fermi liquid; F-FL: fully polarized Fermi liquid. Our study here focuses on the FFLO regime. (c) Possible phases as a function of temperature. N:normal Fermi liquid; CDW: charge density wave; SF^4: charge 4 superfluid.

the small oscillation of the stripe LO order whereas θ the phase fluctuation of the amplitude. In the quasi-1D system, x and z directions are not equivalent. Therefore to the quadratic order the total free energy in terms of generalized elastic fields are described by two anisotropic XY models [5, 17]

$$\Delta F = \int dxdz \left[\frac{A}{2}(Qu_x)^2 + \frac{B}{2}(Qu_z)^2 + \frac{C}{2}\theta_x^2 + \frac{D}{2}\theta_z^2\right] \quad (3)$$

where $f_x = \partial_x f$. In our notation, u and θ/Q have the dimension of length, their first derivatives are dimensionless, and coefficients A, B, C, D have the dimension of energy. For results presented in this paper, we take $f(z) = \cos Qz$ which is $(e^{iQz} + e^{-iQz})/2$. In this case, these two Goldstone modes correspond to phase fluctuations of two Fulde-Ferrell (FF) order $\Delta(x,z) = \Delta_0(e^{iQ(z+u^+)} + e^{-iQ(z+u^-)})/2$. For the FF order, $\Delta(x,z) \propto e^{iQz}$ which does not break translation symmetry. When identifying $u^+ = u + \theta/Q$, $u^- = u - \theta/Q$, $\Delta(x,z) = \Delta_0 \cos(z + u(x,z))\, e^{i\theta(x,z)}$, consistent with the functional form in Eq. (2). To be general, we shall not specify the form of $f(z)$ unless necessary.

In 2D, each elastic field is associated with one topological defect. For u the defect is the (edge) dislocation satisfying $\oint \vec{\nabla} u \cdot d\vec{l} = Ln_d$; for θ the defect is the vortex satisfying $\oint \vec{\nabla}\theta \cdot d\vec{l} = 2\pi n_v$ with n_d, n_v integers. There is another topological defect referred to as a half-vortex half-dislocation (HH) where $(n_d, n_v) = (\pm 1/2, \pm 1/2)$, which originates from the Z_2 symmetry of the order parameter [4, 5] – when circulating around an HH defect, each of the half vortex and the half dislocation introduces a minus sign leaving the order parameter unchanged. The proliferation of topological defects leads to Kosterlitz-Thouless (KT) transition. The KT transition temperature $T^u = \frac{\pi}{2}\sqrt{AB}$ for dislocations, $T^\theta = \frac{\pi}{2}\sqrt{CD}$ for vortices, and $T^{hh} = \frac{\pi}{8}(\sqrt{AB} + \sqrt{CD}) = (T^u + T^\theta)/4$ for HH [5]. The last temperature cannot be highest. When increasing the temperature, there are three distinct possibilities as illustrated in Fig. (1)(c): (I) T^{hh} is the lowest, (II) T^θ the lowest, and (III) T^u the lowest [5]. For (I) there is only one transition from LO to normal state at T^{hh}. For (II), the LO phase first becomes a charge density wave (CDW) state at T^θ and then normal at T^{hh}. For (III),the LO phase first becomes a charge 4 superfluid at T^u and then normal at T^{hh}.

The microscopic model we use is a one band model with attractive contact interaction in a quasi-1D system [10].

$$H = \sum_{\vec{k},\sigma} \xi_{\vec{k},\sigma} c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma} + g_{1D} \sum_{i_x} \int dz\, c^\dagger_{i_x,\uparrow}(z) c^\dagger_{i_x,\downarrow}(z) c_{i_x,\downarrow}(z) c_{i_x,\uparrow}(z) \quad (4)$$

with $\xi_{\vec{k},\sigma} = \frac{\hbar^2 k_z^2}{2m} - 2t_\perp \cos(bk_x) - \mu + h(-1)^\sigma$, where k_z is unbound, $|bk_x| < \pi$ with b the intertube distance, and

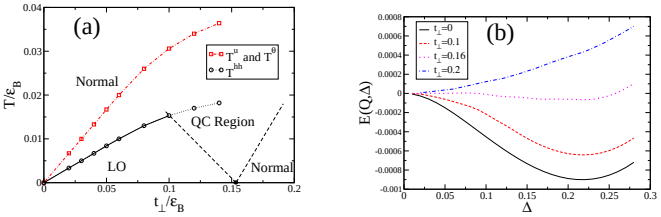

FIG. 2: (a) Phase diagram for $\mu = 2$, $h = 1.145$, $Q = 1.2$, $\Delta_0 = 0.22$. Curves with ligands are computed whereas two dashed lines embracing the quantum critical (QC) region are schematic. The calculated transition temperature T^{hh} within the QC region is not well defined and its plot is switched to the dotted line with circles. (b) Energy as a function of gap amplitude Δ for $\mu = 2$, $h = 1.145$, $Q = 1.2$, $t_\perp = 0 - 0.2$. The energy minimum occurs at $\Delta = 0.22$ for $t_\perp < 0.15$.

$(-1)^\uparrow = 1$, $(-1)^\downarrow = -1$. Following Ref [10, 18], we measure all lengths in the 1D scattering length $a_{1D} = -\frac{2\hbar^2}{m g_{1D}}$ and all energies by the 1D bound energy $\epsilon_B = \frac{\hbar^2}{m}\frac{1}{a_{1D}^2}$. The dimensionless parameters in this model are $t_\perp/\epsilon_B$, μ/ϵ_B, h/ϵ_B, and b/a_{1D}. The attractive interaction implies negative g_{1D} and thus positive a_{1D}. The relation between a_{1D} and a_{3D} is [19] $a_{1D} = -a_\perp\left(\frac{a_\perp}{a_{3D}} - \frac{1.4603}{\sqrt{2}}\right)$ with $a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$. The typical $a_\perp$ is of the order 100nm and a_{3D} can be controlled by the Fashbach resonance. Taking $a_{1D} = 100$nm, $m = 6/(6 \times 10^{23})$g (^{6}Li) [11], the bound state energy is $\epsilon_B \sim 1.5 \times 10^{-6}$K.

To obtain the effective theory at given parameters, we first solve the Hamiltonian by the variational method where the order parameter is assumed to be sinusoidal $\Delta_{i_x}(z) = \Delta(z) = \Delta_0 \cos(Qz) = g_{1D}\langle c_{i_x,\downarrow}(z) c_{i_x,\uparrow}(z)\rangle$. $f(z)$ in Eq. (2) is chosen to be $\cos Qz$. The mean field (Bogoliubov de Gennes) Hamiltonian is

$$H_{mf} = \sum_{\vec{k}} (c^\dagger_{\vec{k},\uparrow}, c_{-\vec{k},\downarrow}) H(\Delta_0.Q) \begin{pmatrix} c_{\vec{k},\uparrow} \\ c^\dagger_{-\vec{k},\downarrow} \end{pmatrix} + \sum_{\vec{k}} \xi_{\vec{k},\downarrow} + L_z N_x \frac{\Delta_0^2}{2|g_{1D}|} \quad (5)$$

where

$$H(\Delta_0, Q) = \begin{pmatrix} \xi_{\vec{k},\uparrow} & B \\ B & -\xi_{-\vec{k},\downarrow} \end{pmatrix} \quad (6)$$

with the block $B = \frac{\Delta_0}{2}\delta_{k_x,-k_x}(\delta_{k_z+Q,-k_z} + \delta_{k_z-Q,-k_z})$. The Δ_0 and Q are determined by minimizing the free energy with respect to Q and Δ_0 given by

$$F[\Delta] = -T\sum_n \log[1 + e^{\frac{-\epsilon_n}{T}}] + \sum_{\vec{k}} \xi_{\vec{k},\downarrow} + L_z N_x \frac{\Delta_0^2}{2|g_{1D}|} \quad (7)$$

where ϵ_n are *all* eigenvalues of the matrix $H(\Delta_0, Q)$. In the $T \to 0$ limit, $-T\sum_n \log[1 + e^{-\epsilon_n/T}] = \sum_n \epsilon_n \Theta(-\epsilon_n)$.

The energy cost for given strain configurations $u(x,z)$, $\theta(x,z)$ of elastic fields is computed by $\Delta F(b) = F[\Delta_0 e^{i\theta} f(z+u)] - F[\Delta_0 f(z)]$, which to the lowest order of u and θ in quasi-1D reduces to the form

$$\Delta F(b) = b\sum_{x_i}\int dz\left[\frac{A(b)}{2}(Qu_x)^2+\frac{B(b)}{2}(Qu_z)^2 + \frac{C(b)}{2}\theta_x^2+\frac{D(b)}{2}\theta_z^2\right] \quad (8)$$

where in the quasi-1D system $u_x(x_i) \equiv \frac{u(x_i+b)-u(x_i)}{b}$, $\theta_x(x_i) \equiv \frac{\theta(x_i+b)-\theta(x_i)}{b}$. Replacing $b\sum_{i_x} \to \int dx$, Eq. (8) is the same as Eq. (3).

We now show that the KT transition temperature is independent of the intertube distance b when $t_\perp$ is fixed. For simplicity we only consider the stripe oscillation field u. The same argument applies to the phase field θ. When $b \to \alpha b$, the total free energy cost due to compression or stretching along z is unchanged (since the energy depends only on $t_\perp$ which is fixed) but the energy density changes. Consequently $\Delta F_z(b) = \Delta F_z(\alpha b)$ implies

$$bN_xL_z\frac{B(b)}{2}(Qu_z)^2 = \alpha bN_xL_z\frac{B(\alpha b)}{2}(Qu_z)^2 \quad (9)$$

leading to $B(\alpha b) = B(b)/\alpha$. The free energy caused by u_x depends only on intertube coupling $t_\perp$ and therefore the elastic field difference between two adjacent tubes $u(x+b)-u(x)$. When $b \to \alpha b$, as long as $u(x+b)-u(x) = u(x+\alpha b)-u(x)$ (same $t_\perp$) the total free energy remains unchanged which leads to

$$\frac{A(b)}{2}L_z b\sum_x Q^2\left(\frac{u(x+b)-u(x)}{b}\right)^2 = \frac{A(\alpha b)}{2}L_z(\alpha b)\sum_x Q^2\left(\frac{u(x+\alpha b)-u(x)}{\alpha b}\right)^2 \quad (10)$$

which leads to $A(\alpha b) = \alpha A(b)$. Their product $A(b)B(b)$ is independent of b, so is the transition temperature $T^{KT} \propto \sqrt{AB}$. Since our main interest is the transition temperature, we take $b = a_{1D} = 1$.

To obtain coefficients A, B, C, D in Eq. (3), we take the following approach. Take B as an example, we choose $u(x,z) = u_z z$, $\theta(x,z) = 0$, compute $\Delta F(b)$ for several u_z, and fit $\delta F(b,u_z) \equiv \Delta F(b;u_z)/(bN_xL_z) = \frac{B(b)}{2}(Qu_z)^2$ as expressed in Eq. (8). The same procedure apply to A, C, D. There is another approach to obtain these coefficients involving Green's function [20] which requires computing the inverse of a matrix and is very time-consuming. Our approach instead only involves the computation of eigenvalues [21] which allows us to include more k-points.

Some key steps of computing $\delta F(b,u_z)$ are are summarized here. To obtain the coefficients C and D requires computing the eigenvalues of

$$H(\theta) = \begin{pmatrix}(h_0-\mu)+h & \Delta_0 f(z)e^{i\theta}\\ \Delta_0 f(z)e^{-i\theta} & -(h_0-\mu)+h\end{pmatrix} \quad (11)$$

where h_0 is diagonalized in momentum space as $\frac{\hbar^2k_z^2}{2m} - 2t_\perp\cos bk_x$. By performing a local gauge transformation $c(z) \to c(z)e^{-i\theta/2}$, $H(\theta)$ in new coordinate becomes $H_1(\theta)$ [20, 22] which is

$$H_1(\theta) = \begin{pmatrix}\xi^+(\vec{k}) & \Delta_0 f(z)\\ \Delta_0 f(z) & -\xi^-(-\vec{k})\end{pmatrix} \quad (12)$$

where $\xi^+(\vec{k}) = \xi_{\vec{k}+\vec{v}/2,\uparrow}$ and $\xi^-(-\vec{k}) = \xi_{-\vec{k}+\vec{v}/2,\downarrow}$ with $\vec{v} = \vec{\nabla}\theta/2$. The eigenvalues of H and H_1 are identical, but this transformation automatically obtains the derivative of θ, i.e. θ_x and θ_z, in the diagonal blocks. We use $\mu = 2$, $h = 1.145$, $t_\perp = 0.1$ as an example. Minimizing the energy functional with respect to Δ_0 and Q leads to $Q = 1.2$, $\Delta_0 = 0.22$. Fig. (3) shows $\delta F(\theta_z)$ for $\theta(x,z) = \theta_z z$ and $\delta F(\theta_x)$ for $\theta(x,z) = \theta_x x$ from which the quadratic fit leads to $C = 0.00168$ and $D = 0.23$.

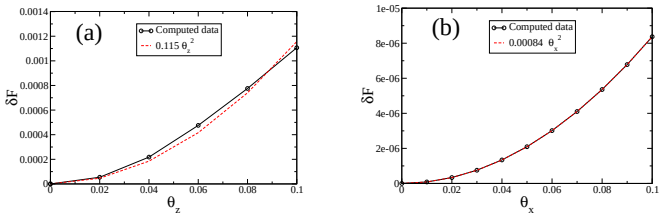

FIG. 3: Energy density as a function of θ_z and θ_x for $\mu = 2$, $h = 1.145$, $Q = 1.2$, $\Delta_0 = 0.22$..

To obtain the coefficients A and B requires computing the eigenvalues of

$$H(u) = \begin{pmatrix}(h_0-\mu)+h & \Delta_0 f(z+u)\\ \Delta_0 f(z+u) & -(h_0-\mu)+h\end{pmatrix}. \quad (13)$$

A useful trick is to do the calculation in a new coordinate whose order parameter is exactly $\Delta_0 f(z)$[20]. We stress here the Jacobian arising from the coordinate transform has to be taken into account because it is the free energy, not the free energy density, which is invariant under coordinate transformation. Again we use parameter $\mu = 2$, $h = 1.145$, $t_\perp = 0.1$ as an example. Fig. (4) shows $\delta F(Qu_z)$ for $u(x,z) = u_z z$ and $\delta F(Qu_x)$ for $\theta(x,z) = u_x x$ from which we can fit $A = 0.00168$ and $B = 0.234$. Note that $f(z)$ is taken to be $\cos Qz$ for the results presented here. However we emphasize that the coefficients A, B, C, D can be obtained for any given order parameters.

We compute the coefficients for several $t_\perp$ and determine all three KT transition temperatures. Our main result is shown in Fig. (2)(a) where the phase diagram as a function of T and $t_\perp$ is plotted for a representative set of parameters $\mu = 2$, $h = 1.145$, $Q = 1.2$, $\Delta_0 = 0.22$. When $t_\perp = 0$, there is no intertube coupling and no correlation along x leading to zero T^{KT}. As the system goes from pure 1D to quasi-1D, $T^{KT}(t_\perp) \propto t_\perp$. More specifically, we found the coefficients associated with z derivative, i.e. B and D, depend very weakly on $t_\perp$ whereas

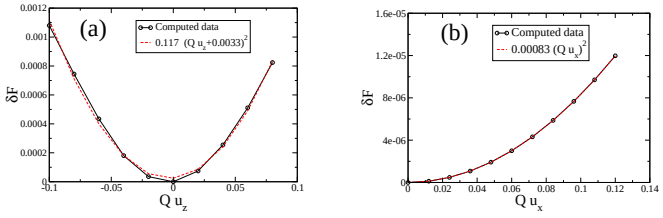

FIG. 4: Energy density as a function of u_z and u_x for $\mu = 2$, $h = 1.145$, $Q = 1.2$, $\Delta_0 = 0.22$.

those with x derivative, i.e. A and C, depend quadratically on $t_\perp$. This explains the linear $t_\perp$ dependence of T^{KT} ($\propto \sqrt{AB}, \sqrt{CD}$). We found T^θ and T^u are very close because the "cosine" ansatz is very close to two *decoupled* FF order with opposite wave vector for small Δ_0 and it is the coupling between u^+ and u^- (the fluctuations of two FF orders) which lifts the degeneracy of u and θ fields. In this case the proliferation of half-vortex half-dislocation costs the least energy and the system only undergoes one transition from LO to normal at T^{hh} when raising the temperature, as shown in the case I of Fig. (1)(c). The transition temperature is of the order of 0.03 ϵ_B which is roughly 8×10^{-8}K for the system of ^{6}Li with $a_{1D} = 100$nm. One also notes that the obtained $T^{KT}(\sim 0.03)$ is an order of magnitude smaller than the mean field gap $\Delta_0(= 0.22)$, so the coefficients computed at $T = 0$ are almost identical (less than 1% difference) to those computed at $T \sim T^{KT}$.

At $T = 0$, our simulation suggests the quantum phase transition from LO to normal phases upon increasing $t_\perp$ is of first order. Fig. (2)(b) shows the energy as a function of Δ for $\mu = 2$, $h = 1.145$, $Q = 1.2$, $t_\perp \in (0, 0.2)$ where the minimum determines the value of Δ_0. We found that as $t_\perp$ increases Δ_0 stays around 0.22 and when $t_\perp > 0.16$ Δ_0 becomes zero. Around $t_\perp = 0.16$, the $E(\Delta)$ is essentially flat with several shallow minima. We note that the FFLO to normal transition as a function of temperature (fixed μ, and h) [8, 9] or h (fixed μ, T) [23] is also of first order. At finite temperature around the critical $t_\perp$, the system is in the quantum critical region whose behavior is under heavy investigations [24, 25] which we will not discuss here.

We have assumed a sinusoidal order parameter in the current calculation. However near BCS/LO transition (Fig. (1)(b)) [10, 18], the order parameter behaves more domain-wall like [23, 26] than sinusoidal. Therefore the ansatz with sinusoidal order parameter does not capture all physics. Close to the BCS/LO transition, we expect that the stripe fluctuation should be stronger than the phase fluctuation ($A < C$, $B < D$) and a two-stage transition with charge 4 superfluid shown as the case III in Fig. (1)(c) can happen. In a cold atom trap where the chemical potential is a position-dependent, the interface between phases shown in Fig. (1)(c) is unavoidable and worth investigating.

In summary, we have computed from a microscopic model the effective theories of Goldstone modes of the LO order for a quasi-1D fermionic system from which the Kosterlitz-Thouless transition temperatures are determined. The transition temperatures are found to depend linearly on the intertube coupling $t_\perp$. As $t_\perp$ increases, the system goes to a quantum critical regime sandwiched by the LO and normal phases. Our approach can generally determine the Goldstone excitations of any stripe order involving charge or spin from a microscopic model which should be useful for comparison between theories and experiments.

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