THE CONCEPT OF AN ORDER AND ITS APPLICATION FOR RESEARCH OF THE DETERMINISTIC CHAINS OF SYMBOLS

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Abstract

The present work is dedicated to searching parameters, alternative to entropy, applicable for description of highly organized systems. The general concept has been offered, in which the system complexity and order are functions of the order establishment rules. The concept of order poles has been introduced. The concept is being applied to definition of the order parameter (OP) for non-random sequences with equal number of zeros and ones. Properties of the OP are being studied. Definition of the OP is being compared to classical definition of amount of information.

1 The complexity and order concept

The determined chain term means the readable object that gives the same sequence of characters for any rereading. These chains are, for example, DNA, RNA, proteins, any book on the shelf. The issue of how to determine degree of order and amount of information in this system is under discussion since publication of Shannon's [1] works. The Shannon's entropy is the parameter that describes uncertainty of the system. Information is regarded as elimination of uncertainty and is calculated provided probabilities of occurrence of symbols from a given alphabet are defined [2]. Applying entropy to determined sequences we simulate it by random sequence of characters. In 1965, A.N. Kolmogorov [3] turned his attention to limitations of the probabilistic approach in determining the amount of information and offered an algorithmic approach, known today as the Kolmogorov's complexity. The Kolmogorov's complexity does not depend on any probabilistic assumptions, but the sequence-generating algorithm.

The foregoing does not detract from the importance of the probabilistic method, in particular, for statistical processing of experimental data in molecular biology. In this way, in works [4, 5, 6] the Logo for certain DNA sections is built based on the Shannon's information.

However, it is very problematic to perceive information contained in biological systems as eliminated uncertainty. Describing the living matter as a complex highly organized system, we nevertheless know [7] that entropy of a piece of rock is not much higher than that of living creatures of the same mass. In quest for other definitions of complexity and order, we offer the following concept.

Consider a system described by a set of discrete parameters that determines its state. The system can be a real physical or abstract. Let us say the system has a complexity if:

- The external system, which, affecting this system according to certain rules or natural laws, transforms it from one state to another, can be built (defined).
- A positive value T, which describes external influence, resulting in the $A \to B$ transition, is defined. Let us regard minimum value of T as complexity of the $A \to B$ transition. For example, complexity for the physical systems can be minimal energy or time necessary for transition from one state to another.

We shall establish that a system may be considered ordered provided it is possible to define specific states, known as poles, defined by the following:

- There is a parameter describing the system, which reaches its extremes at the poles.
- Complexity of transition between the poles is maximal.

In ordered systems, degree of the state order A can be described as a function of transition complexities from state A to the order poles. In the above concept, complexity of transitions and the system's order states depend on ordering of the external influence. Such definition may seem very strange. Let us recall, however, the assertion "living comes from the living only" that nobody has neither proven no refuted as yet. For our purposes, this statement can be formalized as follows:

The system can be ordered only with another ordered system's help.

Now one may easily agree that degree of the system's order depends on who performs the procedure and how.

The order poles can be seen in simple physical systems. Let there be a mixture of solid and liquid phase at the phase transition temperature. Parameters, describing the system, are masses of m_1 and m_2 phases, which are, generally speaking, discrete. The system is closed - the $M = m_1 + m_2$ mass is constant. Let us build an external system - the calorimeter, allowing for heat transfer. Energy, transferred to the system, changes both m_1 and m_2, and its absolute value may be considered the transition complexity. Maximum complexity is achieved upon complete transition from one phase to another. Thus, there are two state poles. Another physical example is an atom in different energy states. The order poles in this case - the ground state and the ion. In systems, described by multiple parameters, there can be a large number of poles. The order poles concept may be of interest when studying biological systems. A very general idea, described in the present study, is realized to determine the order parameter for a finite sequence of symbols with equal numbers of ones and zeros.

2 The minimum information poles in the classical theory

Since the order poles concept is central to this paper, we consider it necessary to present preliminary researches and show that within the framework of classic information theory it is possible to define the poles as states of perfect order with minimum information.

A remark from work [8] brings us to the idea of zero information determination: "DNA can not be periodic, just as any sequence of letters in a meaningful textbook". Intuitive assertion of minimum information for a periodic chain can actually be obtained by strict formulation of the problem.

Let us examine long chains, composed of zeros and ones, assuming it is possible to calculate probabilities of character appearance using a statistical method. Based on these probabilities we can calculate the amount of information that a chain of known length contains. We restrict ourselves to Markov's chains of the first order.

First, let us note that the ideal periodic sequence of symbols can be simulated in the Shannon's classic model. Of course, information in a periodic chain of two characters with a known first character equals zero. However, it is important to figure out behavior of the Shannon's information in case of a small deviation from the chain periodicity. Besides, for comparison with the order parameter we shall need the maximum information condition for the Markov's chain.

Let us consider a sequence in which 0 and 1 occurs with equal probabilities. Let us further suppose that in the $2n$ symbols long chain there are on average k pairs of 00 sequences. (To calculate pairs without restricting generality, we can close the chain into the ring. If $s \geq 2$ zeros follow in sequence, we believe there are $s-1$ pairs of 00. It is then easy to calculate [2] the amount of information in such $2n$ symbol long chain:

$$H(k) = 1 - \frac{2n-1}{n}[k \log_2 \frac{k}{n} + (n-k) \log_2 \frac{n-k}{n}] \tag{1}$$

Analyzing function (1), let us write down the extreme values of $H(k)$ at $n \gg 1$, as well as values in the field $0 \leq k < n$, immediately following the minimum in the ascending order.

$$H_{max} = H(n/2) = 2n\,bit \tag{2}$$

$$H_{min} = H(0) = 1\,bit \tag{3}$$

$$H_1 = H(1) = 2\log_2 n\,bit \tag{4}$$

$$H_2 = H(n-1) = 2\log_2 n\,bit \tag{5}$$

We get the maximum amount of information when pairs of 00 and 11 appear with the same frequency as pairs of 01 or 10. This situation corresponds to a chain without memory.

Let us consider the situation near information minimum. At $k = 0$, we have a deterministic periodic chain (hereinafter S_+) with period 01, starting either from zero or one:

$$0101010101... \tag{6}$$

The amount of information in a periodic chain of $2n$ symbols equals the one carried by the first symbol. If the first character is known, then, as we have already noted, information equals zero.

At $k = 1$, the chain is little different from the previous one, namely, on average for every $2n$ characters there is a 00 and a 11, for example:

$$0101010110010101010 10... \quad (7)$$

At $k = n - 1$ we get a chain in which for $2n$ symbols there is on average one pair 01 and one pair 10, the rest are 00 and 11. At $n \gg 1$ such chain approaches the deterministic chain in which n zeros is followed by n ones:

$$00000...011111...1 \quad (8)$$

Let us denominate chain (8) as S_- and call it the zeros and ones separation chain. This pertains to both open and closed chains. In the latter case, it is of no importance whether all zeros are located on the left or on the right.

Let us point out here, that a sequence like (8) can be obtained by means of the Markov's chain of order n. Examining the deterministic chain, we label the transition probabilities with 0 or 1 values only. Such probabilities result in generating of periodic sequences with different periods. The smallest period contains one character, the largest - $2n$ symbols. It is therefore possible to distribute the transition probabilities in such a way as to generate a strictly periodic chain, in which n zeros should be followed by n ones. Generally speaking, any of the periodic sequences may be regarded as an example of ideal order. If we restrict ourselves to the Markov's chain of the first order and the case with a uniform distribution of zeros and ones, then let us consider three almost ideal orders, corresponding to minimum information and described by formulas (3) - (5).

Order (7) is only a minor violation of order (6). However, in transition from $k = 0$ to $k = 1$, i.e. with minimum periodicity disruption, the quantity of information leaps infinitively within $n \to \infty$. From the information theory viewpoint, this is not surprising - it is the same as the infinite information of a black point on a white background. But when describing a real physical object such result would appear unsatisfactory.

The sequences of ideal order (6) and (8), obtained within the framework of the discussed problem, are examined below as order poles.

3 Determination of order poles and parameter based upon the connection function

Let us define the function of connection between two adjacent symbols $u_{i,i+1}$. Such function may have a real sense, for example, the binding energy between pairs of complementary nucleotides in the DNA. Not so long ago, these energies have been measured [9]. Let us build a sum, properties of which relate to the idea of order poles and parameter.

$$F = \sum_{i=1}^{2n} u_{i,i+1}, \tag{9}$$

where in case of a closed circular chain we have $u_{2n,2n+1} = u_{2n,1}$, whereas for an open chain, adding up in (9) exists only until $2n - 1$.

For a chain, consisting of zeros and ones, the connection function argument for its next-door neighbors are four possible sequences 11, 10 ,01, 00. Accordingly, we'll define 4 values for $u_{i,i+1}$: ε_{11}, ε_{10} ,ε_{01}, ε_{00}. For the sake of symmetry let us put $\varepsilon_{10} = \varepsilon_{01}$. Let us break a chain into blocks. The block here and below is a sequence of identical symbols, each of them surrounded by another symbol from both sides. Let the number of blocks of one of the symbols be equal to b. It is clear that the number of blocks of the second character and the number of 01 or 10 pairs in the chain also equal b. Since there is an equal number of zeros and ones in the chain, the number of 00 and 11 pairs in it equals $k = n - b$. Then, from (9) it should be:

$$F = n\left(\varepsilon_{11} + \varepsilon_{00}\right) + b\Delta\epsilon \tag{10}$$

$$\Delta\epsilon = 2\varepsilon_{10} - \left(\varepsilon_{11} + \varepsilon_{00}\right) \tag{11}$$

For the long chains, the entered total function of connections of the next-door neighbors (9) by structure equals result (1), obtained based upon the first-order Markov chain's model. Conformity of formulas (10) and (1) is reached assuming that

$$\varepsilon_{11} = \varepsilon_{00} = -\log_2 \frac{n-b}{n} \tag{12}$$

$$\varepsilon_{10} = -\log_2 \frac{b}{n} \tag{13}$$

Assuming that the order formula, based on a probabilistic model, corresponds to the general structure (9), we can try to determine the order parameter, based on the latter. Unlike probabilistic model, where $u_{i,i+1}$ are determined statistically and depend on the state of the entire chain, we assume that parameters ε_{11}, ε_{10} ,ε_{01} are constant, which is justified, because the adjacent elements of the chain are "unaware" of the chain's general status. First and foremost, let us find the extremums of function (9), being limited to the closed chain. In two cases, when $F(b)$ differs from the constant, we have:

$$\Delta\epsilon < 0 \Rightarrow F_{max} = F(0),\ F_{min} = F(n) \tag{14}$$

$$\Delta\epsilon > 0 \Rightarrow F_{max} = F(n),\ F_{min} = F(0) \tag{15}$$

Thus, the state of ideal periodicity $b = n$ and the state of complete separation $b = 0$ (disintegration on two) appear to be very specific indeed - in these states either minimum or maximum of function (9) is achieved.

Function (9) has an additivity property almost relative to the cross-linking operation of two chains A and B (not necessarily with the equal number of zeros and ones):

$$F(A\_B) = F(A) + F(B) + \gamma \tag{16}$$

where A_B denotes the chain cross-linked together, γ is a parameter that depends on symbols in the cross-link point . If two chains with open ends are cross-linked, then $\gamma = u_{i,i+1}$, where $i, i+1$ are numbers of the cross-linked elements in chain A_B. It is also possible to cross-link chains, closed in a ring. For this purpose let us cut each in some place and cross-link the ends. If, when cross-linked, all four elements are identical, or if one of A bonds with zero of B, and one of B bonds with zero of A, then $\gamma = 0$. If two zeros of A bond with two ones of B, then $\gamma = \Delta\epsilon$. If zero of A bonds with zero of B, and one of A bonds with one of B, then $\gamma = -\Delta\epsilon$. In order to introduce the order parameter, you must determine what properties it should possess. We can act by analogy with the way in the classic theory a formula for the Shannon's information is being derived from the axioms. May we remind that one of the axioms is the information additivity. The role of similar axiom in our case will be played by the "near-additivity" property (16). A set of functions, defined on closed chains A_n with equal numbers of zeros and ones and possessing property (16), can be represented as:

$$f(A_n) = Cn + b\Delta\epsilon \tag{17}$$

where C is an arbitrary constant. Let us define the order parameter $K(b,n)$ in the form of renormalized function (17):

$$K(b,n) = \beta f(A_n) \tag{18}$$

We want the order parameter to possess certain symmetry at the poles. Let us note that the symmetry requirement $K(0,n) = K(n,n)$ is insoluble for the C constant, it is therefore natural to demand the antisymmetry property of on the poles:

$$K(0,n) = -K(n,n) \tag{19}$$

Determining the C constant in accordance with (19) and choosing β for considerations of convenient normalization, we obtain:

$$K(b,n) = 2b - n \tag{20}$$

For an ideally periodic chain we get $K = n$, in case of separation of zeros and ones $K = 2 - n$ and at disintegration into two $K = -n$. As we shall see below, result (20) is a particular case of another order parameter determination.

4 Basic idea. Measure of order for a finite sequence, containing equal number of zeros and ones.

Classic definitions of information amount are based on a postulate: there is a mechanism of symbol sequence generation. In the Shannon's model there must be an apparatus that generates characters with certain probabilities. The Kolmogorov's complexity [10] is determined by existence of the algorithm, which can create a specified sequence, and the amount of information depends on the algorithm proper. It is clear that if some sequence of symbols is set, then without defining its generation mechanism or research by some procedures it is impossible to define what we would like to call the amount of information or degree of a symbol sequence order. We shall proceed from the idea whereby instead of a generating mechanism a transformation mechanism is being used, which does not change the chain composition. The idea of comparing binary words by means of transformation is known from works [11], [12]. Let us consider a set of sequences, in which there are n ones and n zeros. All such sequences differ in the location of symbols, we may call the S_i state, where i is a state index. Under transformation we understand transition of sequence from S_i to S_j by means of transfer of one or several adjacent symbols from one part of sequence to the other. Let us call each such transition a step. We believe that transitions should be executed under certain rules, which we hereinafter call the ordering method Ω. Let us define following requirements to Ω:

- The ordering method must determine what kind of symbol transfers are allowed.
- The ordering method must allow transition from any state S_i to any state S_j in a certain number of steps. The minimum number of steps required for this transition, we hereby denote as $T(S_i, S_j, \Omega) = t_{ij}$. For symmetry sake , we also require $t_{ij} = t_{ji}$.

The examined system assumes introduction of order poles S_+ (6) and S_- (8) in accordance with the above concept. Indeed, S_+ is maximum and S_- is minimum in terms of number of 01 and 10 pairs. Relative complexity of the S_+ and S_- poles appears maximum for the Ω rules, considered below, i.e. the minimum number of steps necessary for transition between poles, is no less than minimum number of steps for transition between any other two states.

$$t = T(S_+, S_-, \Omega) \geq T(S_i, S_j, \Omega) \tag{21}$$

(in (21) S_i or S_j not pole).

Now we want to build a function, which will compare every state with a number, the meaning of which is degree of the symbol chain ordering. Since we have identified two states of the ideal order S_+ and S_- , an arbitrary state of S_i according to the general idea is characterized by two parameters, determining the number of steps you need to reach the poles from the S_i state applying the Ω rules.

$$t_{i+} = T(S_i, S_+, \Omega) \qquad t_{i-} = T(S_i, S_-, \Omega) \tag{22}$$

Let us look for the order parameter $K(S_i)$ as a function $K(t_{i+}, t_{i-})$ that satisfies the next properties (for brevity sake, dependence on Ω in certain places is being ignored):

- First, unlike properties of classic entropy, we would like small changes in a sequence to always result in small changes of the order parameter. We shall formulate a stricter requirement. Let one step to have been accomplished according to the Ω rule and the sequence transferred from state S_i to S_j . In the new state we have an order parameter $K(t_{j+}, t_{j-})$. If the sequence is long enough, then a one-step transition is local, i.e. changing order in small part of the sequence only. It is therefore natural to require that change in the order parameter under such transition from any state would not depend on the arrangement of all sequence in general, i.e. would not depend on t, t_{i+} and t_{i-} , but exclusively on $\Delta t_+ = t_{j+} - t_{i+}$ and $\Delta t_- = t_{j-} - t_{i-}$:

$$K(t_{j+}, t_{j-}) - K(t_{i+}, t_{i-}) = C(\Delta t_+, \Delta t_-) \tag{23}$$

- The second requirement is the anti-symmetry of the poles' values:

$$K(0, t) = -K(t, 0) \tag{24}$$

(Should we require symmetry of the order parameter on the poles instead of anti-symmetry (24), then from (23) we get $K \equiv Const.$)

Property (23) is the necessary and sufficient condition for linearity of function $K(t_{i+}, t_{i-})$, whereas consideration of (24) allows us to build a formula for the calculation of the order parameter.

Necessity. Suppose function $K(t_{i+}, t_{i-})$ is linear.

$$K(t_{i+}, t_{i-}) = \alpha t_{i+} + \beta t_{i-} + c, \tag{25}$$

where values α, β, c - are constants. Then :

$$K(t_{j+}, t_{j-}) - K(t_{i+}, t_{i-}) = \alpha \Delta t_+ + \beta \Delta t_-, \tag{26}$$

at that, due to minimality of $t_{i+}, t_{i-}, t_{j+}, t_{j-}$, variables $\Delta t_+, \Delta t_-$ can only have values of 0,1,-1.

Sufficiency. Suppose difference $K(t_{j+}, t_{j-}) - K(t_{i+}, t_{i-})$ is independent of t_{i+}, t_{i-} for all i states. In particular, it should be true for all states the chain passes along its shortest way from S_+ to S_- . For every such transition we have:

$$t_{j+} = t_{i+} + 1 \qquad t_{j-} = t_{i-} - 1 \tag{27}$$

Then, during this transition

$$K(t_{j+}, t_{j-}) - K(t_{i+}, t_{i-}) = a, \tag{28}$$

where a is constant. From (27) and (28) we get:

$$a = \frac{1}{t}(K(t,0) - K(0,t)) \tag{29}$$

$$K(t_{i+}, t_{i-}) = \frac{1}{t}(K(0,t)t_{i-} + K(t,0)t_{i+}) \tag{30}$$

From the locality requirement it follows that a in (29) does not depend on t. Using the antisymmetry requirement for the poles' order parameter and choosing a suitable scale, we get a formula to calculate the order parameter:

$$K(t_{i+}, t_{i-}) = t_{i-} - t_{i+} \tag{31}$$

Now enter the relative order parameter of the S chain.

$$k(S, \Omega) = \frac{K(S, \Omega)}{t(\Omega)} \tag{32}$$

Then for any ordering method, we get:

$$k(S_+, \Omega) = 1, \quad k(S_-, \Omega) = -1 \tag{33}$$

Enter determination of chaos. We talk about the state of chaos, if:

$$k(S, \Omega) = 0 \tag{34}$$

or about the near-chaos state, if:

$$|k(S, \Omega)| \ll 1 \tag{35}$$

Let us also define a value we call Ω-information in the S chain:

$$I = I(S, \Omega) = \begin{cases} T(S, S_+, \Omega), & K(S, \Omega) \geq 0 \\ T(S, S_-, \Omega), & K(S, \Omega) < 0 \end{cases} \tag{36}$$

That is, the amount of Ω-information equals minimum number of transfers for transition to the nearest pole at a given Ω. Note, that the absolute order parameter and the Ω-information are not additive values in terms of bonding operations:

$$K(A\_B, \Omega) \neq K(A, \Omega) + K(B, \Omega) \tag{37}$$

5 The order parameter for some methods of transfer.

Considering the various ways of transfer further, we shall be interested in finding such symbol location conditions that lead to the state of chaos. Another issue is to find such Ω, for which condition of the order parameter's additivity is met with respect to chain bonding operations. It is also interesting to see how the order parameter and the amount of Ω-information will change with a small chain deviation from its ideal order.

5.1 Method of a single symbol arbitrary transfer.

Let us assume it is allowed to take any symbol and to transfer it by inserting between any two symbols or to the end of the line. Let us denote such sequencing method as Ω_1. The number of transfers required for transition between the poles (6), (8) by means of Ω_1 is:

$$t(\Omega_1) = n - 1 \tag{38}$$

It is easy to verify that (21) holds, and in the case of closed chains, it is strict. For the poles (6), (8) we get:

$$K(S_+) = n - 1, \quad K(S_-) = -n + 1 \tag{39}$$

For brevity, in this section we present results for open chains only.

Small deviation from the ideal order. Let us consider chain S_1 , near to periodic, similar to (7), in which there are $2n - 2$ pairs of 01 or 10, and once we have both 00 and 11. (Although we discuss an open chain, when counting pairs it is more convenient to regard the chain as closed in the ring.)

$$T(S_1,S_+)=1=I,\quad T(S_1,S_-)=n-i,\quad K(S_1)=n-i-1 \tag{40}$$

In (40) $i=1$, in case 00 and 11 are inside the chain and $i=2$, if 00 or 11 are on its edge. The relative order parameter for the examined chain:

$$k(S_1)=1-\frac{i}{n-1} \tag{41}$$

Note also that

$$K(S_+)-K(S_1)=i,\quad k(S_+)-k(S_1)=\frac{i}{n-1} \tag{42}$$

Thus, unlike the Shannon's entropy, a small deviation of an arbitrarily long chain from the perfectly periodic one causes a finite change of the order parameter and the Ω-information amount, whereas difference in relative order parameters tends to zero.

Let us consider a near-separation chain (8). We assume one of the ones has been transferred and introduced between zeros. Let us denote such chain as S_2. Then:

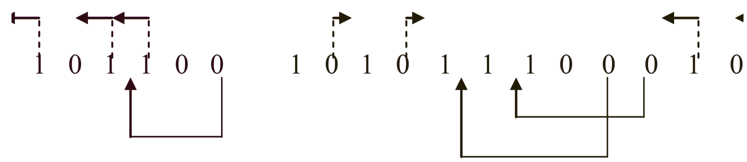
$$K(S_2)=3-n,\quad K(S_-)-K(S_2)=-2. \tag{43}$$

In this case, for an arbitrarily long chain the change of the absolute order parameter is finite as well, whereas the amount of Ω-information in such chain is $I(S_2)=1$.

The order parameter calculation formula. Let us give an example of calculation of the order parameter and the Ω-information amount for the chain S in the figure below. Upper pointers show transfer to the state of separation, lower ones - to the state of ideal periodicity.

$$T(S,S_+)=3=I(S),\quad T(S,S_-)=6,\quad K(S)=3,\quad k(S)=\frac{3}{8}$$

It is clear that $T(S,S_+)$ always equals the number of intervals between consecutive ones or zeros (for exact calculation the chain should be closed into a ring). Determination of $T(S,S_-)$ is not that obvious. Let us break up the chain into blocks, number the blocks from left to right by means of the



i index and denote the s_i number of symbols in a block as ($s_i \geq 1$). Let us count the number of zeros and ones to the block's left. Let l_{si} be the number of symbols to the block's left (same as in the block proper), and $\bar{l}_{si}$ - the number of other symbols to the left. Then, to the block's right there are $n - s_i - l_{si}$ symbols of the block and $n - \bar{l}_{si}$ of other symbols. In order to pass to the state of complete separation it is necessary to transfer symbols of one kind (on the block's left) to the right, and symbols of other kind (on the block's right) to the left. We get two possible numbers of transitions:

$$x_1 = \bar{l}_{si} + n - s_i - l_{si} \tag{44}$$

$$x_2 = n + l_{si} - \bar{l}_{si} \tag{45}$$

Let us choose the smallest of the two variables x_1, x_2 by means of the function:

$$f_i = \min(x_1, x_2) = \begin{cases} n + l_{si} - \bar{l}_{si} & s_i \leq 2(\bar{l}_{si} - l_{si}) \\ \bar{l}_{si} + n - s_i - l_{si} & s_i > 2(\bar{l}_{si} - l_{si}) \end{cases}$$

Then:

$$T(S, S_-) = \min_i(f_i)$$

A few additional examples of calculation of the order parameter and Ω-information.

$$A: \ 0111111010000010011$$

$$T(A, A_-) = 5, \quad T(A, A_+) = 5, \quad K(A) = 0, \quad I(A) = 5,$$

$$B: \ 11101011100000$$

$$T(B, B_-) = 2, \quad T(B, B_+) = 4, \quad K(B) = -2, \quad I(B) = 2.$$

The "bonding" operation and discussion of additivity. Although operation of cross-linking chains A and B is not additive, as we have noted already (37), it is possible to discuss the near-additivity property.

$$K(A\_B) = K(A) + K(B) + s, \tag{46}$$

where s is a small number compared with each of the chains' length. Clearly, for the near-periodicity chains (6), condition (46) holds. The additivity is quintessentially violated for the near-separation chains. Let us consider bounding of two type (8) chains. Let chains A and B have length of $2m$ and $2k$ correspondingly and $k \geq m$:

$$A: \underbrace{111...11000...00}_{2m}, \ T(A, A_+) = m - 1, \ T(A, A_-) = 0, \ K(A) = 1 - m,$$

$$B: \underbrace{111...11000...00}_{2k}, \ T(B, B_+) = k - 1, \ T(B, B_-) = 0, \ K(B) = 1 - k,$$

$$C = A\_B: \underbrace{111...11000...00}_{2m}\underbrace{111...11000...00}_{2k},$$

$$T(C, C_+) = m + k - 2, \ T(C, C_-) = m, \ K(C) = -k + 2.$$

In the studied case:

$$K(A\_B) = K(A) + K(B) + m, \tag{47}$$

The "bonding" operation and struggle of orders. Let us consider the bonding operation of n identical A chains, each in the state of complete separation (s of ones followed by s of zeros ($s > 1$)):

$$A: \underbrace{111...11000...00}_{2s}, \quad k(A) = -1.$$

Let us call the resulting chain nA:

$$\underbrace{A\_A\_A...\_A}_{n} = nA = \underbrace{\underbrace{1...110...00}_{2s}\underbrace{1...110...00}_{2s}...\underbrace{1...110...00}_{2s}}_{n} \quad (48)$$

It easily to calculate, that:

$$T(nA, (nA)_+) = n(s-1)$$

$$T(nA, (nA)_-) = s(n-1)$$

The absolute order parameter equals:

$$K(nA) = n - s, \quad (49)$$

Ω-information:

$$I(nA) = \begin{cases} ns - n, & n \geq s \\ ns - s, & n < s \end{cases} \quad (50)$$

The relative order parameter equals:

$$k(nA) = \frac{n-s}{ns-1}, \quad (51)$$

At that

$$\lim_{n\to\infty} k(nA) = \frac{1}{s}, \quad (52)$$

Results (51) - (52) describe the struggle of orders. At $n = 1$, we have the order of separation. With n increasing, the relative order parameter increases, passes a near-chaos point, changes its sign when the chain periodicity begins to dominate and in the extreme reaches a value, determined solely by the period composition.

A more general result for the struggle of orders can be obtained if we consider the bounding operation of n identical chains B, $2s$ long and with equal number of zeros and ones, for each of which:

$$T(B, B_+) = m \leq s - 1 \quad (53)$$

We obtain:

$$T(nB, (nB_+)) = nm \quad (54)$$

$$T(nB, (nB_-)) = sn - l \quad (55)$$

(where $l \leq s$).

$$K(nB) = n(s - m) - l \tag{56}$$

$$\lim_{n\to\infty} k(nB) = 1 - \frac{m}{s} \tag{57}$$

5.2 Method of transfer by blocks.

Let us set the ordering rule Ω_2 that allows transferring the entire block or its part as a whole. It is easily understood that $t(\Omega_2) = t(\Omega_1)$. The calculation rule for $T(S, S_+)$ is the same, as in Ω_1, however it is different for $T(S, S_-)$. For an open chain when it begins and ends with the same symbol:

$$T(S, S_-) = b, \tag{58}$$

If the ends have different symbols:

$$T(S, S_-) = b - 1. \tag{59}$$

If the chain is closed, we have (59) and in all cases $T(S, S_+) = n - b$. The order parameter for the closed chain :

$$K(S) = 2b - n - 1. \tag{60}$$

Up to an irrelevant unit, formula (60) corresponds to result (20), obtained earlier in Section (3). Therefore, for Ω_2 the order parameter has the property of additivity almost under the bonding operation (46), where $s \in \{0, \pm 1\}$. Ω-information has this property only when $K(A)$ and $K(B)$ have the same sign or equal zero.

$$K(A)K(B) \geq 0 \Rightarrow I(A\_B) = I(A) + I(B) + s,\ \ s \in \{0, \pm 1\} \tag{61}$$

If $K(A)$ and $K(B)$ have different signs, the A_B cross-link generates new Ω-information. For example, if we take two chains of equal length $2n$, one of which is in the state of separation and the other one is perfectly periodic, the Ω-information in each of them equals zero, but after bonding we obtain $I = n$.

Both for the absolute order parameter and the amount of Ω-information, the property of near-commutativity holds:

$$K(A\_B) = K(B\_A) + s \tag{62}$$

$$I(A\_B) = I(B\_A) + s \tag{63}$$

where, as above, $s \in \{0, \pm 1\}$.

Ordering with blocks has one more important property, which we formulate as the chaos theorem.

Theorem of Ω_2 chaos. From (58)-(60) it follows that for the Ω_2 rules the state of chaos occurs when $n = 2b$ or $n = 2b - 1$.

Hence, for the chain in question the state of chaos in relation to transfer by blocks is achieved under the same conditions that exist when the Shannon's information maximum is reached for the first-order Markov's chain, i.e. when frequencies of 00 *and* 01 *pairs' occurrence are equal.*

Struggle of orders. Let us calculate the Ω_2 order parameter for chain (48).

We calculate the order parameter Ω_2 for the chain (48).

$$T(nA, (nA)_+) = n(s-1)$$
$$T(nA, (nA)_-) = n - 1$$

$$K(nA) = 2n - 1 - sn, \tag{64}$$

$$k(s,n) = k(nA) = \frac{2n - 1 - sn}{ns - 1}, \tag{65}$$

$$\lim_{n\to\infty} k(nA) = \frac{2}{s} - 1, \tag{66}$$

Similar to the case of a single symbol transfer, in the Ω_2 model the struggle of orders is also observed during cross-linking of identical sites with separation (65)-(66). At $s = 1$, regardless of n, we obtain the ideal periodicity state, for $s \to \infty$ we obtain the state of separation. The near-chaos state is reached already at $s = 2$. Difference from a single symbol transfer is clear - transfer by blocks shortens the path to the state of complete separation.

5.3 The adjacent elements' permutation method for the Ω_3 ring chains.

We shall consider closed chains with the equal number of ones and zeros. (We shall write these chains in a row and assume that the last element is joined to the first). Note, that in this S_- (8) is equivalent, for example, to the chain like:

$$\underbrace{00...0}_{r}\underbrace{11...1}_{n}\underbrace{00...0}_{n-r}$$

We shall introduce the Ω_3 ordering rule, which allows swapping two adjacent characters. In this case, for the chains in question ($n \geq 2$):

$$t(\Omega_3) = T(S_+, S_-) = T(S_-, S_+) = \begin{cases} 0.25n^2, & n = 2k, \\ 0.25(n^2 - 1), & n = 2k - 1, \end{cases} \tag{67}$$

where k is a natural number. (To get the minimum number of steps required for transition from (6) to (8), zeros should be transferred simultaneously from two opposite sides of the ones' block).

First of all, let us investigate sequence (7) denoted below as G. To do this we need an additional parameter l, denoting the minimal number of characters in a circle between 00 and 11 pairs. It is obvious, that l is even, at that $l \leq n - 2$. The number of steps we require to return the sequence to a perfectly periodic state is:

$$T(G, G_+) = 0.5l + 1 \tag{68}$$

The number of transfers for the transition to a complete separation state:

$$T(G, G_-) = t(\Omega_3) - 0.5l - 1 \tag{69}$$

From (68) -(69) and definition (31) we have:

$$K(G) = t(\Omega_3) - l - 2 \tag{70}$$

$$k(G) = 1 - \frac{l + 2}{t(\Omega_3)} \tag{71}$$

$$K(G_+) - K(G) = l + 2 \tag{72}$$

$$I(G,l) = 0.5l + 1 \tag{73}$$

$$k(G_+) - k(G) = \frac{l+2}{t(\Omega_3)} \tag{74}$$

Change of the relative order parameter, as before, tends to zero at $n \to \infty$ and constant l. However, change of the absolute order parameter and the amount of Ω-information increases linearly with increasing l. In particular, if 00 and 11 divide the chain into two equal parts, then $l = n-2$ and hence:

$$I(G, n-2) = 0.5n \tag{75}$$

Let us analyze the previously examined chain, obtained from the state of separation through transfer of one of the ones in a zeros' block.

$$E : \underbrace{000010...0}_{n+1}\underbrace{1111...1}_{n-1} \tag{76}$$

Suppose this one is located l zeros from the nearest interface of zeros and ones. Then:

$$T(E, E_-) = l, \quad T(E, E_+) = t(\Omega_3) - l \tag{77}$$

$$K(E) = 2l - t(\Omega_3), \quad k(E) = \frac{2l}{t(\Omega_3)} - 1 \tag{78}$$

$$K(E) - K(E_-) = 2l \tag{79}$$

$$k(E_)) - k(E_-) = \frac{2l}{t(\Omega_3)} \tag{80}$$

$$I(E,l) = l \tag{81}$$

General conclusions, made for chain G, are also true for E.

By specific examples one can easily verify that the near-additivity property is not met in relation to Ω_3 . To verify struggle of orders let us calculate the order parameter for the chain (48) assuming it is ring-shaped. Below we use the notion of $t(\Omega, j)$ - number of transitions from the ideal periodicity under the state of separation for a $2j$-long chain. For the adjacent symbols' permutation method we get:

$$k(nA) = \frac{s^2 t(\Omega_3, n) - nt(\Omega_3, s)}{t(\Omega_3, ns)} \tag{82}$$

:

$$k(nA) = \begin{cases} 1 - \frac{1}{n}, & s = 2i, \quad n = 2m, \\ 1 - \frac{s^2 - 1}{s^2 n}, & s = 2i - 1, \quad n = 2m, \\ 1 - \frac{1}{n} - \frac{1}{n^2}, & s = 2i, \quad n = 2m - 1, \\ 1 + \frac{(1 - s^2)(1 + n)}{s^2 n^2 - 1}, & s = 2i - 1, \quad n = 2m - 1 \end{cases} \tag{83}$$

Here, if s = 1, then $n \geq 2$, for the rest of s we have $n \geq 1$.

Result (83), generally speaking, is weakly dependent on s. In case $n = 1, s \geq 2$, we have the state of separation: $k = -1$. For $n = 2$ and any s, we already have $k > 0$, i.e. we skip the state of chaos, and with increasing n we have $k \to 1$. Thus, for large n in terms of the order parameter the chain behaves as an ideally periodic one (6).

6 Conclusion

Development of the suggested idea is subject to the following problems' solving:

- Generalization of the order parameter determination in case of unequal number of zeros and ones.
- Expansion of theory in case of any number of symbols in an alphabet.
- Establishing criteria for distinguishing one ordering method from another and selection of optimal ordering method.

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