SIMULATION OF COMPLEX PNEUMATIC SYSTEMS M.

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1. Introduction

Modern pneumatic systems consist of various functional elements such as compressors, engines, valves, cylinders, filters, pipelines etc. The processes occurring in these systems quickly vary, i.e. speed and pressure quickly vary in time. The speed of a sound in a pneumatic system is much less in comparison with the speed of a sound in a hydraulic system, therefore the dynamic processes in pneumatic system occur much slowly. The account of physical processes in pneumatic systems depending on the properties of the system has the significant applied importance.

The movement of gas in a pneumatic pipeline is accepted as one dimension and unsteady i.e. all local speeds are considered equal to average speed and depend on time. The pressure also is considered identical in all points of cross section and depends on longitudinal coordinate of a pipeline and on time. Such movement of gas is characterized by the occurrence of a wave of increased and lowered pressure which is distributed from the place of change of pressure and the deformation of walls of the pipeline.

The differential equations of the movement of gas in pipelines are solved by the finite difference method and more precisely - by a characteristics method [1, 2, 3].

The theory of characteristics is of paramount importance in the treatment of gas dynamics equations.

It is helpful in the solution of problems and in the physical interpretation of associated phenomena.

Let us consider the following set of quasi-linear partial differential equations. The term quasi-linear refers to the fact that the equations are linear in the derivates of the dependent variables, but in general are nonlinear.

2. Theoretical results

The system of continuity, momentum equations and perfect gas equation of state is [4, 5]:

$$\frac{\partial(\rho S)}{\partial t} + \frac{\partial(\rho v S)}{\partial x} = F_1(x), \quad (1)$$

$$\frac{\partial(\rho v S)}{\partial t} + \frac{\partial}{\partial x}\left[S(p + \rho v^2)\right] + \tau\Pi + S\rho a_x = F_2(x), \quad (2)$$

$$p = \rho RT, \quad c^2 = \gamma RT, \quad (3)$$

where P- gas density; p - pressure; v- velocity; $S = S(x)$- cross section area of a pipeline; τ -shear stresses on the inner surface of a pipeline; a_x- acceleration along x axis; c- wave speed in medium (gas); Y - ration of specific heat;

$F_1(x) = \Pi(x)m_{in}(x)$, $m_{in}(x)$ - distributed tributary mass through surface of a pipeline;

$F_2(x) = m_{in}(x)v_{in}(x)\frac{dS}{dx}$; $\Pi(x)$ - perimeter of inner surface of a pipeline; R - gas constant; j- temperature.

When the speed of the movement of gas is larger than the speed of a sound (v « c)the member pv^2 in the equation of a momentum can not be taken into account.

The system of the equations of dynamics of gas in the variable mass charge $Q = \rho v$ and pressure ρ looks like:

$$\frac{\partial p}{\partial t} + c^2\frac{\partial Q}{\partial x} = R_1, \quad (4)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial p}{\partial x} = R_2, \quad (5)$$

where

$$R_1 = c^2\left(\frac{F_1}{S} - Q\frac{d\ln S(x)}{dx}\right),$$

$$R_2 = \frac{F_2}{S} - \frac{\lambda(\mathrm{Re})\Pi Q|Q|}{S\rho} - \rho a_x,$$

$$\lambda(\mathrm{Re}) = \begin{cases} \left[\dfrac{0{,}556}{\lg\left(\dfrac{\mathrm{Re}}{7}\right)}\right]^2, & when \quad \mathrm{Re} \le 2300; \\ 0{,}1\left(\dfrac{\Delta}{d} + \dfrac{100}{\mathrm{Re}}\right)^{0.25}, & when \quad \mathrm{Re} > 2300; \end{cases}$$

Re - Reynolds number, $\mathrm{Re} = \frac{vd}{\nu} = \frac{RTdQ}{\nu p}$,

ν- kinematics viscosity of gas (air) [5]:

$$\nu = 10^{-5}\rho\left[1{,}712 + 5{,}8\cdot 10^{-3}(T-273)\right] = b(T)p;$$

$$b(T) = \frac{10^{-5}\left[1{,}712 + 5{,}8\cdot 10^{-3}(T-273)\right]}{RT};$$

Δ - roughness of a wall of a pipeline; d - diameter of a pipeline.

System of equations (4) and (5) in the matrix form is:

$$[A]\left\{\frac{\partial u}{\partial t}\right\} + [B]\left\{\frac{\partial u}{\partial x}\right\} = \{f\}, \qquad (6)$$

where $\{u\} = \begin{Bmatrix} p \\ Q \end{Bmatrix}$,

$$[A] = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}; \quad [B] = \begin{vmatrix} 0 & c^2 \\ 1 & 0 \end{vmatrix}; \quad \{f\} = \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix}.$$

Equating the determinant to zero, that is

$$\left|[B] - [A]\frac{dx}{dt}\right| = 0, \qquad (7)$$

we shall receive the equation which allows to determine $\frac{dx}{dt}$ determining characteristic directions.

If this equation has two various real roots $\frac{dx}{dt} = \delta_i, \ (i=1,2)$, the initial system of the differential equations refers to as hyperbolic. The inclination tangent to the characteristic depends not only on independent coordinates (x, t), but also from the decision (ρ, Q). For this problem we have:

$$\frac{dx}{dt} = z = \pm c. \qquad (8)$$

However, for existence of not unique solution of the system of the equations (6) the fulfillment of one condition (8) is not enough. Sufficient condition is the reference in a zero of the extended determinant. This condition refers to as by a condition of compatibility. In our case the equation looks like:

$$\frac{dQ}{dt} + \frac{1}{z}\frac{dp}{dt} = \frac{R_1}{z} + R_2. \qquad (9)$$

By substituting two meanings of the characteristics from expression (8) we shall receive system of two equations:

$$\frac{dQ}{dt} + \frac{1}{c}\frac{dp}{dt} = \frac{R_1}{c} + R_2, \qquad (10)$$

$$\frac{dQ}{dt} - \frac{1}{c}\frac{dp}{dt} = -\frac{R_1}{c} + R_2. \qquad (11)$$

Whole length of a pneumatic pipeline is divided into elements of length Δx. Unknown variable -mass charge and pressure of gas at the moment of time $t + \tau$ are determined by values of these parameters at the moment of time t (Fig 1).

Pressure and speed in a point D at the moment of time $t + x$ are determined from system of nonlinear algebraic equations of the following kind:

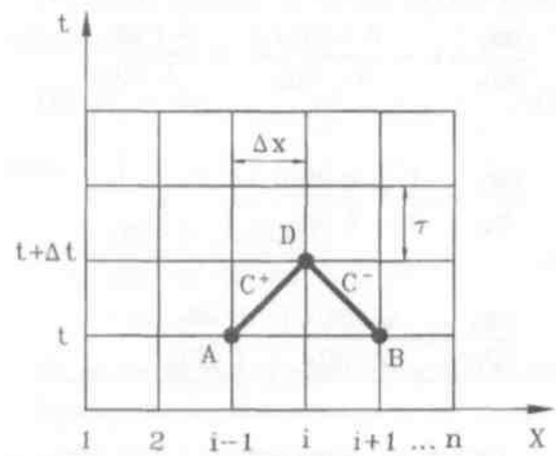

Fig 1. The circuit of determination of parameters of a point D by a method of characteristics

$$C^{+}:\ \Phi_1 = Q_D - Q_A + \frac{1}{c}(p_D - p_A) - \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_A + \left(\frac{R_1}{c}\right)_D\right] - \frac{\Delta t}{2}\left[\left(R_2\right)_A + \left(R_2\right)_D\right] = 0 \qquad (12)$$

$$C^{-}:\ \Phi_2 = Q_D - Q_B - \frac{1}{c}(p_D - p_B) + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_B + \left(\frac{R_1}{c}\right)_D\right] - \frac{\Delta t}{2}\left[\left(R_2\right)_B + \left(R_2\right)_D\right] = 0. \qquad (13)$$

The system of nonlinear algebraic equations (12) and (13) is solved by the Newton method in the matrix form have the following kind:

$$[J]_i\{\Delta Y\}_i = -\{\Phi\}_i , \qquad (14)$$

where $\{\Delta Y\}_i^T = [\Delta p_i , \Delta Q_i]$; $\{\Phi\}_i^T = [\Phi_{1i}, \Phi_{2i}]$; i – number of iteration; $[J]$ – Jacobi matrix,

$$[J] = \begin{bmatrix} \frac{\partial \Phi_1}{\partial p_D} & \frac{\partial \Phi_1}{\partial Q_D} \\ \frac{\partial \Phi_2}{\partial p_D} & \frac{\partial \Phi_2}{\partial Q_D} \end{bmatrix},$$

$$\frac{\partial \Phi_1}{\partial p_D} = \frac{1}{c} - \frac{\Delta t}{2}\frac{\partial R_2}{\partial p_D};$$

$$\frac{\partial \Phi_1}{\partial Q_D} = 1 - \frac{\Delta t}{2}\frac{\partial (R_1/c)}{\partial Q_D} - \frac{\Delta t}{2}\frac{\partial R_2}{\partial Q_D};$$

$$\frac{\partial \Phi_2}{\partial p_D} = -\frac{1}{c} - \frac{\Delta t}{2}\frac{\partial R_2}{\partial p_D};$$

$$\frac{\partial \Phi_2}{\partial Q_D} = 1 + \frac{\Delta t}{2}\frac{\partial (R_1/c)}{\partial Q_D} - \frac{\Delta t}{2}\frac{\partial R_2}{\partial Q_D};$$

$$\frac{\partial (R_1/c)}{\partial Q_D} = c\frac{d\ln(S(x))}{dx};$$

$$\frac{\partial (R_2)}{\partial p_D} = -\frac{\Pi R T Q|Q|}{8S}\left[\frac{\left(\frac{\partial \lambda}{\partial p}\right)p - \lambda(p,T)}{p^2}\right] - \frac{a_x}{RT};$$

$$\frac{\partial (R_2)}{\partial Q_D} = -\frac{\Pi RT}{8Sp}\left[\frac{\partial \lambda}{\partial Q_D}\right]Q|Q| + 2\lambda(p,T)|Q|;$$

$$\frac{\partial \lambda}{\partial p_D} = \frac{\partial \lambda}{\partial \mathrm{Re}}\frac{\partial \mathrm{Re}}{\partial p_D};\quad \frac{\partial \lambda}{\partial Q_D} = \frac{\partial \lambda}{\partial \mathrm{Re}}\frac{\partial \mathrm{Re}}{\partial Q_D};$$

$$\frac{\partial \mathrm{Re}}{\partial p_D} = -\frac{RTd}{\nu(p,T)}\frac{Q}{p^2};\quad \frac{\partial \mathrm{Re}}{\partial Q_D} = \frac{RTd}{\nu(p,T)p};$$

$$\frac{\partial \lambda}{\partial \mathrm{Re}} = \begin{cases} -\frac{0.6183}{\ln(10)\mathrm{Re}\lg^3\left(\frac{\mathrm{Re}}{7}\right)}, & \text{when } \mathrm{Re}\le 2300; \\ -\frac{2,5}{\mathrm{Re}^2\left(\frac{\Delta}{d}+\frac{100}{\mathrm{Re}}\right)^{0.75}}, & \text{when } \mathrm{Re} > 2300; \end{cases} \qquad (15)$$

By solving the system of the equations (14), we determine a new value of variable:

$$\{Y\}_{i+1} = \{Y\}_i + \{\Delta Y\}_i . \qquad (16)$$

We shall consider a few numbers of units of a branching of model of pneumatic system representing practically interest of technical applications. _Unit of a branching of pipelines_ (Fig 2):

General unit of pneumatic system is considered in which input n_1 and output n_2 are in pipelines. In the unit inflow of gas m_{in} and outflow of gas m_{out} are available. The system of the equations of the movement of gas in general unit κ accepts the following kind:

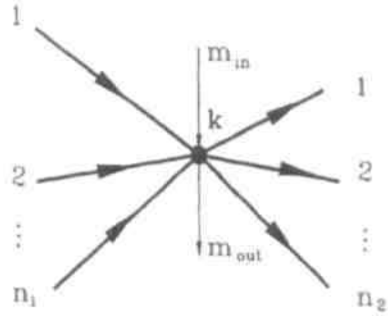

Fig 2. The circuit of unit of a branching of pipelines with inflow and selection of gas

$$\Phi_i = Q_{i,k} - Q_{i,k-1} + \frac{1}{c}\left(p_{i,k} - p_{i,k-1}\right) - \\ - \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{i,k-1} + \left(\frac{R_1}{c}\right)_{i,k}\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_{i,k-1} + (R_2)_{i,k}\right] = 0; \quad (17)$$

$$\Phi_j = Q_{j,k} - Q_{j,k+1} - \frac{1}{c}\left(p_{j,k} - p_{j,k+1}\right) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{j,k} + \left(\frac{R_1}{c}\right)_{j,k+1}\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_{j,k} + (R_{j,k+1})_D\right] = 0. \quad (18)$$

For the unit of connection of pipelines the dependence of material balance is fair:

$$\Phi_{n_1+n_2+1} = \sum_{i=1}^{n_1} S_i Q_i - \sum_{j=1}^{n_2} S_j Q_j + m_{in} - m_{out} = 0, \quad (19)$$

where m_i - inflow of gas; m_{out} - selection of gas; S_i, S_j - area of cross sections of pipelines.

The pressure in general unit is determined by the next expression:

$$p_{i,k} = p_{j,k} = p_k. \quad (20)$$

The system of nonlinear algebraic equations (17)-(19) is solved by the Newton method:

$$[J]_i \{\Delta Y\}_i = -\{\Phi\}_i, \quad (21)$$

where

$$\{\Delta Y\}_i^T = \begin{bmatrix} \Delta Q_{1,k}, \ \Delta Q_{2,k}, \ \dots, \ \Delta Q_{n_1,k}, \ \Delta Q_{1,k}, \ \Delta Q_{2,k}, \\ \dots, \ \Delta Q_{n_2,k}, \ \Delta p_k. \end{bmatrix}$$

On an input in a pipeline the charge is given On an input in a pipeline the charge of gas $q(t, x = 0) = Q_1$ is given.

The equations for the determination of pressure on an input in a pipeline (node 1) have the following kind:

$$\Phi = Q_1 - Q_2 - \frac{1}{c}(p_1 - p_2) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_1 + \left(\frac{R_1}{c}\right)_2\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_1 + (R_2)_2\right] = 0. \quad (22)$$

The equation (22) is solved by the Newton method:

$$[J]_i \Delta p_{1,i} = -\Phi_i,$$

where

$$[J]_i = \frac{\partial \Phi}{\partial p_1} = -\frac{1}{c} + \frac{\Delta t}{2}\frac{\partial (R_1/c)}{\partial p_1} - \frac{\Delta t}{2}\frac{\partial (R_2)}{\partial p_1}. \quad (23)$$

On an input in a pipeline the pressure is given

On an input in a pipeline the pressure of gas $P(t, x = 0) = p_1$ is given.

The equation for the determination of charge on an input in a pipeline (node 1) is (22).

The equation (22) is solved by the Newton method:

$$[J]_i \Delta Q_{1,i} = -\Phi_i,$$

where:

$$[J]_i = \frac{\partial \Phi}{\partial Q_1} = 1 + \frac{\Delta t}{2}\frac{\partial (R_1/c)}{\partial Q_1} - \frac{\Delta t}{2}\frac{\partial (R_2)}{\partial Q_1}. \quad (24)$$

On an output from a pipeline charge is given Equation for the determination of the pressure on an output from a pipeline (node κ) has the following kind:

$$\Phi = Q_k - Q_{k-1} + \frac{1}{c}(p_k - p_{k-1}) - \\ - \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{k-1} + \left(\frac{R_1}{c}\right)_k\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_{k-1} + (R_2)_k\right] = 0. \quad (25)$$

where

The given equation is solved by the Newton method:

$$[J]_i \Delta p_{k,i} = -\Phi_i, \quad (26)$$

$$[J]_i = \frac{\partial \Phi}{\partial p_k} = \frac{1}{c} - \frac{\Delta t}{2}\frac{\partial (R_2)}{\partial p_k}. \quad (27)$$

On an output from a pipeline pressure is given

Equation for the determination of the charge on an output from a pipeline (node k) is (25).

The equation (25) is solved by the Newton method:

$$[J]_i \Delta Q_{k,i} = -\Phi_i , \tag{28}$$

where

$$\frac{\partial \Phi}{\partial Q_k} = 1 - \frac{\Delta t}{2}\frac{\partial (R_1 / c)}{\partial Q_k} - \frac{\Delta t}{2}\frac{\partial R_2}{\partial Q_k}; \tag{29}$$

The expiration of gas from a cavity of constant volume (Fig 3)

Fig 3. The circuit of expiration of gas from a cavity of constant volume

The change of pressure of constant volume *(V = const)* of a cavity is determined from the following equation:

$$\frac{dp_v}{dt} = \frac{\mu_1 S_1 \gamma K_1(T_v) p_v}{V} \varphi\left(\sigma = \frac{p_v}{p_1}\right) \tag{30}$$

where $K_1(T) = \sqrt{\frac{2\gamma RT}{\gamma - 1}}$ (31)

$$\varphi(\sigma) = \sqrt{\sigma^{\frac{2}{\gamma}} - \sigma^{\frac{\gamma+1}{\gamma}}} . \tag{32}$$

The differential equation (30) is solved by a trapezoid method:

$$\left(\frac{dp_v}{dt}\right)_{t+\Delta t} = \frac{2}{\Delta t}(p_{v,t} + p_{v,t+\Delta t}) - \left(\frac{dp_v}{dt}\right)_t . \tag{33}$$

Applying a trapezoid method to the equation (30) we shall receive the following equation:

$$\Phi_1 = \frac{2}{\Delta t}(p_{t+\Delta t} - p_t) - \left(\frac{dp_v}{dt}\right)_t - \left(\frac{\mu_1 S_1 \gamma K_1 p_{v,t+\Delta t}}{V} \varphi(\sigma = \frac{p_{v,t+\Delta t}}{p_{1,t+\Delta t}})\right) = 0 . \tag{34}$$

In node 1 local losses of pressure are taken into account. The dependence between pressure p_v and pressure p_l is determined from the following relation:

$$\Phi_2 = p_v - p_1 - \frac{\xi R T_1 Q_1^{\,2} sign(Q_1)}{2 p_1} = 0 \tag{35}$$

Dependence between pressure p_l and mass charge Q_l is:

$$\Phi_3 = Q_1 - Q_2 - \frac{1}{c}(p_1 - p_2) + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_1 + \left(\frac{R_1}{c}\right)_2\right] - \frac{\Delta t}{2}\left[(R_2)_1 + (R_2)_2\right] = 0. \tag{36}$$

System of nonlinear algebraic equations (34)-(36) is solved by the Newton method:

$$[J]_i \{\Delta Y\}_i = \{\Phi\}_i , \tag{37}$$

$$\{\Delta Y\}_i^T = \left[\Delta p_v \; \Delta p_1 \; \Delta Q_1\right].$$

where

During gas in a cavity of constant volume (Fig 4)

Fig 4. The circuit of during gas in a cavity of constant volume The change of pressure of constant volume *(V = const*) of a cavity is determined from the following equation:

where μ_k - ration of charge in the κ node.

$$\frac{dp_v}{dt} = \frac{\mu_k S_k \gamma K_1(T_k) p_k}{V} \varphi\left(\sigma = \frac{p_v}{p_k}\right), \tag{38}$$

The differential equation (38) is also solved by a trapezoid method. Applying a trapezoid method to the equation (38) we shall receive the following equation:

In node A- local losses of pressure are taken into account. The dependence between pressure p_k. and

$$\Phi_1 = \frac{2}{\Delta t}(p_{v,t+\Delta t} - p_{v,t}) - \left(\frac{dp_v}{dt}\right)_t - \left(\frac{\mu_k S_k \gamma K_1(T_k) p_{k,t+\Delta t}}{V} \varphi\left(\sigma = \frac{p_{v,t+\Delta t}}{p_{k,t+\Delta t}}\right)\right) = 0. \tag{39}$$

pressure p_v is determined from the following relation:

$$\Phi_2 = p_{k,t+\Delta t} - p_{v,t+\Delta t} - \frac{\xi R T_{k,t+\Delta t} Q^2{}_{k,t+\Delta t} sign(Q_{k,t+\Delta t})}{2 p_{k,t+\Delta t}} = 0 . \tag{40}$$

Dependence between pressure p_k and mass charge Q_k is:

$$\Phi_3 = Q_k - Q_{k-1} - \frac{1}{c}(p_k - p_{k-1}) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{k-1} + \left(\frac{R_1}{c}\right)_{2}\right]_k - \\ - \frac{\Delta t}{2}\left[(R_2)_{k-1} + (R_2)_k\right] = 0. \quad (41)$$

System of nonlinear algebraic equations (39)-(41) is solved by the Newton method:

$$[J]_i \quad \{\Delta Y\}_i^T = \left[\Delta p_v \ \Delta p_k \ \Delta Q_k\right].$$

where:

<u>*During gas and the expiration of gas from a cavity of constant volume*</u> (Fig 5)

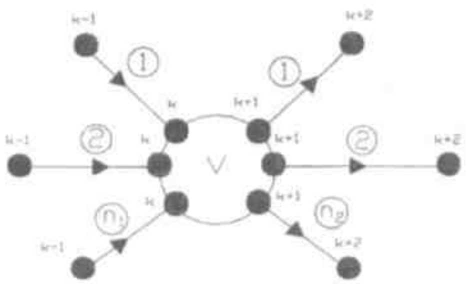

Fig 5. The circuit of during gas and the expiration of gas from a cavity of constant volume

Dependence between pressure p_k and mass charge Q_k is:

$$\Phi_i = Q_{i,k} - Q_{i,k-1} - \frac{1}{c}(p_{i,k} - p_{i,k-1}) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{i,k-1} + \left(\frac{R_1}{c}\right)_{i,k}\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_{i,k-1} + (R_2)_{i,k}\right] = 0, \\ i = 1,\ldots,n_1. \quad (43)$$

where n_1 - number of input pipeline to cavity.

In nodes k local losses of pressure are taken into account. The dependence between pressures p_k and pressure p_v are determined from the following relations:

$$\Phi_{n_1+i} = p_{i,k,t+\Delta t} - p_{v,t+\Delta t} - \\ \frac{\xi_{i,k} RT_{i,k,t+\Delta t} Q^2_{i,k,t+\Delta t}\, sign(Q_{i,k,t+\Delta t})}{2p_{i,k,t+\Delta t}} = 0, \quad (44) \\ i = 1,\ldots,n_1.$$

The change of pressure of constant volume (V = *const)* of a cavity is determined from the following equation:

$$\frac{dp_v}{dt} = \sum_{i=1}^{n_1} \frac{\mu_{i,k} S_{i,k} \gamma K_1(T_{i,k}) p_{i,k}}{V} \varphi\left(\sigma = \frac{p_v}{p_{i,k}}\right) - \\ - \sum_{j=1}^{n_2} \frac{\mu_{j,k+1} S_{j,k+1} \gamma K_1(T_v) p_v}{V} \varphi\left(\sigma = \frac{p_{j,k+1}}{p_v}\right). \quad (45)$$

where n_2 - number of output pipeline from the cavity.

The differential equation (45) is solved by a trapezoid method. Applying a trapezoid method to the equation (45) we shall receive the following equation:

$$\Phi_{2n_1+1} = \frac{2}{\Delta t}(p_{v,t+\Delta t} - p_{v,t}) - \left(\frac{dp_v}{dt}\right)_t - \\ - \sum_{i=1}^{n_1} \frac{\mu_{i,k} S_{i,k} \gamma K_1(T_{i,k}) p_{i,k}}{V} \varphi\left(\sigma = \frac{p_v}{p_{i,k}}\right) + \\ + \sum_{j=1}^{n_2} \frac{\mu_{j,k+1} S_{j,k+1} \gamma K_1(T_v) p_v}{V} \varphi\left(\sigma = \frac{p_{j,k+1}}{p_v}\right) = 0. \quad (46)$$

In nodes k +1 local losses of pressure are taken into account. The dependence between pressures p_v and pressure P_{k+1} are determined from the following relations:

$$\Phi_{2n_1+1+j} = p_{v,t+\Delta t} - p_{j,k+1,t+\Delta t} - \\ \frac{\xi_{j,k+1} RT_{j,k+1,t+\Delta t} Q^2_{j,k+1,t+\Delta t}\, sign(Q_{j,k+1,t+\Delta t})}{2p_{j,k+1,t+\Delta t}} = 0, \\ j = 1,\ldots,n_2. \quad (47)$$

Dependence between pressure P_{k+1} and mass charge Q_{k+1} is:

$$\Phi_{1+2n_1+n_2+j} = Q_{j,k+1} - Q_{j,k+2} - \frac{1}{c}(p_{j,k+1} - p_{j,k+2}) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{j,k+1} + \left(\frac{R_1}{c}\right)_{j,k+2}\right] - \\ - \frac{\Delta t}{2}\left[(R_2)_{j,k+1} + (R_2)_{j,k+2}\right] = 0, \\ j = 1,\ldots,n_2. \quad (48)$$

System of nonlinear algebraic equations (44)-(48) is solved by the Newton method:

$$[J]_i \{\Delta Y\}_i = \{\Phi\}_i , \qquad (49)$$

$$\{\Delta Y\}_i^T = \begin{bmatrix} \Delta Q_{1,k} \ldots \Delta Q_{n_1,k} \Delta p_{1,k} \ldots \Delta p_{n_1,k} \Delta p_v \\ \Delta p_{1,k+1} \ldots \Delta p_{n_2,k+1} \Delta Q_{1,k+1} \Delta Q_{n_2,k+1} \end{bmatrix} .$$

where

<u>*Node of a pipeline arc connected to cavities of the cylinder* (Fig 6)</u>

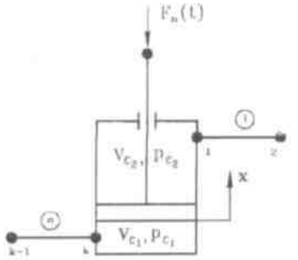

Fig 6. The circuit of n working cavity of the cylinder with a pipeline

The equation for determination of the charge of gas in node k pipeline looks like:

$$\Phi_1 = Q_k - Q_{k-1} - \frac{1}{c} \ (p_k - p_{k-1}) + \\ + \frac{\Delta t}{2}\left[\left(\frac{R_1}{c}\right)_{k-1} + \left(\frac{R_1}{c}\right)_{2}\right]_k - \\ - \frac{\Delta t}{2}\left[(R_2)_{k-1} + (R_2)_k\right] = 0 \ . \qquad (50)$$

In node κ local losses of pressure are taken into account. The dependence between pressure p_k and pressure p_c is determined from the following relation:

According to the first law of thermodynamics the

$$\Phi_2 = p_{k,t+\Delta t} - p_{c_1,t+\Delta t} - \\ \frac{\xi_k RT_{k,t+\Delta t} Q^2{}_{k,t+\Delta t} \, sign(Q_{k,t+\Delta t})}{2 p_{k,t+\Delta t}} = 0. \qquad (51)$$

whole thermal energy moved with gas is spent for the change of internal energy and for the work of expansion of gas in a cavity of the cylinder. The dependence of pressure in a working cavity of the cylinder looks like:

$$\frac{dp_{c_1}}{dt} = \frac{\gamma RT_{m} G_k}{V_{c_1}} - \frac{\gamma p_{c_1}}{V_{c_1}} \frac{dV_{c_1}}{dt} , \qquad (52)$$

where G_k - mass charge of gas, determined on the formula Scn-Vcnan and Vencel [4]:

when $p_k > p_{c_1}$:

$$G_k = \mu_k S_k p_k K_2(T_k) \varphi\left(\sigma = \frac{p_{c_1}}{p_k}\right), \qquad (53)$$

where μ_k- factor of the charge in node; S_k - cross section area of node κ ;

$$K_2(T) = \sqrt{\frac{2\gamma}{(\gamma - 1)RT}} \ ; \qquad (54)$$

V_{c1} - volume of gas in a working cavity of the cylinder,

$$V_{c_1} = V_{c_{10}} + S_{c_1} q , \qquad (55)$$

V_{C10} - initial volume of gas in a working cavity of the cylinder;S_{c1} - area of the piston; q- displacement of the piston.

The differential equation (52) is also solved by a trapezoid method. Applying a trapezoid method to the equation (52) we shall receive the following equation:

$$\Phi_3 = \frac{2}{\Delta t}\left(p_{c_1,t+\Delta t} - p_{c_1,t}\right) - \left(\frac{dp_{c_1}}{dt}\right)_t - \\ - \frac{\gamma RT_k G_k}{V_{c_1}} + \frac{\gamma p_{c_1}}{V_{c_1}} \frac{dV_{c_1}}{dt} = 0 \ . \qquad (56)$$

where $G_{c_2} = \mu_1 S_1 p_{c_2} K_2(T_{c_2}) \varphi\left(\sigma = \frac{p_1}{p_{c_2}}\right),$

$$\frac{dp_{c_2}}{dt} = - \frac{\gamma RT_{c_2}}{V_{c_2}} G_{c_2} - \frac{\gamma p_{c_2}}{V_{c_2}} \frac{dV_{c_2}}{dt} , \qquad (57)$$

$$p_{c_2} > p_1 ; \qquad (58)$$

$$V_{c_2} = V_{c_{20}} - S_{c_2} q , \qquad (59)$$

The dependence of pressure in the second cavity of the cylinder looks like:

when

V_{c2} - volume ot gas in the second cavity of the cylinder,

V_{c20} , S_{C2} - initial volume of gas and area of the cylinder in the second cavity.

The differential equation (57) is also solved by a trapezoid method. Applying a trapezoid method to the equation (57) we shall receive the following equation:

$$\Phi_4 = \frac{2}{\Delta t}\left(p_{c_2,t+\Delta t} - p_{c_2,t}\right) - \left(\frac{dp_{c_2}}{dt}\right)_t +$$
$$+\frac{\gamma RT_{c_2} G_{c_2}}{V_{c_2}} + \frac{\gamma p_{c_2}}{V_{c_2}}\frac{dV_{c_2}}{dt} = 0 . \qquad (60)$$

In node *k+1* local losses of pressure are taken into account. The dependence between pressure p_{c2} and pressure P_{k+1} is determined from the following relation:

$$\Phi_5 = p_{c_2,t+\Delta t} - p_{c_2,t+\Delta t} -$$
$$\frac{\xi_{k+1} RT_{k+1,t+\Delta t} Q^2{}_{k+1,t+\Delta t}\, sign(Q_{k+1,t+\Delta t})}{2 p_{k+1,t+\Delta t}} = 0 . \qquad (61)$$

The equation of a movement of the piston has the following kind:

$$M_p \frac{d^2 q}{dt^2} = F\left(1 - k_p\right) - F_f k_d , \qquad (62)$$

where F - general force, working on the piston,

$$F = S_{c_1} p_{c_1} - S_{c_2} p_{c_2} - F_{ext}(t) - F_h , \qquad (63)$$

$F_{ext}(t)$- external force; F_h- damping force,

$$F_h = h\frac{dq}{dt} , \qquad (64)$$

k_p, k_d- coefficients of a condition of rest and movement [6],

$$k_p = sign\left(1 - sign\left(|F| - F_{f0}(1 - k_d)\right)\right); \qquad (65)$$

$$k_d = sign\left(1 + sign\left(\left|\frac{dq}{dt}\right| - \varepsilon_{\frac{dq}{dt}}\right)\right); \qquad (66)$$

$$F_f = F_{f\,max}\, sign(q) , \qquad (67)$$

$F_{f\,max}$ – maximum force of friction,

F_{f0} – friction force on the rest,

$\varepsilon_{\frac{dq}{dt}}$ – threshold value of velocity of piston.

The differential equation (62) is also solved by a trapezoid method. Applying a trapezoid method to the equation (62) we shall receive the following equation:

$$\Phi_6 = \left(\frac{dq}{dt}\right)_{t+\Delta t} - \frac{2}{\Delta t}\left(q_{t+\Delta t} - q_t\right) + \left(\frac{dq}{dt}\right)_t = 0 ; \qquad (68)$$

$$\Phi_7 = M_p\left(\frac{4}{\Delta t^2}\left(q_{t+\Delta t} - q_t\right) - \frac{2}{\Delta t}\left(\frac{dq}{dt}\right)_t - \left(\frac{d^2 q}{dt^2}\right)_t\right) -$$

$$[J]_t \{\Delta Y\}_t = \{\Phi\}_t , \qquad (70)$$

where: (69)

System of nonlinear algebraic equations (50)-(69) is solved by the Newton method:

3. Conclusions

$$\{\Delta Y\}^T = \left[\Delta p_k\ \Delta Q_k\ \Delta p_1\ \Delta Q_1\ \Delta p_{c_1}\ \Delta p_{c_2}\ \Delta q\ \Delta\left(\frac{dq}{dt}\right)\right].$$

The movement of gas in pneumatic systems is described by the differential equations with a partial derivative which allows to investigate wave processes in these systems. The occurring wave processes depend on physical properties of gas, mechanical parameters of pneumatic system, external influence. The theory of mathematical modelling of dynamic processes in pneumatic systems is offered. It will allow more precisely to describe physical processes in pipelines connected by cavities and cylinders. At small and variable speeds of the movement of the piston in the pneumatic cylinder it is necessary to take into account the force of friction. Depending on external forces and the force of friction the movement of the piston can be with stops, that is a discrete movement. The common external force working on the piston depends on dynamic parameters of a pneumatic system, therefore it is necessary to investigate dynamic processes of separate units together with a common pneumatic system. The developed mathematical models of separate parts of pneumatic systems will add to the created library of mathematical models of pneumatic, hydraulic and mechanical systems.

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Įteikta redakcijos kolegijai 1999 09 06 Pateikta spaudai 1999 12 03

SUDĖTINGŲ PNEUMATINIŲ SISTEMŲ MODELIAVIMAS

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S a n t r a u k a

Pateikiama sudėtingų pneumatinių sistemų matematinio modeliavimo teorija. Pneumatinė sistema sudaryta iš vamzdyno, pastovių tūrių talpyklų, pneumatinių cilindrų. Žinomos kraštinės sąlygos - slėgio ar masės debito kitimas laike. Dujų judėjimas vamzdyne aprašomas diferencialinėmis lygtimis su dalinėmis išvestinėmis. Tai leidžia modeliuoti slėgio bangų kitimą vamzdynuose, įvertinant dujų spūdumą, trinties nuostolius tarp vamzdyno sienelės ir dujų, vietinius slėgio nuostolius dėl skirtingų vamzdyno geometrinių parametrų. Sudaryti dujų judėjimo matematiniai modeliai pneumatinėje sistemoje: vamzdynas ir pastovaus tūrio talpykla, vamzdyno bendras mazgas, į kurį gali įtekėti ar ištekėti tam tikras dujų masės kiekis, vamzdynas ir pneumatinis cilindras. Pneumatiniame cilindre taip stūmoklio ir cilindro veikia trinties jėga, kurios skaitinė reikšmė kinta priklausomai nuo stūmoklio judėjimo greičio. Sudarytas stūmoklio judėjimo matematinis modelis, kuriuo įvertinama rimties trinties jėga. Tai leidžia nagrinėti stūmoklio judėjimą nedideliais greičiais ir jo judėjimą su sustojimais. Tai labai svarbu precizinėse pneumatinėse sistemose, kai reikia tiksliai nustatyti stūmoklio padėtį įvertinant dinaminius procesus pneumatinėje sistemoje. Dujų judėjimo lygtys sprendžiamos baigtinių skirtumų metodu, t. y. charakteristikų metodu. Šis metodas leidžia gauti netiesinių algebrinių lygčių sistemą, kuri sprendžiama Niutono metodu. Pateikta pneumatinių sistemų modeliavimo teorija leidžia dideliu tikslumu nagrinėti fizikinius procesus, vykstančius šiose sistemose.

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